Equalization Methods in DMT and FMT Systems for Broadband Wireless Communications*

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Abstract

Multicarrier systems are adopted in several standards for their ability to achieve optimal performance in very dispersive channels. In particular, discrete multitone (DMT) and filtered multitone (FMT) systems are two examples where the modulation filter has an ideal rectangular amplitude characteristic in time and frequency domains, respectively.

In this letter we compare DMT and FMT with various equalization schemes in the presence of a multipath channel. The performance measure is the achievable bit rate, given by the sum of the bit rates each active subchannel is able to deliver with a given error probability. Although FMT requires a fixed structure with a higher computational complexity than DMT, it turns out that FMT, even with the simplest one-tap per subchannel adaptive equalizer, has an efficiency similar or higher than DMT. Hence, FMT can be a valid alternative to DMT also for broadband wireless applications.

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1 INTRODUCTION

In this letter we consider multicarrier systems where interpolating filters are frequency shifts of a given prototype filter and the spacing between adjacent subcarriers is the same for all subcarriers. In fact, only in this case the system may be efficiently implemented by means of a fast Fourier transform (FFT) and a network of polyphase filters [1, 2]. In particular, we restrict our analysis to two cases\(^1\): i) DMT, where the prototype filter has an ideal rectangular time-domain amplitude characteristic, and ii) FMT, where the prototype filter has a nearly rectangular frequency domain amplitude characteristic. This choice implies inevitable intersymbol interference (ISI) at the receiver, while intercarrier interference (ICI) between subchannels is almost negligible when an appropriate design of the prototype filter is carried out. Another possible choice of the prototype filter is the square-root Nyquist shape [3]. However, ICI is eliminated by increasing the spacing between adjacent subcarriers at the expense of a lower bandwidth efficiency.

We recall that DMT is generally intended as an effective modulation technique for high bit rate applications in very dispersive channels. First applications in this direction encompass Asymmetric Digital Subscriber Lines and the digital audio and video broadcasting of signals; in these last two cases the term Orthogonal Frequency Division Multiplexing (OFDM) is usually used. More recently, DMT has been also considered for the development of Broadband Radio Access Networks (BRAN) like HIPERLAN Type 2 [4]. The peculiarity of DMT systems is that equalization of dispersive channels is performed simply by multiplying the signal at the output of each subchannel by a coefficient that is related to the channel frequency response. However, this simple scheme works only if redundancy, in terms of a prefix of suitable length [5, 6], is inserted in the transmitted signal, which, on the other hand, lowers the system spectral efficiency. For this reason FMT has been considered and some simple equalization schemes for wireless applications are investigated.

In this framework we should mention the work by Vandendorpe et al. (see [7, 8] and references therein) where a DMT system is equalized by a multidimensional DFE. However, here this approach has not been considered due to the computational complexity of its design; moreover, it must be updated whenever the channel changes.

This letter is organized as follows. In Section 2 we briefly outline the system model. In Section 3 we investigate some possible equalization methods for FMT systems. In Section 4 we give the computational complexity of the proposed equalization schemes while in Section 5 some performance comparisons are presented.

\(^1\)By adopting the same terminology as in [1], in this letter we refer to the multicarrier systems denoted as DMT and FMT. However, while in [1] such systems are for wired applications, here they are intended for slowly time-varying wireless channels.
The general baseband equivalent multicarrier scheme is represented in Fig. 1. Here $M$ data signals $d_m(k)$, $m = 0, 1, \ldots, M - 1$, $k \in \mathbb{Z}$, with symbol rate $1/T$, are multiplexed in frequency by using a bank of bandpass FIR interpolating filters $\{h^{(m)}(n)\}$, $n \in \mathbb{Z}$, to produce the transmitted signal $s(i)$, $i \in \mathbb{Z}$, at time $iT_c = i\frac{T}{M}$. Here we consider critically sampled (or maximally decimated) filter bank modulation [1, 2], so that the subcarrier spacing is equal to $\frac{1}{F}$. At the receiver, a bank of FIR decimation filters, $\{g^{(m)}(n)\}$, selects the information for each subchannel. We assume that $\{h^{(m)}(n)\}$ and $\{g^{(m)}(n)\}$ are non-zero only for $n = 0, 1, \ldots, N_h - 1$, and $n = 0, 1, \ldots, N_g - 1$, respectively.

Here the transmit and receive filters are, respectively, the frequency shift of two prototype filters $\{h(n)\}$ and $\{g(n)\}$, namely

$$
\begin{align*}
    h^{(m)}(n) &= e^{j2\pi \frac{m}{M}n}h(n), \\
g^{(m)}(n) &= e^{j2\pi \frac{m}{M}n}g(n).
\end{align*}
$$

Moreover, the prototype receive filter is matched to the transmit one, i.e. $g(n) = h^*(N_h - 1 - n)$. The choice of $\{h(n)\}$ determines the particular type of multicarrier system; in particular, when it is an ideal rectangular pulse the system is called DMT [5]. In this case the frequency responses of all subchannels overlap but neither ISI nor ICI arise for ideal transmission channels. Instead, when $\{h(n)\}$ is designed to minimize the overlap between the frequency responses of two adjacent subchannels, the corresponding system is called filtered multitone (FMT) [1]. In this system negligible ICI is present at the receiver, whatever the transmission channel is; however, ISI is always present and an equalizer is needed at the receiver.

We remark that our analysis takes into account also the effect of the analog filters included in the digital-to-analog (D/A) and analog-to-digital (A/D) converters (see Fig. 1) which are designed to approximate an ideal square-root raised cosine shape with Nyquist frequency $1/(2T_c)$ and a roll-off factor of 0.07. This particular value has been chosen as a trade-off between minimization of the number of virtual carriers and the distortion of the spectrum of active subcarriers [6, 11] \(^4\). We denote by $\{c_o(i)\}$ the equivalent discrete-time channel impulse response which is the sampled version of the composed channel given by the cascade of the D/A converter, the analog radio channel $c(t)$ and the A/D converter. In particular, we assume that $c_o(i) \neq 0$ at most for $i = 0, 1, \ldots, N_c - 1$. Additive complex-valued zero-mean white Gaussian noise (AWGN), $w(i)$, with variance $\sigma_w^2$ is assumed to be superimposed to the received signal.

\(^2\)Notation: signals are indicated with lowercase letters, while their Fourier transforms are indicated with the corresponding uppercase letters. $E[\cdot]$, $'$ and $\dagger$ denote expectation, complex conjugate and transposition, respectively; $\mathbb{R}$ and $\mathbb{Z}$ are the sets of real and integer numbers, respectively, and $j = \sqrt{-1}$.

\(^3\)Without loss of generality, we assume that both $N_h$ and $N_g$ are integer multiples of $M$.

\(^4\)The power spectrum of a DMT signal is composed of shifted versions of $\text{sinc}^2(\cdot)$ functions centered on each subcarrier frequency while for FMT it is roughly composed of shifted versions of rectangular functions. Hence the analog filter included in the D/A converter affects the edge subcarriers more significantly in DMT rather than in FMT.
3 Equalization Schemes

In this section we present some simplified schemes and, for comparison purposes, we also review other known equalization procedures. Let’s introduce the $M$-point DFT of the channel impulse response, i.e.

$$
\psi_m = \sum_{i=0}^{N_c-1} e^{-j2\pi \frac{m}{M} i} c_e(i), \quad m = 0, 1, \ldots, M - 1.
$$

(2)

A DMT

A well-known equalization scheme for DMT systems consists of multiplying the $m$-th sample at the output of the FFT by the coefficient (zero-forcing criterion [9])

$$
C_m = \frac{1}{\psi_m}, \quad m = 0, 1, \ldots, M - 1.
$$

(3)

This technique implicitly implies the use of a cyclic prefix at the transmitter [6] (and the system is denoted as DMT-CP). By introducing a time-domain FIR filter (also called channel shortening filter) immediately after the A/D converter, the length of the effective channel impulse response may be reduced and a shorter cyclic prefix may be used. In this way, we can design DMT systems also for low values of $M$ and still obtain a good spectral efficiency. Algorithms to design channel shortening filters have been developed in [10] where several solutions have been presented. However, these algorithms still have a significant computational complexity and must work at the high speed $1/T_c$. Hence, this approach is not suitable for wireless (possible mobile) applications because of the fast time-varying nature of the channel.

B FMT

In this system almost no ICI arises whatever the transmission channel is, while ISI is always present in each subchannel, even if the transmission channel is ideal. Hence, it is mandatory to face two problems: equalization of the transmit filters and equalization of the transmission channel.

Decision feedback multichannel equalizer (DFME). Assuming no ICI, the feedforward filters work on a subchannel base. A general solution, with an efficient receiver implementation [1, 2], is reported in Fig. 2a, where a DFE is inserted at the output of each subchannel. Although this scheme may not be convenient for high rate applications because of its computational complexity and the fact that the filter coefficients must be updated at least at regular time intervals to cope with the time varying nature of the channel, we now derive its equations for an upper bound on the system performance.

The overall impulse response of the $m$-th subchannel is given by

$$
h_{\text{tot},m}(n) = \sum_{n_1=0}^{N_t-1} g^{(m)}(nM - n_1) \sum_{n_2=0}^{N_c-1} h^{(m)}(n_1 - n_2)c_e(n_2).
$$

(4)
Let’s indicate with \( \{ z_{FF,m}(n) \} \), \( n = 0, 1, \ldots, N_{FF} - 1 \), the feedforward filter of the \( m \)-th subchannel and with \( \{ z_{FB,m}(n) \} \), \( n = 1, 2, \ldots, N_{FB} \), the corresponding feedback filter. Let’s define two vectors containing, respectively, the feedforward and feedback filters coefficients:

\[
Z_{FF,m} = [z_{FF,m}(0), z_{FF,m}(1), \ldots, z_{FF,m}(N_{FF} - 1)]',
\]

\[
Z_{FB,m} = [z_{FB,m}(1), z_{FB,m}(2), \ldots, z_{FB,m}(N_{FB})]'.
\]

From the autocorrelation matrix with entries

\[
[R_{x,m}]_{i,l} = \sigma_d^2 \sum_{n_1=0}^{N_{FF}} h_{tot,m}(n_1) h^*_\delta,m(n_1 - (i - l)) - \]

\[
\sigma_d^2 \sum_{n_2=1}^{N_{FB}} h_{tot,m}(n_2 - l + \Delta) h^*_\delta,m(n_2 - i + \Delta) + \sigma_w^2 \delta(i - l),
\]

\( i, l = 0, 1, \ldots, N_{FF} - 1 \),

where \( \sigma_d^2 \) is the power of each zero-mean independent and identically distributed signal \( \{ d_m(k) \} \), \( \Delta \) is a suitable delay, and the cross correlation vector \( P_{x,m} \) with entries \( [P_{x,m}]_l = \sigma_d^2 h^*_\delta,m(\Delta - l) \), \( l = 0, 1, \ldots, N_{FF} - 1 \), the feedforward and feedback filter can be designed as follows:

\[
Z_{FF,m} = R_{x,m}^{-1} P_{x,m},
\]

\[
z_{FB,m}(n) = - \sum_{l=0}^{N_{FB}-1} z_{FF,m}(l) h_{tot,m}(i + \Delta - l), \quad n = 1, 2, \ldots, N_{FB}.
\]

We should notice that this scheme has a high computational complexity because at each channel estimate it requires the inversion of \( M N_{FF} \times N_{FF} \) matrices.

**Post-DFT simplified DFME (postDFME).** A simplified DFME is now derived. Let us assume that over each subchannel the frequency response of the transmission channel is flat, i.e. it has both a constant amplitude and a constant phase. This condition is related to the number of subchannels of the multicarrier system and to the coherence bandwidth of the channel [12]. Under this assumption, from (1) and (2) the convolution between the \( m \)-th transmit filter and the transmission channel yields

\[
\sum_{n_1=0}^{N_h-1} c_\psi(n - n_1) h^{(m)}(n_1) \simeq \psi_m h^{(m)}(n) = \psi_m e^{j2\pi \frac{F_m}{M} n} h(n), \quad n = 0, 1, \ldots, N_h - 1.
\]

Hence, the transmission channel can be adaptively equalized by a one tap per subchannel equalizer, \( C_m \), (as for DMT systems with cyclic prefix) while the transmit filters can be equalized by a fixed DFE. Moreover, for the \( m \)-th subchannel the cascade of transmit and receive filters turns out to be independent of the subchannel index. In fact from (1) we have that

\[
h_{eq}(n) = \sum_{n_1=0}^{N_h-1} h^{(m)}(n_1) g^{(m)}(nM - n_1) = \sum_{n_1=0}^{N_h-1} h(n_1) g(nM - n_1).
\]

4
Hence, the feedforward and feedback filters ($\{q_{FF}(n)\}$ and $\{q_{FB}(n)\}$) are the same for all subchannels and they can be designed by using the standard DFE technique outlined in (8)–(10) where $h_{tot}$ is replaced by $h_{eq}$. The resulting equalization scheme is represented in Fig. 2b for the $m$-th subchannel.

We can expect to obtain some performance improvement by assuming a transmission channel with a linear phase within each subchannel. This linear term corresponds to the delay of the $m$-th subchannel and can be estimated easily from the coefficients $\{\psi_m\}$. As a first-order approximation of the delay $\tau_m$ on the $m$-th subchannel, we assume

$$\tau_m = \angle \psi_m - \angle \psi_{m+1} \frac{T}{2}, \quad m = 0, 1, \ldots, M - 1,$$

(13)

where $\angle$ denotes the angle of a complex number in the range $[-\pi, \pi]$.

To simplify interpolation, each signal $\tilde{d}_m(\cdot)$ is derived at the oversampled rate $T/2$. Then it is delayed of the quantity $\tau_m$ by a variable phase interpolator to yield [9]

$$\tilde{d}_m(kT + \tau_m) = \sum_{n=-N_{i}/2}^{N_{i}/2 - 1} \text{sinc} \left( n + \frac{2\tau_m}{T} \right) \tilde{d}_m \left(\frac{(2k - n)T}{2}\right), \quad m = 0, 1, \ldots, M - 1.$$  

(14)

In our simulations the number of coefficients of the interpolator filter is $N_i = 12$.

In order to obtain the oversampled signal $\tilde{d}_m(nT/2)$, the receiver has two parallel structures composed of a serial to parallel converter, a filter bank and a DFT. The input to the first structure is signal $r(i)$ and the output yields the even samples $\tilde{d}_m(2kT/2)$. The input to the second structure is a delayed version of the received signal, $r(i + M/2)$, and the relative output are the odd samples, $\tilde{d}_m((2k + 1)T/2)$. A receiver with the added feature of time interpolator will be named fractionally spaced (FS) equalizer.

**Pre-DFT simplified DFME (preDFME).** In the previous scheme, equalization is based on the signal at the output of the receiver matched filters. In the preDFME the DFME is instead directly applied to the received signal. In this case receive filters equalize the transmit filters while the channel is equalized by the one-tap per subchannel structure. In particular, the feedforward and feedback subchannel filters, $\{p_{FF,m}(n)\}$, $n = 0, 1, \ldots, N_{FF} - 1$, and $\{p_{FB,m}(n)\}$, $n = 1, 2, \ldots, N_{FB}$, are computed as DFE of the corresponding polyphase component of the transmit prototype filter, $\{h_m(n)\}$. The resulting equalization scheme is represented in Fig. 2c. If we denote by $\{\xi_m(k)\}$, $m = 0, 1, \ldots, M - 1$, the input of the DFT, and by $\{D_m(k)\}$, $m = 0, 1, \ldots, M - 1$, the DFT of the transmit data, this structure minimizes the mean square error $E[|D_m(k) - \xi_m(k)|^2], m = 0, 1, \ldots, M - 1$. The design of filters $\{p_{FF,m}(n)\}$ and $\{p_{FB,m}(n)\}$ can be realized using (8)–(10) with the substitution of $\{h_{tot,m}(n)\}$ with $\{h_m(n)\}$.

As it will be seen, for the same filter length, the preDFME is more efficient than the postDFME, because it needs to equalize only the transmit filters. Let us note that in the preDFME the feedforward
and feedback filters are different for each branch of the receiver filter bank. Also for the preDFME, a FS approach can be used.

## 4 Computational Complexity

Table 1 summarizes the computational complexity, in terms of the number of complex multiplications per subchannel, of various equalization structures. Here we assume that the transmission channel is known. Moreover, the reported computational complexity does not account for the channel estimate (i.e. its DFT), nor for the transmitter.

**DMT.** Equalization based on cyclic prefix is the simplest one and apparently has a per-subchannel complexity of the order of $O(1)$.

**FMT.** If $N_{FF}$ ($N_{FB}$) is the number of coefficients of the feedforward (feedback) filter of each subchannel, the computational complexity per subchannel of the postDFME is $O(N_{g} + N_{FF} + N_{FB} + 1)$. Instead, the computational complexity of the preDFME is $O(N_{FF} + N_{FB} + 1 + M \log_2 M)$, where the logarithmic term is due to the additional IDFT at the receiver. DFME has the same computational complexity of postDFME. The FS feature yields an additional computational term of $O(N_{x})$, due to the interpolation filter, and the receiver complexity almost doubles with respect to the structure working at $T$, due to the generation of $\tilde{d}_m(\cdot)$ at $T/2$.

Now we account for the operations required to design the channel equalizer. Since they depend on the transmission channel estimate, this computation must be performed at each new channel estimate. DMT-CP, FMT-preDFME and postDFME have all zero cost, since they just use the DFT of the channel estimate.

The FS feature has an unitary cost, as (13) requires one multiplication. The design of the DFME requires instead the inversion of $M N_{FF} \times N_{FF}$ hermitian matrices, with a per subchannel computational complexity of $O(N_{FF}^3)$. 

<table>
<thead>
<tr>
<th>multicarrier structure</th>
<th>Computational complexity (per subchannel) of receiver structure. $M = 64$</th>
<th>$M = 128$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>DMT</td>
<td></td>
<td>99</td>
</tr>
<tr>
<td></td>
<td>$O(N_{1} + 1 + 2(N_{g} + N_{FF}) + N_{FB})$</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>$O(1 + \log_2 M + N_{FF} + N_{FB})$</td>
<td>55</td>
</tr>
<tr>
<td>FMT</td>
<td></td>
<td>83</td>
</tr>
<tr>
<td>FS preDFME</td>
<td>$O(N_{1} + 1 + 2(N_{g} + N_{FF}) + N_{FB})$</td>
<td>94</td>
</tr>
<tr>
<td>FS postDMFT</td>
<td>$O(N_{1} + 1 + 2(N_{g} + N_{FF}) + N_{FB})$</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Computational complexity of various receiver structures for two values $M$ of subchannels, and for filters parameters of Section 7.
The performance of the various modulation and equalization techniques have been tested on a channel whose features defined in the HIPERLAN Type 2 standard [4]. In particular we have assumed: \( M = 64 \) and \( M = 128 \), \( 1/T_c = 25 \text{ MHz} \), a frequency selective Rayleigh fading radio channel with a root mean square (rms) delay spread \( \sigma = 0.25 \mu \text{s} \) and \( 1 \mu \text{s} \), and an exponentially decaying power profile [12]. Hence the normalized, to the received signal bandwidth, rms delay spread is \( \sigma_r = \frac{\sigma}{T_c} = 1 \) and \( 4 \). The first value of \( \sigma_r \) models an indoor channel, while the second models an outdoor highly dispersive channel.

The channel impulse response is assumed to be known at the receiver (i.e. perfect channel estimation) and static, at least for the duration of one symbol. The average signal-to-noise ratio (SNR), namely the ratio between the power of the signal \( s_c(i) \) and the power of the noise \( w(i) \), is assumed 25 dB. The length of the prototype transmit filter is \( N_h = N_g = M \cdot 16 \). The performance is evaluated in terms of achievable bit-rate [13]. We indicate by \( SDR_m \) the signal-to-disturbance ratio at the decision device for the \( m \)-th subchannel.

Following the considerations made in [13], using a trellis code with gain of 3.5 dB, the modified \( SDR_m \) of the \( m \)-th subchannel, \( m \), is given in dB by \( SDR_{m,\text{dB}} = SDR_{m,\text{dB}} - \gamma \), where \( \gamma = 12.8 \text{ dB} \). The achievable bit rate per subchannel is given by \( \beta_m = \frac{1}{3} \log_2(SDR_{m} + 1) \), \( m = 0, 1, \ldots, M - 1 \), where \( SDR_m = 10^{SDR_m,\text{dB}/10} \). The achievable bit-rate for transmission is therefore obtained by summing up the \( \beta_m \) values over the active subchannels allocated for transmission, i.e. \( \beta = \sum_{m=0}^{M-1} \beta_m \). The complementary cumulative distribution function (cCDF) of \( \beta \) is used to compare the performance of different systems. For DMT we have used the following equalizer parameters:

- **DMT–CP**: length of the cyclic prefix \( C = \lceil 3\sigma_r \rceil + 5 \), to include the A/D and D/A interpolation filters length.

while for **FMT**:

- **DFME, postDFME and preDFME**: number of coefficients of the feedforward and feedback filter, \( N_{FF} = 19 \) and \( N_{FB} = 19 \), respectively.

In order to obtain a fair comparison with the simplified FSpreDFME and FSpostDFME, the FSDFME is also considered. Before reporting performance we recall the major features of the various schemes. Firstly, FMT has a better bandwidth efficiency than DMT–CP, because it does not use any cyclic prefix. Moreover, FMT needs fewer virtual carriers than DMT since in this case the spectrum of the transmitted signal is less distorted by the filter included in the D/A converter, as observed in [11]. On the other end, FMT equalization, both with preDFME and postDFME, is based on the assumption of subchannel flatness. Hence these schemes show a performance degradation for higher values of the ratio \( \sigma_r \).
In Fig. 3 these effects are clearly depicted. For a lower value of $\frac{\sigma^2}{\sum}$ ($\sigma = 4$ and $M = 128$), FMT–FSpreDFME outperforms DMT–CP. However for $M = 64$ and the same value of $\sigma$, the channel cannot be considered flat on each subchannel anymore, and FMT suffers a performance degradation approximately equal to the bandwidth efficiency improvement due to the absence of cyclic prefix. In this case FMT–FSpreDFME and DMT–CP exhibit similar performance at higher cCDF values.

In Fig. 4 we have compared all equalizer structures for $\sigma = 4$ and $M = 64$. Because of its extreme flexibility, FSDFME yields the best performance. The introduction of the subchannel delay recovery both in FSpreDFME and in FSpostDFME, allows for a performance improvement with respect to the simpler preDFME and postDFME schemes.

In Fig. 5 the two best-performing equalization methods are considered against DMT–CP for $\sigma = 1$ and $M = 64$. In particular, we note that for $\sigma = 1$ the subchannel flatness assumption is well verified and, while equalization is similar in DMT–CP and FMT–FSpreDFME, FMT benefits from its bandwidth efficiency and outperforms DMT. For the same reason, FSDFME yields similar performance of FSpreDFME. In fact almost all the interference is due to the transmit filters which are equalized in the same way by both systems.

6 CONCLUSIONS

In this letter we have presented equalization techniques for wireless FMT systems. From the numerical results we can conclude that FMT–FSpreDFME yields better performance (in terms of achievable bit rate) than DMT–PC and also than FMT–FSpostDFME.

Hence, although FMT requires a moderately higher computational complexity than DMT, it may be considered as a possible candidate for high speed transmission in dispersive indoor/outdoor wireless channels. However, when the number of subcarriers is very high (e.g. 256 or more) and the receiver must be very simple (e.g. for consumer electronics), and the whole system is synchronous, DMT with cyclic prefix may still represent the most convenient solution.
References


Figure 1: Baseband block diagram of a multicarrier system.
Figure 2: Equalizers for FMT systems. 

a) DFME; b) postDFME; c) preDFME.
Figure 3: Performance comparison of DMT–CP and FMT–FSpreDFME for two values of $M$. Transmission channel with SNR = 25 dB and $\sigma_r = 4$. 
Figure 4: Performance of FMT using five different equalizer structures: $M = 64$, SNR = 25 dB and $\sigma_r = 4$. 


distributable CDF of the achievable rate

achievabale rate (Mbit/s)

0.1

0.2

0.3

0.4

0.5

0.6

0.7

0.8

0.9

1

postDFME

preDFME

FSpostDFME

FSpreDFME

FSDFME
Figure 5: Performance comparison of FMT and DMT, each using the best two equalizer structures: $M = 64$, $\text{SNR} = 25 \text{ dB}$ and $\sigma_r = 1$. 