Frequency-Domain Equalization and Channel Estimation for Broadband Wireless Communications

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AA. AA. 1999-2002
Ho fatto una gran fadiga,
ho fatto anca dei mancamenti,
ma spero che, per rason della stravaganza,
tutti sti siori me perdonerà.

(C. Goldoni)
Acknowledgments

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Beyond the people that worked directly with me during these years, many other relatives and friends have supported me, sharing with me joys and difficulties of the research activity. To all of them my deep gratitude. A very big thank to my parents Alberto and Elena and my brother Lorenzo, who always helped me through many problems, respected and supported my choices with admirable love.
Abstract

New services such as high-speed data transmission and multimedia applications have tremendously increased the bandwidth requirements for communications in the last two decades. On the other hand, the emerging need for access to information everywhere at every time (ubiquity) has focused attention over wireless communications. At the physical layer, wireless broadband transmission is characterized by dispersion of the channel and its possibly time-varying nature. When more transmitters share the same radio bandwidth, co-channel interference arises. This thesis addresses three disturbances affecting the received broadband signal: the distortion introduced by the channel dispersion, the distortion due to the time-variations of the channel and the interference due to the simultaneous transmission of various devices. In order to face these phenomena we consider both the equalization of the channels, and the use of multiple antennae. We also devise new methods to estimate the channel parameters needed for the equalization. In all the proposed techniques, operations on the signals are performed in the frequency domain, in order to obtain efficient solutions.

For a static channel, a new transmission format is proposed for a single carrier transmission, which allows a reduced complexity implementation of the decision feedback equalizer, due to the frequency-domain operations of the feedforward filter. For a time-varying channel scenario, a new iterative interference cancellation scheme is considered for an orthogonal frequency division multiplexing modulation. Another contribution of the thesis is the derivation of the optimum adaptation of a multiple antennae transmitter in order to minimize the impact of co-channel interference in a co-channel interference scenario. For both the time-invariant and the time-varying cases, new reduced complexity channel parameter estimators are derived. Lastly, the thesis proposes also new channel estimators. For the static case, two estimators are presented: one is based on the efficient realization of the inverse Fourier transform, while the latter performs a polynomial interpolation of the frequency response of the channel. For the time-varying scenario, a multistage estimator is proposed, where the estimate is performed according to a model based on the Taylor expansion of the time-varying channel taps.
Sommario

Nuovi servizi, quali la trasmissione dati ad alta velocità e le applicazioni multimediali, hanno aumentato notevolmente la banda richiesta per le comunicazioni negli ultimi vent’anni. D’altra parte, il bisogno emergente di accedere alle informazioni in qualsiasi luogo e in qualsiasi momento (ubiquità) ha concentrato l’attenzione sulle comunicazioni radio. A livello di strato fisico, la trasmissione radio a banda larga è caratterizzata dalla distorsione del segnale ricevuto, a causa della dispersione del canale e della sua possibile natura tempo variante. Nella tesi si occupa di tre tipi di disturbo relativi a segnali a banda larga: la distorsione dovuta alla dispersione del canale, la distorsione dovuta alla tempo-varianza del canale e quella dovuta alla trasmissione simultanea da parte di più apparecchi. Per fronteggiare questi fenomeni, consideriamo sia l’equalizzazione del canale che l’uso di più antenne. Vengono ricavate anche nuove tecniche per la stima dei parametri di canale necessari per l’equalizzazione. In tutte le tecniche proposte, le operazioni sui segnali sono realizzate nel dominio della frequenza, così da ottenere soluzioni efficienti.

Per la modulazione a singola portante, si propone un nuovo formato di trasmissione che permette la realizzazione efficiente di un decision feedback equalizer, grazie alle operazioni nel dominio della frequenza del filtro di feedforward. In una situazione di tempo-varianza del canale trasmissivo si considera invece uno schema di cancellazione iterativa dell’interferenza per sistemi di modulazione Orthogonal Frequency Division Multiplexing. Per un sistema di comunicazione che utilizza due o più antenne sia al trasmettitore che al ricevitore in uno scenario con interferenza di co-canal, si ricava l’adattamento ottimo dei guadagni di trasmissione che minimizza l’impatto dell’interferenza di co-canal. Infine, sia per il caso di canale tempo-variante che quello tempo-invariante, vengono derivati nuovi stimatori dei parametri di canale, aventi complessità ridotta. Per il caso statico, si considerano due stimatori: uno è basato su una realizzazione efficiente della trasformata inversa di Fourier, mentre il secondo realizza un’interpolazione polinomiale della risposta in frequenza del canale. Per lo scenario tempo-variante, si propone invece uno stimatore a stadio multiplo, in cui la stima è realizzata secondo un modello basato sulla espansione in serie di Taylor dei tappi tempo-varianti del canale.
Notations and abbreviations

Notations
Lowercase is reserved for time-domain signals. Uppercase is reserved for frequency-domain signals.

Italics is reserved for scalar signals. Bold is reserved for vector and matrix signals. Scalar time-domain signals are indicated with lowercase italics, i.e. \( x \).
Scalar frequency-domain signals are indicated with uppercase italics, i.e. \( X \).
Time-domain vector and matrix signals are indicated with lowercase bold, i.e. \( \mathbf{x} \).
Frequency-domain vector and matrix signals are indicated with uppercase bold, i.e. \( \mathbf{X} \).
Components of vectors are indicated with italics with indexes, i.e. \( x_n \). Components of matrices are indicated with indexes of squared brackets, i.e. \( [x]_{r,c} \).
Complex conjugate is indicated with \( ^* \), i.e. \( x^* \).
Real and imaginary parts of \( x \) are denoted as \( \text{Re}[x] \) and \( \text{Im}[x] \), respectively.
Transpose is indicated with \( ^T \), i.e. \( x^T \).
Determinant and trace of matrix \( X \) are denoted as \( \det[X] \) and \( \text{trace } X \), respectively.
Hermitian operator (conjugate and transpose) is indicated with \( ^H \), i.e. \( x^H \).
The squared norm of vector \( a \) is denoted as
\[
||a||^2 = \sum |a_k|^2.
\]

Data signals
- \( d^{(n)}(k) \): for a single carrier system, complex data signal sent on the antenna \( n \), at time \( k \).
- \( D_m^{(n)}(k) \): for an OFDM system, complex data signal sent on the antenna \( n \), at time \( k \) on subcarrier \( m \).

Signals on the channel
- \( T \): sample rate.
- \( s^{(n)}(k) \): sampled time-domain signal sent on the antenna \( n \), at time \( k \).
- \( h_{n_T,n_R}(t, \tau) \): impulse response of the channel filter at time \( t \) from antenna \( n_T \) to antenna \( n_R \). This filter include the transmit and receive filters.
• \( h^{(n_T,n_R)}_t(t) \): sampled impulse response of the channel filter at time \( t \) from antenna \( n_T \) to antenna \( n_R \). This filter includes the transmit and receive filters.

• \( w^{(n)}(k) \): sampled time-domain noise on the antenna \( n \), at time \( k \).

• \( r^{(n)}(k) \): sampled time-domain received signal on the antenna \( n \), at time \( k \).

• \( H^{(n_T,n_R)}_m(t) \): frequency response of the channel from antenna \( n_T \) to antenna \( n_R \) at time \( t \), on blocks of size \( M \).

• \( \mathcal{H}^{(n_T,n_R)}_m(t) \): frequency response of the channel from antenna \( n_T \) to antenna \( n_R \) at time \( t \), on blocks of size \( P \).

• \( S^{(n_T)}_m(k) \): frequency response of the transmitted signal from antenna \( n_T \) at the block \( k \).

• \( R^{(n_R)}_m(k) \): frequency response of the signal received by antenna \( n_R \) at the block \( k \).

• \( W^{(n_R)}_m(k) \): frequency response of the noise on antenna \( n_R \) at the block \( k \).

Indexes \( n, n_T \) and \( n_R \) are omitted for single-antenna systems.

For time-invariant channels the index \( (t) \) is omitted.

From the above notation we have

\[
    r^{(n_R)}(k) = \sum_{\ell=0}^{N_R-1} \sum_{n_T=0}^{N_T-1} h^{(n_T,n_R)}(kT) s^{(n_T)}(\ell) + w^{(n_R)}(k), \quad n_R = 0, 1, \ldots, N_R - 1. \tag{2}
\]

For an OFDM system with \( M \) subcarriers and a cyclic prefix of \( L \) samples, we have

\[
    R^{(n_R)}_m(k) = \sum_{n_T=0}^{N_T-1} H^{(n_T,n_R)}_m(k(M+L)T) S^{(n_T)}_m(k) + W^{(n_R)}_m(k(M+L)T), \tag{3}
\]

where \( n_R = 0, 1, \ldots, N_R - 1 \).

Other symbols

• \( F \): Discrete Fourier Transform matrix

• \( M \): number of subcarriers of both OFDM system and SC systems with frequency-domain equalization

• \( L \): number of taps of the cyclic prefix or PN-extension

• \( N_T \): number of transmit antennae

• \( N_R \): number of receive antennae

• \( j = \sqrt{-1} \).

• \( \angle x \): the phase of the complex number \( x \)
Abbreviations
ABR: Achievable Bit Rate
ATP: Adjacent Tone Partitioning
AWGN: Additive White Gaussian Noise
BER: Bit Error Rate
CA: Constant Amplitude
ccdf: Complementary Cumulative Distribution Function
cdf: Cumulative Distribution Function
CP: Cyclic Prefix
CTP: Comb Tone Partitioning
DVB-T: Digital Video Broadcasting-Terrestrial
DFE: Decision Feedback Equalizer
DF-ICI: Decision Feedback ICI cancellation scheme
DFT: Discrete Fourier Transform
EA: Equal Amplitude
EP: Equal Phase
FD: Frequency-Domain
FEC: Forward Error Control
FER: Frame Error Rate
FFT: Fast Fourier Transform
GSM: Global System for Mobile communications
ICI: Inter Channel Interference
IDFT: Inverse Discrete Fourier Transform
IFFT: Inverse Fast Fourier Transform
IS: Interference Suppression
ISI: Inter Symbol Interference
LAN: Local Area Network
LE: Linear Equalizer
LMMSE: Linear Minimum Mean Square Error
LOS: Line of Sight
LS: Least Square
MC: Multi Carrier
MI: Minimum Intereference
MIMO: Multiple Input Multiple Output
ML: Maximum Likelihood
MMSE: Minimum Mean Square Error
MSE: Mean Square Error
MSNIR: Minimum Signal to Noise plus Interference Ratio
NLOS: No Line of Sight
OC: Optimum Combining
OFDM: Orthogonal Frequency Division Multiplexing
pdf: probability density function
PER: Packet Error Rate
POL: Polynomial
PN: Pseudo Noise
RC: Repetition Code
rms: root mean square
SC: Single Carrier
SIR: Signal to Interference power ratio
SNR: Signal to Noise Ratio
SNIR: Signal to Noise plus Interference Ratio
ST: Space-Time
SVD: Singular Value Decomposition
TD: Time-Domain
QoS: Quality of Service
ZF: Zero Forcing
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Introduction

New services such as high-speed data transmission and multimedia applications have tremendously increased the bandwidth required for communications in the last two decades. On the other hand, the emerging need for access to information everywhere at every time (ubiquity) has focused attention over wireless communications. The combination of these needs has driven the development of new broadband wireless standards, which include, among others, wireless local area networks (WLAN), [1, 2], home-networks (e.g. the European project WindFlex, [3]) and digital audio and video broadcast (DAB/DVB), [4, 5]. Limits of radio resources have also pushed for extreme applications of existing standards: for example, the terrestrial DVB (DVB-T), originally designed for home reception, is now considered for broadcast of digital contents to moving cars. At the physical layer, all broadband wireless communications have some common characteristics. The channel has a dispersive impulse response, which is reflected into a selectivity of the channel frequency response. Moreover, if the receiver is moving, and/or the environment is not static, the channel turns out to be time-varying. As a result, interference among the transmitted symbols arises, which significantly degrades the system performance.

In this thesis we consider another scenario including interference among different devices that transmit on the same bandwidth at the same time. This may happen for the proximity of two wireless devices belonging to different piconets.

This thesis addresses three disturbances affecting the received broadband signals: the distortion introduced by the channel dispersion, the distortion due to the time-variations of the channel and the interference due to the simultaneous transmission of various devices. In order to face these phenomena we consider the equalization of both time-invariant and time-varying channels, and the use of multiple antennae to combat co-channel interference. For these applications we also devise new methods to estimate the channel parameters. A common feature of the techniques proposed in this thesis, is that operations on the signals are performed in the frequency domain, in order to obtain efficient solutions.

The interference generated by dispersive channels (intersymbol interference, ISI) can be coped with various approaches. While the best performance is given by the maximum likelihood (ML) detection, commonly used suboptimal approaches comprise linear and non-linear equalizers, which revert the distortion of the dispersive channel by means of linear filtering and cancellation of the interference, [6]. Equalization may be performed on blocks of the received signals, by means of the efficient fast Fourier transform (FFT), using either an orthogonal frequency division multiplexing (OFDM) transmission, or a frequency-domain (FD) equalization
INTRODUCTION

in single carrier (SC) transmissions. We recall that in OFDM the bandwidth is divided into many low-rate channels and detection is performed in the frequency domain; for dispersive channels, a cyclic-prefix (CP) is inserted in the transmitted data frame, [7], which slightly reduces the bandwidth efficiency. In SC communication, instead, a high-rate signal is transmitted over the entire bandwidth and detection is performed in the time-domain. Both FD-linear equalizer [8–10] and FD-non linear equalizers (decision feedback equalizer, DFE) [11] have been proposed for SC transmission. However, the better performing FD-DFE of [11] is suitable only for channels with a very long impulse response. In Chapter 2, we present a new FD-DFE architecture, suitable also for moderately dispersive channels, which yields a better performance than time-domain DFEs, while having a much reduced complexity. This is possible at the cost of a slightly reduced bandwidth efficiency, similar to that of OFDM. The new transmission format used for FD-DFE is presented in Chapter 1. The design of the filters is performed according to both the zero-forcing and minimum mean square error criteria and performance of the new scheme is compared both to OFDM and to classical single carrier transmission with time-domain (TD) equalization.

The combination of OFDM and mobile reception poses further problems at the physical layer. In particular, the received signal turns out to be affected by interference among different carriers (inter-carrier interference, ICI) and various techniques have been devised to limit this problem. Some solutions model the interference as additional noise, without considering its correlation with the data, [12]. More effective self-cancellation techniques require instead the use of particular precoded signals [13]. When intervention is limited to the receiver, a linear channel-dependent matrix equalizer can be used in order to restore the orthogonality among the subcarriers, [14]. However, the computational complexity required by the matrix multiplication as well as the matrix adaptation is excessive for systems with a large number of subcarriers. In Chapter 3 we propose a scheme for the cancellation of ICI, which progressively increases the reliability of the detected data. This scheme relies on a simplified channel model, based on the Taylor expansion of the time-varying channel impulse response, as described in Chapter 1.

When the broadband communication devices are sharing the same radio resource at the same time, interference may arise. For devices belonging to the same network, interference is limited by appropriate techniques of medium access, such as frequency- or code-division multiple access. For devices belonging to different networks, no coordination is available among different transmitters and co-channel interference may be a limiting factor. This is the case for home broadband networks that are organized in piconets. In order to cope with this phenomenon, multiple antennae can be used both at the transmitter and the receiver. Also in this case we focus our attention on efficient schemes based on FFT and in particular we consider a multi-antenna system using OFDM modulation, [15]. We recall that in literature transmit adaptation has been derived in scenarios with no co-channel interference [16–18], while when interference is present, only receive equalizers have been considered [19, 20]. In Chapter 4 for a co-channel interference scenario we present various design techniques to optimize transmit gains for an OFDM system equipped with multiple antennae. We focus on two coding schemes for multi-antenna OFDM: the first is a simple repetition code, while the latter is the space-time block
coding scheme. To limit complexity, the receiver adopts maximum ratio combining whose optimization depends only on the channel and not on the interference signal. For both schemes we derive the transmit gains that maximize the signal to interference power ratio at the receiver. As suboptimal schemes we also consider cases where gains have the same phase or the same amplitude. In order to have an upper bound on the system performance we derive a novel expression of the capacity of a system with two receive antennae and adaptive transmit gains. In fact, previous results are limited to the system where each transmitted signal is the linear combination of all space-coded data [21].

All the considered frequency-domain equalizers or transmit adaptation schemes require a good estimate of the channel parameters. For OFDM, various linear channel estimators have been studied [22–25]. These include the MSE criterion and other suboptimal approaches. Under the assumption of knowing the statistical properties of the channel, the estimation is performed in the time-domain, where the energy of the channel is more concentrated. For static channels, in Chapter 5, we propose two new schemes to estimate the channel impulse response. While the first method is based on a reduced complexity time-domain estimate, the latter is a simple interpolator in the frequency domain. As a result, complexity and reliability of the estimate can be traded off, according to the system requirements. Also for the channel estimate, time- variations of the impulse response require more elaborate solutions, to account for a larger number of parameters and a different impact on the received signal. For this case, in Chapter 5 we propose a new multistage method for the estimate of the parameters that describe the time-varying channel, according to the model of Chapter 1.
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Chapter 1

Broadband Wireless Modulation Models

Wireless broadband communication systems are characterized by dispersive channels, which have a frequency-selective and possibly time-varying response. In this chapter we describe two channel models for broadband wireless communications that describe wireless indoor and mobile communications, respectively: the first is a quasi-static model, while the latter is time-varying. Both models are then extended to the general case of communications involving multiple antennae at the receiver and/or at the transmitter.

In order to allow a correct transmission on the broadband channel, two modulation techniques are considered here: single carrier (SC) modulation with broadband equalization and multicarrier modulation with orthogonal frequency division multiplexing (OFDM). For both systems, linear equalizers (LE) have been proposed [6, 7], that compensate for the dispersive channels. However, single carrier transmission with LE yields a lower performance than OFDM.

A contribution of this chapter is the introduction of the PN-extended transmission for SC communications, that allows a reduced-complexity frequency domain equalization, at the expense of a lower bandwidth efficiency. This transmission structure will be used in Chapter 2 for the derivation of the frequency-domain DFE that yields performance similar to OFDM.

Moreover, for an OFDM system we give a mathematical description of the received signal on mobile communication. In [26] an approximation of the OFDM received signal based on the Taylor-expansion has been derived for a Rayleigh fading channel. Here we extend the approximation to a general time-varying channel.

1.1 Communication model

In a digital communication system, the message is coded and modulated before transmission. The transformations that the signal $s(k)$ undergoes from the transmitter to the receiver are depicted in Fig. 1.1.a.

For transmission on the channel, a Transmitter front-end (Tx fe) with filter $g_{\text{Tx}}$, performs the interpolation from the discrete-time domain signal $s(k)$ at time $T$ to the continue-time domain
CHAPTER 1. BROADBAND WIRELESS MODULATION MODELS

Figure 1.1: Channel models of a broadband wireless system, including the transmit and receive front-end. a) Continuous-time model; b) Discrete-time equivalent model.

signal \( s(t) \). Moreover, this block also modulates the signal on the carrier frequency. Similarly, a Receiver front-end (Rx fe) performs the frequency demodulation, in order to obtain a baseband signal and the resulting signal is sampled by the decimation filter \( g_{Rx} \), to obtain \( r(k) \) at rate \( 1/T \). The equivalent baseband signal model, where the frequency modulation at the transmitter and the frequency demodulation at the receiver are omitted, is shown in Fig. 1.1.b.

As it will be shown in the following section, the transmission channel can be described as a linear filter, which performs the convolution of the transmitted signal \( s(t) \) with the time-varying channel impulse response \( h_{Ch}(t, \tau) \), where \( t \) is the time of the resulting signal. On the receive antenna, thermal noise and other sources of disturbance generate additive Gaussian noise, which will be indicated as \( w(t) \).

Under the assumption that the cascade of the transmit and receive front-end

\[
g(t) = \int_{-\infty}^{\infty} g_{Tx}(t - \tau) g_{Rx}(\tau) d\tau
\]

has a bandwidth shorter than \( 1/T \), no alias is generated in the transmission process and the communication channel can be described by an equivalent discrete-time model. As shown in Fig. 1.1.b, the discrete-time signal \( s(k) \) is convolved by the channel filter with impulse response

\[
h(t, nT) = \int g(nT - \eta) h_{Ch}(t, \eta) d\eta,
\]

which includes the interpolating and sampling filters. The continuous-time noise is replaced by the discrete-time noise

\[
w(k) = \int g_{Rx}(kT - t) w(t) dt,
\]
which is added to the filtered signal to obtain \( r(k) \).

For a static scenario, the equivalent channel is denoted also as \( h(\tau) \), while the discrete taps are denoted as

\[
h_n = h(t, nT),
\]

(1.4)

### 1.2 Channel models

In a wireless communication, the transmitted signal \( s(t) \) is reflected by obstacles that generate replicas of \( s(t) \) with different delays, amplitudes and phases. For a broadband transmission, the delays are larger than the duration of the transmitted symbols, so that interference among different symbol arises (inter symbol interference, ISI). In the frequency domain this phenomenon yields a frequency-selective response of the channel where each frequency is characterized by a different amplitude and phase. Moreover, when the transmitter and/or the receiver are in motion, or the position of the objects changes (e.g. a man walking nearby the receiver), the channel is time-variant.

We will now provide models for both time-invariant and time-varying channels.

#### 1.2.1 Time-invariant channel model

When considering the propagation of a signal on a wireless static environment, the main phenomenon is the reflection of the transmitted signal on objects surrounding the transmitter and the receiver. These reflections generate replicas of the transmitted signal with different delays, amplitudes and phases. The resulting propagation is called multipath propagation and the impulse response of the channel turns out to be the sum of \( N_\tau \) impulses, i.e.

\[
h(\tau) = \sum_{n=0}^{N_\tau-1} h(\tau_n) \delta(\tau - \tau_n),
\]

(1.5)

where \( \delta(x) \) is the Dirac impulse, \( \{\tau_n\} \) and \( \{h(\tau_n)\} \) are the delays and complex gains of each path. An example of a discrete-time multipath channel impulse and frequency responses are shown in Fig. 1.2.

Both theoretical considerations and experimental data confirm that in most cases the different reflections are statistically independent, both in their complex amplitudes and delays. Moreover, a common assumption is that the delays \( \tau_n \) are uniformly distributed random variables in the interval \([0, \tau_{\text{max}}]\). Two important characterizations of a multipath channel are given by the rms delay spread and the power delay profile. The root mean square (rms) delay spread is the standard deviation of the delays weighted proportionally to the energy of the reflected waves, i.e.

\[
\tau_{\text{rms}} = \sqrt{\frac{\sum_n E[|h(\tau_n)|^2](\tau_n^2 - \sum_k E[|h(\tau_k)|^2]\tau_k)}{\sum_n E[|h(\tau_n)|^2]}}.
\]

(1.6)
Figure 1.2: An example of a discrete-time multipath channel impulse and frequency responses.
1.2. CHANNEL MODELS

Table 1.1: WindFlex channel model pdfs, LOS case

<table>
<thead>
<tr>
<th>Tap</th>
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<tbody>
<tr>
<td>1</td>
<td>Frechet</td>
<td>4</td>
<td>Exp. (µ = 1.45e7)</td>
<td>7</td>
<td>Exp. (µ = 0.41e7)</td>
</tr>
<tr>
<td></td>
<td>(σ = 2.66e8, λ = 7)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Exp. (µ = 5.44e7)</td>
<td>5</td>
<td>Exp. (µ = 1.03e7)</td>
<td>8</td>
<td>Exp. (µ = 0.27e7)</td>
</tr>
<tr>
<td>3</td>
<td>Exp. (µ = 2.51e7)</td>
<td>6</td>
<td>Exp. (µ = 0.79e7)</td>
<td>9</td>
<td>Exp. (µ = 0.71e7)</td>
</tr>
</tbody>
</table>

It describes the overall dispersion of the channel. The power delay profile of the channel describes the behavior of the power of the channel at different delays. For example, an exponential power profile models a channel where reflections with smaller delays have (on average) an exponentially greater power than reflections with longer delays, i.e.

\[
E[|h(τ)|^2] = \begin{cases} 
\frac{1}{τ_{\text{rms}}} e^{-\frac{τ}{τ_{\text{rms}}}} & τ ≥ 0 \\
0 & τ < 0 
\end{cases}.
\] (1.7)

For the statistical properties of each random variable \( h(τ_n) \) various models are considered. In this thesis we consider the Rayleigh [6] and the WindFlex [27] models.

- **Rayleigh model.** In the Rayleigh model each complex tap gain \( h(τ_n) \) is a Gaussian random variable, with zero mean and variance according to the power delay profile. This model gives a good description of wireless propagation in many applications, including for example wireless LAN, such as HIPERLAN [1], which operates at 2.4 GHz.

- **WindFlex model.** WindFlex is a wireless indoor communications systems for small office/home office applications. It operates at a carrier frequency of 17 GHz with a 50 MHz-bandwidth. A test campaign has been conducted to assess the channel characteristics. Two models have been derived: one for line-of-sight propagation (LOS), and one for no line-of-sight propagation (NLOS), with a \( τ_{\text{rms}} \) of 27ns and 33ns, respectively. For each model, a finite number of taps with delays of 10ns has been considered, with the following distributions:

  - LOS case: a Frechet distribution for the first tap, while the remaining taps have an exponential pdf.
  - NLOS case: the first tap is characterized by a combination of an exponential and Weibull variable, while the others have an exponential distribution.

Tables 1.1 and 1.2 show the parameters of the pdf used for each tap, where \( µ \) and \( σ \) denote the mean and the standard deviation of the variables, respectively.
Table 1.2: WindFlex channel model pdfs, NLOS case

<table>
<thead>
<tr>
<th>Tap</th>
<th>Type</th>
<th>Tap</th>
<th>Type</th>
<th>Tap</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5[Exp(σ = 4.378e6) + Weibull (σ = 4.207e7, λ = 5)]</td>
<td>7</td>
<td>Exp. (μ = 1.88e5)</td>
<td>13</td>
<td>Exp. (μ = 9.21e4)</td>
</tr>
<tr>
<td>2</td>
<td>Exp. (μ = 3.04e6)</td>
<td>8</td>
<td>Exp. (μ = 2.51e5)</td>
<td>14</td>
<td>Exp. (μ = 1.27e5)</td>
</tr>
<tr>
<td>3</td>
<td>Exp. (μ = 2.47e6)</td>
<td>9</td>
<td>Exp. (μ = 5.69e5)</td>
<td>15</td>
<td>Exp. (μ = 2.76e4)</td>
</tr>
<tr>
<td>4</td>
<td>Exp. (μ = 2.14e6)</td>
<td>10</td>
<td>Exp. (μ = 1.53e5)</td>
<td>16</td>
<td>Exp. (μ = 6.71e4)</td>
</tr>
<tr>
<td>5</td>
<td>Exp. (μ = 1.1e6)</td>
<td>11</td>
<td>Exp. (μ = 3.29e5)</td>
<td>17</td>
<td>Exp. (μ = 6.42e4)</td>
</tr>
<tr>
<td>6</td>
<td>Exp. (μ = 3.71e5)</td>
<td>12</td>
<td>Exp. (μ = 2.67e5)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1.2.2 Time-varying channel model

When the receiver is moving with respect to the transmitter, the channel becomes time-varying and its impulse response is dependent on the particular time of the transmission. Also in this case the impulse response is the sum of delayed impulses with different complex amplitudes, i.e. the channel impulse response at time \( t \) can be written as

\[
\hat{h}_{Ch}(t, \tau) = \sum_{n=0}^{N_T-1} h_{Ch}[t, \tau_n(t)] \delta[\tau - \tau_n(t)],
\]  

(1.8)

where now both the delays and the amplitudes are time-dependent.

A good description of the time-varying channel for a wireless broadband transmission is provided by the Jakes model [28], which describes the impulse response of the channel filter at time \( t \) as the sum of \( N_T \) reflections, each characterized by a delay \( \tau_\ell \) and \( N_J \) oscillatory coefficients: \( \{ h'_\ell,q \} \) are complex channel gains, while \( \{ f_\ell,q \} \) are frequency shifts. The resulting channel response is

\[
\hat{h}_{Ch}(t, \tau) = \sum_{\ell=0}^{N_T-1} \delta(\tau - \tau_\ell) \sum_{q=0}^{N_J-1} h'_\ell,q e^{2\pi j f_\ell q t}.
\]  

(1.9)

In order to simplify the forthcoming analysis, (1.9) can be rewritten as

\[
\hat{h}_{Ch}(t, \tau) = \sum_{\ell=0}^{N_T-1} h(\tau_\ell) \delta(\tau - \tau_\ell) e^{2\pi j f_\ell t},
\]  

(1.10)

obtained by an appropriate reordering of \( \{ h_{\ell,q} \} \) and \( \{ f_{\ell,q} \} \).

For a mobile receiver which is moving at speed \( v \) and transmitting on the \( f_c \) carrier frequency, the maximum frequency Doppler shift is

\[
f_d = \max\{f_\ell\} = \frac{v}{c} f_c,
\]  

(1.11)
1.3 Multicarrier Transmission

The scheme of the OFDM transmitter is shown in Fig. 1.3. In OFDM modulation, a discrete complex data stream \( a(n) \) with rate \( R = 1/T \) is divided into \( M \) sub-streams at rate \( R/M \) by a serial to parallel converter (S/P). Each of the low-rate data streams

\[
D_m(k), \quad m = 0, 1, \ldots, M - 1.
\]

where \( c \) is the light’s speed. According to the Jakes model, the statistical description of the random variables \( f_\ell \) is given by

\[
f_\ell = f_d \cos \theta_\ell,
\]

where \( \theta_\ell \) is a uniform random variable in the interval \([0, 2\pi)\).

1.2.3 Multiantenna communications model

When the receiver or the transmitter are equipped with two or more antennae, the communication system is denoted as a Multiple Input/Multiple Output (MIMO) system. In order to obtain a complete description of the communications, the channels between each couple of transmit/receive antenna must be characterized, i.e. for the link between antenna \( n_T \) and antenna \( n_R \), the channel \( h^{(n_T,n_R)}(t, \tau) \) must be provided. For each of these links, the channel models described in Sections 1.2.1 and 1.2.2 hold.

For a complete statistical description, the correlation among the different channels should be provided. As a common assumption, the channels between each couple are assumed independent, i.e.

\[
E[h^{(n_1,n_2)}(t_1, \tau_1)h^{(n_3,n_4)}(t_2, \tau_2)] = 0, \quad n_1 \neq n_3, n_2 \neq n_4, \forall t_1, t_2, \tau_1, \tau_2.
\]

1.3 Multicarrier Transmission

Figure 1.3: OFDM transmitter.
is modulated with rectangular filters on equally spaced sub-carriers with a spacing \( f_s = \frac{1}{TM} \), by means of a Inverse Discrete Fourier Transform (IDFT), to obtain the samples

\[
d_m(k) = \sum_{\ell=0}^{M-1} D_\ell(k)e^{2\pi j \frac{m\ell}{M}}, \quad m = 0, 1, \ldots, M-1.
\] (1.15)

From (1.15) it can be seen that the modulation is efficiently implemented with an inverse discrete Fourier transform (IDFT) of size \( M \). At the receiver, a discrete Fourier transform (DFT) is applied to recover the transmitted signal and, for a flat fading channel, neither ISI among samples of the same subcarrier nor intercarrier interference (ICI) among samples of different subcarriers arise.

When the channel is frequency dispersive and time-invariant, an ICI- and ISI-free reception is obtained by extending each transformed block with a cyclic prefix [29] before transmission, so that the last \( L \) samples of the extended block coincide with its first \( L \) samples. The transmitted block of length \( M + L \) samples becomes

\[
s(k) = [d_{M-L}(k), d_{M-L+1}(k), \ldots, d_{M-1}(k), d_0(k), d_1(k), \ldots, d_{M-1}(k)],
\] (1.16)

where the first \( L \) samples coincides with the last samples of the block.

At the receiver, each block \( r(k) \) of \( M + L \) samples is split into two parts, where the first \( L \) samples are discarded, while DFT is performed on the remaining \( M \)-size block, as shown in Fig. 1.4.

If the channel impulse response is static and shorter than the duration of the cyclic extension \( (N_\tau \leq L) \), the convolution between the transmitted signal and the channel becomes circulant, and the signal \( R_m(k) \) is a noisy version of the product between the corresponding data symbol \( D_m(k) \) and the DFT coefficient of the channel. By indicating the \( M \)-size DFT of the equivalent discrete-time channel as

\[
H_m = \sum_{n=0}^{N_\tau-1} h_n e^{-j2\pi \frac{nm}{M}},
\] (1.17)
1.3. MULTICARRIER TRANSMISSION

The received signal on the $m$th subcarrier is

$$R_m(k) = D_m(k)H_m + W_m(k), \quad m = 0, 1, \ldots, M - 1,$$

(1.18)

where $W_m(k)$ is the DFT of the noise, i.e.

$$W_m(k) = \sum_{\ell=0}^{M-1} w(\ell + k(M + L))e^{-j2\pi \ell m / M}.$$

(1.19)

1.3.1 OFDM transmission on time-varying channel

On a time-varying channel, (1.18) does not hold anymore and the received signal on the $m$th subcarrier becomes

$$R_m(k) = \sum_{k_1=0}^{M-1} e^{-j2\pi k_1 n} \sum_n h(k_1T, n)s(k(M + L) + k_1 - n) + W_m(k).$$

(1.20)

As a step toward a simplified description of the time-varying channel, the channel impulse response $h(t, \ell T)$ is expanded into a Taylor series as a function of $t$ around $t = nMT/2$. Let’s indicate with $\beta^{(p)}_\ell$ the $p$th derivative of $h_\ell(t)$ for $t = 0$, i.e.

$$\beta^{(p)}_\ell = [\frac{\partial^{(p)} h(t, \ell T)}{\partial^{(p)} t}]_{t=MT/2}.$$

(1.21)

By expanding $h(t, nT)$ into Taylor series, the received signals (1.20) becomes

$$R_m(k) = \sum_{k_1=0}^{M-1} s(k(M + L) + k_1) \sum_{q=0}^{M-1} e^{-j2\pi (m-k_1)q/M} \cdot \sum_{\ell \geq 0} \sum_p H^{(p)}_\ell(q - M/2)^p + W_m(k),$$

(1.22)

where $m = 0, 1, \ldots, M - 1$ and

$$H^{(p)}_k = \frac{1}{p!} \sum_{\ell=0}^{L-1} \beta^{(p)}_\ell T^p \sum_q g(t_0 - T_\ell + qT)e^{-j2\pi kq/M}, \quad p \geq 0,$$

(1.23)

for each subcarrier $k = 0, 1, \ldots, M - 1$.

A matrix form notation of (1.22) is now derived. We collect the information on the $p$th derivative in the vectors $H^{(p)} = [H^{(p)}_1, H^{(p)}_2, \ldots, H^{(p)}_M]$, and define the $M \times M$ matrices $\Xi^{(p)}$ with entries

$$\Xi^{(p)}_{m,k} = \frac{1}{M} \sum_{i=0}^{M-1} (i - M/2)^p e^{-j2\pi (m-k)j/M}, \quad m, k = 0, 1, \ldots, M - 1,$$

(1.24)
which describe the leakage from the signal transmitted on the subcarrier $k$ onto the subcarrier $m$. By collecting $\{R_m(k)\}$, $\{s(k(M+L)+M+L-1)\}$ and $\{W_m(k)\}$ in the $M \times 1$ vectors $R(k)$, $s(k)$ and $W(k)$, respectively, (1.22) can be rewritten as

$$R(k) = \left( \sum_{p \geq 0} \Xi^{(p)} \text{diag}\{H^{(p)}\} \right) s(k) + W(k), \tag{1.25}$$

where diag$\{a\}$ is the diagonal matrix having as diagonal the elements of the vector $a$. From this expression we note that the matrices $\Xi^{(p)}$ describe the leakage of the data signal among the subcarriers, while all information on the channel are collected in the vectors $H^{(p)}$.

In the following we will assume that the time-variations of the channel can be factorized as

$$h(k(M+L)T, \tau) = \mu_k \nu_k(\tau), \tag{1.26}$$

where $\nu_k(\tau)$ are i.i.d. zero-mean random processes and $\mu_k$ are constants that determine the power of each tap. This assumption yields a simplified expression of the correlation matrix of the channel response at different frequencies and for different derivatives. We define the $NP \times 1$ vector

$$H = [H^{(0)T}, H^{(1)T}, \ldots , H^{(P-1)T}]^T, \tag{1.27}$$

and the correlation matrix among the elements of the same derivative

$$R_f = E[H^{(p)H^{(p)H}}], \tag{1.28}$$

and among all elements of the derivatives

$$R_H = E[HH^H]. \tag{1.29}$$

If (1.26) holds, by defining the $NP \times 1$ vector

$$H = [H^{(0)T}, H^{(1)T}, \ldots , H^{(P)T}]^T, \tag{1.30}$$

there exists two matrices $R_f$ and $R_c$, of size $(N \times N)$ and $(P \times N)$, respectively, such that

$$E[HH^H] = R_c \ast R_f, \tag{1.31}$$

where $\ast$ denotes the Kronecker product. Note that $R_f$ describes the correlation among various frequencies of the same derivative, while $R_c$ describes the correlation among different derivatives.

### 1.3.2 Simplified model for OFDM mobile communications

An approximation of the model is obtained by truncating the summation (1.25) to the first $P$ terms, thus obtaining an expression of the output which relies on $M(P + 1)$ parameters that
describe the channel. The approximation is justified if the error introduced by ignoring the higher-order derivatives is negligible.

Now the error introduced by the approximation can be evaluated through the power of the neglected terms. In particular, from (1.25), the autocorrelation of the residual interference is

\[
R_e = E\left[\sum_{q \geq P} \Xi(q) \text{diag}\{H^{(q)}\} s(k) \right] \left(\sum_{p \geq P} \Xi(p) \text{diag}\{H^{(p)}\} s(k)\right)^H]
\]

\[
= \sigma_a^2 \sum_{q \geq P} \sum_{p \geq P} [R]_{p,q} \Xi(p+q). \quad (1.32)
\]

From (1.32) and (1.24) we obtain the variance of error due to model mismatch

\[
\sigma_e^2 = \frac{\sigma_a^2}{M} \sum_{q \geq P} \sum_{p \geq P} [R]_{p,q} \Xi(p+q). \quad (1.33)
\]

A further reduction of parameters is achieved by considering the eigenvalue decomposition of the autocorrelation matrix of \( H^{(p)} \). The decomposition of \( R_f \) into singular values (SVD), yields

\[
R_f = U^H \Lambda U,
\]

where \( U \) is the eigenvector matrix and \( \Lambda \) is the diagonal matrix having as elements the eigenvalues of \( R_f \) in decreasing order i.e.

\[
0 \leq [\Lambda]_{k_1,k_1} \leq [\Lambda]_{k_2,k_2}, \quad k_1 \leq k_2 . \quad (1.35)
\]

A reduced set of parameters is obtained by ignoring the lowest \( M - M_e \) eigenvalues and approximating each derivative vector \( H^{(p)} \) with the \( M_e \)-size vector \( \overline{H}^{(p)} \)

\[
H^{(p)} \approx \overline{F} H^{(p)}, \quad (1.36)
\]

where \( \overline{F} \) is the \( M \times M \) matrix containing the eigenvectors associated with the \( M_e \) most significant eigenvalues. Let’s indicate the vectors of the reduced of parameters as

\[
\overline{H} = [\overline{H}^{(0)}, \overline{H}^{(1)}, \ldots, \overline{H}^{(P-1)}], \quad (1.37)
\]

which is related to the full set of parameters as follows

\[
H \approx (I_P * \overline{F}) \overline{H}, \quad (1.38)
\]

where \( I_P \) is an identity matrix of size \( P \times P \).
Rayleigh-fading channel

For a Rayleigh-fading channel with the Jakes time-variation model (1.10), the description of the ICI in terms of expanded Taylor series has been considered in [26]. In particular, by indicating the discrete \(N\)-size Fourier transform of the cascade of the transmit and receive filters \(g(t)\) with the \(\ell\)th tap of the channel filter at the sampling delay \(t_0\) as

\[
\tilde{H}_\ell[k] = h_\ell e^{j2\pi f_\ell t_0} \sum_i g(t_0 - \tau_\ell + iT) e^{-j2\pi ki/N}, \quad k = 0, 1, \ldots, N - 1. \tag{1.39}
\]

from (1.23) and the computation of the derivatives of (1.10) we obtain

\[
H^{(p)}_k = \left(\frac{i2\pi f_d/f_s}{p!}\right)^p \sum_{\ell=0}^{N_r-1} (f_\ell/f_d)^p \tilde{H}_\ell[k], \quad k = 0, 1, \ldots, M - 1, \tag{1.40}
\]

where

\[
f_s = 1/MT. \tag{1.41}
\]

The received signal on the \(m\)th subcarrier can be written as

\[
R_m(k) = \sum_{k_1=0}^{M-1} s(k(M + L) + k_1) \sum_{q=0}^{M-1} e^{-j2\pi(m-k_1)q/M} \sum_{\ell=0}^{N_r-1} \tilde{H}_\ell[k] e^{-j2\pi f_\ell Tq} + W_m(k), \tag{1.42}
\]

for \(m = 0, 1, \ldots, M - 1\). Note that in (1.42) the time-varying nature of the channel is given by the factor \(e^{-j2\pi f_\ell Tq}\). For a static receiver there is no frequency shift for each path \((f_\ell = 0, \forall \ell)\) and the value of \(\sum_{\ell} \tilde{H}_\ell[k] e^{-j2\pi f_\ell Tq}\) does not depend on \(q\). In this case, (1.42) becomes (1.18).

The derivatives \(H^{(p)}\) are zero-mean Gaussian vectors and (1.26) holds. The normalized autocorrelation matrix of each vector \(H^{(p)}\) has entries [26]

\[
[R_f]_{m,\ell} = (1 + j2\pi f_s \tau_{\text{rms}}(m - \ell))^{-1}, \quad m, \ell = 0, 1, \ldots, M - 1, \tag{1.43}
\]

while the correlation among the derivatives is provided by the matrix \(R_c\), with elements

\[
[R_c]_{p,q} = \begin{cases} 
\frac{(-1)^{p+q} (p+q-1)!!((2\pi f_d/f_s)^2)^{p+q}}{p!q!} & \text{when } (p + q) \text{ is even} \\
0 & \text{otherwise}.
\end{cases} \tag{1.44}
\]

In this case, from (1.23) and (1.44) we observe that the power of the vectors \(\Xi^{(p)}H^{(p)}\) is exponentially decaying with \(p\), i.e.

\[
\sigma^{(p)^2} = E[\|\Xi^{(p)}\text{diag}\{H^{(p)}\}S(k)\|_m^2] = \frac{(2p - 1)!!}{(2p)!!p!} \left(\frac{2\pi f_d}{f_s}\right)^{2p}. \tag{1.45}
\]
1.3.3 Multiantenna OFDM system

We consider the combination of multicarrier modulation and multiple antennae. In particular, we focus our attention on a wireless system with two transmit antennae and two receive antennae.

Provided that the cyclic prefix is sufficiently long, the transmission channel is flat on each OFDM subcarrier. We indicate with $H_{m}^{(k,\ell)}$ and $G_{m}^{(k,\ell)}$ the frequency response of the transmit and interference channel, respectively, from antenna $k$ to antenna $\ell$ of the $m$th OFDM subcarrier. We denote as $\alpha_{1}^{(m)}$ and $\alpha_{2}^{(m)}$, respectively, the gain of subcarrier $m$ transmitted on antennae 1 and 2, respectively, as shown in Fig. 1.5.

1.4 Single Carrier Transmission

In single carrier transmission the digital data stream $a(n)$ is modulated on a single carrier frequency and sent on the channel. On a dispersive channel, intersymbol interference arises among different transmitted symbols. In order to face ISI, various solutions can be adopted. The minimization of the error probability of the detected signal, given the received signal, is obtained by filtering $r(n)$ with a filter matched to the channel filter and then applying a Maximum Likelihood (ML) detector, which can be efficiently implemented by the Viterbi algorithm, [6]. However, this solution may be exceedingly complex, and other suboptimal solutions include the linear equalizer or the decision feedback equalizer. In the first case the distortion introduced by the channel is compensated by an appropriate filtering of the received signal, while in the latter solution, a filter partially compensates for the channel and past decisions are used to remove the residual interference. For channels with long impulse responses, the realization of filtering as a
convolution in the time domain may still be computationally expensive. Instead, the frequency-domain implementation of filtering yields a reduced complexity. Hence, in order to develop the frequency domain DFE of Chapter 2 we introduce in this section a new transmission format for SC modulation, that allows a reduced-complexity frequency domain equalizer.

### 1.4.1 PN-extended transmission

Frequency domain equalization is based on the equivalence of the convolution of two sequences in the time domain and the product of their Fourier transforms. However, in general, the Fourier transform can not be performed on the entire received signal and frequency domain equalization must be performed on blocks, using the DFT. In this case, the equivalence between the time domain convolution and the frequency domain multiplication holds only if the transmitted signal forces the linear convolution with the channel impulse response to be circular [30]. We recall that, given the sequences \( \{h_\ell\}_{\ell=0}^{L} \) and \( \{s(n)\}_{n=0}^{M+L-1} \), their convolution (limited to the first \( M+L \) samples)

\[
z_n = \sum_{\ell=0}^{L} h_\ell s(n-\ell), \quad n = 0, 1, \ldots, M+L-1,
\]

is circular on blocks of size \( P \) if

\[
s(n) = s(n+P), \quad n = 0, 1, \ldots, L-1.
\]

In this case the element by element product of the \( P \)-size DFT of \( \{h_\ell\} \) and of the first \( P \) samples of \( \{s(n)\} \) yields the \( P \)-size DFT of samples \( \{z_L, z_{L+1}, \ldots, z_{L+P-1}\} \).

In order to force the linear convolution (1.46) to become circular is to extend each data block with a fixed sequence of symbols [31], for example a pseudo noise (PN) [6, pp. 724] sequence \( \{p_n\}_{n=0}^{L-1} \). As shown in Fig. 1.6, the new data block of \( M+L \) symbols is

\[
s(k) = [s(k(M + L)), s(k(M + L) + 1), \ldots, s(k(M + L) + (M + L) - 1)]
\]

\[
= [a(kM), a(kM + 1), \ldots, a(kM + M - 1), p_0, p_1, \ldots, p_{L-1}],
\]

where now the last \( L \) symbols are the PN sequence. Moreover, a PN extension is transmitted before the first data block. As indicated in Fig. 1.7, while the cyclic extension copies part of the information data at the beginning of each block, the PN extension philosophy is to interleave information blocks with a known PN sequence. Since in a continuous transmission

\[
s[k(M + L) + M + n] = s[k(M + L) + (M + L) + M + n] = p_n,
\]

\[
n = 0, 1, \ldots, L-1,
\]

we have (1.47) verified for blocks of size \( P = M + L \) of the signal \( \{s(n)\} \). Hence, if we indicate the \( P \)-size DFT of \( r(k) = [r(kP), r(kP + 1), \ldots, r(kP + P - 1)] \) with

\[
R(k) = [R_{kP}, R_{kP+1}, \ldots, R_{kP+P-1}],
\]

(1.50)
1.4. SINGLE CARRIER TRANSMISSION

Figure 1.6: SC transmitter with PN-extension.

<table>
<thead>
<tr>
<th>$d_{n-1}(k)$</th>
<th>$d_n(k)$</th>
<th>$d(k)$</th>
<th>$d(k+1)$</th>
<th>$d(k+1)$</th>
</tr>
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<tbody>
<tr>
<td>$d_{n-1}(k)$</td>
<td>$d_n(k)$</td>
<td>$d(k)$</td>
<td>$d(k+1)$</td>
<td>$d(k+1)$</td>
</tr>
</tbody>
</table>

a)  

<table>
<thead>
<tr>
<th>$p_0$</th>
<th>$p_i$</th>
<th>$p_{L-1}$</th>
<th>$a(kM)$</th>
<th>$a(kM+1)$</th>
<th>$a(kM+M-1)$</th>
<th>$a((k+1)M)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_0$</td>
<td>$p_i$</td>
<td>$p_{L-1}$</td>
<td>$a(kM)$</td>
<td>$a(kM+1)$</td>
<td>$a(kM+M-1)$</td>
<td>$a((k+1)M)$</td>
</tr>
</tbody>
</table>

b)  

Figure 1.7: Data block structure for a) cyclic extension and b) PN extension.
the DFT of $s(k)$ with $S(k)$ and the DFT of $\{h_\ell\}$ with $H = [H_0, H_1, \ldots, H_{P-1}]$. $R(k)$ is the multiplication of $S(k)$ and $H$, element by element.

Now, the PN extension is suitable for use with a DFE, since in a continuous transmission, the interference on the first $L$ symbols of each block $r(k)$ is generated by the PN extension symbols, which are fixed and known at the receiver.

On the other hand, in the case of the cyclic-extended transmission, the interference on the first $L$ samples of the cyclic-extended signal is generated by the symbols $\{a_{kM+M-L}, \ldots, a_{kM+M-1}\}$. Hence, if we apply a scalar DFE [6, pp. 621–22] to remove this interference, the feedback filter should already know the last symbols of the current data block, which is impossible. Elaborate DFE, operating on a per-block basis have been proposed [32,33] and they can be used also for a cyclic extended transmission. However, even if they can give a better performance than OFDM, their signal processing and design complexity is much greater (see Section 2.4.3).

Note that the cyclic extension and the PN extension have approximately the same bandwidth efficiency $\frac{M}{M+T}$, for a transmission with numerous blocks. Indeed, the PN extension format includes one more block of $L$ symbols than the cyclic extension format, at the beginning of the transmission.

For both extensions, a higher $M$ yields a greater bandwidth efficiency, but also a higher latency. In general, for time-varying channels, $M$ must be chosen so that the channel could be considered static on each frame. Furthermore, the PN extension can be reduced to zero-padded block transmission [33]. The choice of the sequence may be done according to other criteria, such as the optimization of the peak to average power ratio.
Chapter 2

Frequency-Domain Decision Feedback Equalizer for Single Carrier Transmission

Frequency-domain equalization of single-carrier transmitted signals has been often been considered as an efficient way to perform equalization. However, up to now, only linear equalization has been considered [8–10], while the better performing decision feedback equalizer has not been efficiently realized in the frequency domain.

In this chapter, we present a frequency-domain DFE (FD-DFE) for SC systems which makes use of the PN-extended transmission format presented in Section 1.4.1. The scheme is a non-adaptive DFE where the feedforward part is implemented in the frequency domain, while feedback operates in the time-domain. While gaining the benefits of a DFE in terms of performance, the new scheme has also the advantage of using DFT to reduce complexity of signal processing. For the design of the FD-DFE we propose an efficient algorithm, which directly operates in the frequency domain and is optimal in terms of mean square error.

Another contribution of this chapter is the comparison of FD-DFE and OFDM in two scenarios. The first comparison is in terms of achievable bit rate (ABR) [34]. This corresponds to a scenario where, for a given transmission channel, bit loading is performed on both systems. By computer simulations, we conclude that OFDM and SC FD-DFE yield a similar ABR. This result is also confirmed in a particular case by analytical derivations. When adaptive modulation is considered for OFDM, FD-DFE has a performance close to OFDM. In the second scenario, no channel loading is assumed and performance is compared in terms of frame error rate. In this case, with fixed rate modulation, it is seen that FD-LE significantly outperforms OFDM with fixed modulation [9]. Here we observe that FD-DFE yields an additional 3dB gain over FD-LE.
2.1 Frequency domain DFE

As we have shown in Section 1.4.1, the insertion of a PN-extension of length $L$ at the end of each block of $M$ data symbols, reduces the linear convolution to a circular convolution, thus allowing the frequency-domain equalization at the receiver. In this chapter we assume a PN-extended transmission on a single data link, having both the receiver and the transmitter one antenna.

The FD-DFE structure is shown in Fig. 2.1. The feedforward (FF) filter of the FD-DFE operates in the frequency domain on blocks of size $P$, while the feedback (FB) filter operates in the time domain. In fact, the frequency domain filtering must be performed on a per-block basis, while the feedback section must be fed with the previous detected symbols whose decision is performed in the time domain. Note that also for block equalization [33], the feedback transformation must be a strictly upper triangular matrix.

As shown in Fig. 2.1, after the DFT of the received samples, the FD feedforward filter, with coefficients $\{G_{\text{FF},n}\}$, $n = 0, 1, \ldots, P - 1$, is applied to yield the block signal $Y(k)$ with elements

$$ Y_{kP+n} = R_{kP+n}G_{\text{FF},n}, \quad n = 0, 1, \ldots, P - 1. \quad (2.1) $$
Through the inverse DFT, block $Y(k)$ is then transformed in the time domain to give

$$y(k) = [y_{kP}, y_{kP+1}, \ldots, y_{kP+P-1}] . \tag{2.2}$$

Let’s indicate with $\{H_n\}$ the $P$-size frequency response of the channel. Let’s define the impulse response of the cascade of the equivalent discrete-time channel impulse response and the feedforward equalizer as the IDFT\(^1\) of the product of $\{H_n\}$ and $\{G_{FF,n}\}$, i.e.

$$u_\ell \equiv \sum_{n=0}^{P-1} G_{FF,n} e^{j2\pi n \frac{\ell}{P}}, \quad \ell = 0, 1, \ldots, L_u, \tag{2.3}$$

assuming it has a support of $L_u + 1$ coefficients. Then, each block $y(k)$ can be written as the circular convolution of $s(k)$ and $\{u_\ell\}$ plus noise, namely

$$y_{kP+n} = (s \otimes u)_{kP+n} + \tilde{w}_{kP+n}, \quad n = 0, 1, \ldots, P-1, \tag{2.4}$$

where, from Fig. 2.1 and (2.1),

$$\tilde{w}(kP + n) = \sum_{\ell=0}^{L_u} s(kP + \ell)u_\ell e^{j2\pi n \frac{\ell}{P}} e^{j2\pi n \frac{\ell}{P}} \quad n = 0, 1, \ldots, P-1. \tag{2.5}$$

In (2.4) we note that if $L_u \leq L$ then the condition (1.47) holds for the useful part of $\{y_n\}$, which can be written as a linear convolution, and

$$y_{kP+n} = \sum_{\ell=0}^{L_u} s(kP + n - \ell)u_\ell + \tilde{w}(kP + n), \quad n = 0, 1, \ldots, M-1. \tag{2.6}$$

Note that in (2.6) the last $L$ samples of each block $y(k)$ have not been considered, since they are a noisy and interfered version of the PN sequence, which is already known. On the other hand, if $L_u > L$ then (2.6) does not hold and $\{y_{kP+n}\}$ depends also on $\{s[kP + (n - \ell) \mod P]\}$, $\ell = L+1, L+2, \ldots, L_u$. In this case the feedback filter will not be able to cancel all the interference.

As mentioned previously, the feedforward section operates in the frequency domain, while the feedback section operates in the time domain. Let’s indicate the estimated data sequence with $\{\hat{a}(n)\}$ and the extended estimated sequence with $\{\hat{s}(n)\}$. From (1.48) it holds

$$\hat{s}(kP + n) = \begin{cases} \hat{a}(kM + n), & n = 0, 1, \ldots, M-1, \\ p(n-M), & n = M, M+1, \ldots, P-1. \end{cases} \tag{2.7}$$

Then, if $\{g_{FB,\ell}\}, \ell = 1, 2, \ldots, N_{FB}$, are the coefficients of the FB filter, the signal at the input of the decision element is

$$\hat{a}(kM + n) = y_{kP+n} + \sum_{\ell=1}^{N_{FB}} g_{FB,\ell}\tilde{s}_{kP+n-\ell}, \quad n = 0, 1, \ldots, M-1. \tag{2.8}$$

---

\(^{1}\)To simplify the notation, the normalization factor $1/P$ has been included in the sequence $\{G_{FF,n}\}$. 

Note that, as indicated in Fig. 2.1, for each block the first $N_{FB}$ data symbols, which initialize the feedback part of the DFE, should coincide with the PN symbols $\{p_n\}$.

Since the FD-DFE operates on a per-block basis, error propagation through the FB filter is limited to one block. This is an important advantage over the time-domain DFE (TD-DFE), [35, pp. 365], when the system operates at low $SNR$ values. Moreover, it has a reduced complexity when compared with other frequency domain equalizers [11, 30] that need $2M$-size DFTs. On the other hand, it also has all drawbacks of block-based systems in time-varying channels.

Note also that the FD-DFE scheme is an efficient realization of a block DFE since the DFT, the feedforward equalization and the IDFT can be combined into one single matrix multiplication.

### 2.2 DFE design

We start from the FD channel estimate which can be obtained directly as in [22] or by taking the $P$-size DFT of the channel impulse response estimate.

**Zero Forcing FD-DFE.** According to the ZF criterion, all interferers must be canceled by the feedback part. If the support of $\{u_\ell\}$ is $L_u \leq L$, (2.6) holds true and interference can be canceled by the feedback filter. Hence, let’s set $L_u = L$. The zero-forcing condition can be expressed as

$$u_0 = 1, \quad u_\ell = 0, \quad \text{for} \quad \ell < 0, \ell > L,$$

and only coefficients $u_\ell$, $\ell = 1, 2, \ldots, L$, can be chosen freely. Once the $\{u_\ell\}$ coefficients are known, by selecting $N_{FB} = L$ and

$$g_{FB,\ell} = -u_\ell, \quad \ell = 1, 2, \ldots, L,$$

the feedback filter cancels all residual interferers. In turn, from (2.3) assuming $\mathcal{H}_n \neq 0$ and using (2.10), the coefficients of the feedforward filter $\{G_{FF,n}\}$ can be computed as

$$G_{FF,n} = \frac{1}{\mathcal{H}_n} \sum_{\ell=0}^{L} u_\ell e^{-j2\pi \frac{\ell n}{P}} = \frac{1}{\mathcal{H}_n} \left( 1 - \sum_{\ell=1}^{L} g_{FB,\ell} e^{-j2\pi \frac{\ell n}{P}} \right),$$

where $n = 0, 1, \ldots, P - 1$.

Here, the $L$ coefficients $g_{FB,\ell}$, $\ell = 1, 2, \ldots, L$, are chosen to minimize the power of the filtered noise, which, from (2.3) and under the condition (2.9), can be written as

$$J_{ZF} = \frac{\sigma_w^2}{P} \sum_{n=0}^{P-1} |G_{FF,n}|^2 = \frac{\sigma_w^2}{P} \sum_{n=0}^{P-1} \left| \frac{1}{\mathcal{H}_n} \left( 1 - \sum_{\ell=1}^{L} g_{FB,\ell} e^{-j2\pi \frac{\ell n}{P}} \right) \right|^2.$$

Indicating with $g_{FB,\ell}^{(R)}$ and $g_{FB,\ell}^{(I)}$ the real and imaginary parts of $g_{FB,\ell}$, respectively, since $J_{ZF}$ is a strictly convex function of $g_{FB,\ell}^{(R)}$ and $g_{FB,\ell}^{(I)}$, its minimum is attained when the gradient is set at
2.2. DFE DESIGN

zero, namely
\[ \frac{\partial J_{ZF}}{\partial g_{FB,\ell}} = \frac{\partial J_{ZF}}{\partial g_{FB,0}} + j \frac{\partial J_{ZF}}{\partial g_{FB,\ell}} = 0, \quad \ell = 1, 2, \ldots, L. \] (2.13)

Let \( g_{FB} = [g_{FB,1}, g_{FB,2}, \ldots, g_{FB,L}]^T \) (where \((\cdot)^T\) denotes the transpose) and let’s define the following matrix \( A_{ZF} \) and column vector \( b_{ZF} \)
\[ [A_{ZF}]_{m,\ell} = \sum_{n=0}^{P-1} e^{-j2\pi n(\ell-m)/P} \frac{|H_n|^2}{|H_n|^2}, \quad 1 \leq m, \ell \leq L, \] (2.14)
\[ [b_{ZF}]_m = \sum_{n=0}^{P-1} e^{j2\pi nm/P} \frac{|H_n|^2}{|H_n|^2}, \quad 1 \leq m \leq L, \] (2.15)
then from (2.13) we obtain the linear system of \( L \) equations with \( L \) unknowns \( A_{ZF}g_{FB} = b_{ZF} \).

Since \( A_{ZF} \) is a Toepliz matrix, a reduced complexity algorithm can be used to solve the problem. Additionally, observe that the elements of both \( A_{ZF} \) and \( b_{ZF} \) can be computed as the \( P \)-size DFT of \( \left\{ \frac{1}{|H_n|^2} \right\} \), and FFT can be used. As a matter of fact, \( 2L \)-size DFTs can be considered.

Minimum MSE FD-DFE. According to the MMSE criterion, the coefficients of the FF and FB filters are chosen to minimize the sum of the power of the filtered noise, and the power of the residual interference. In particular, the mean square error at the detector is given by
\[ J_{MMSE} = E[|\tilde{a}(n) - a(n)|^2]. \] (2.16)

By assuming that the past decisions are correct and that \( N_{FB} \leq L \) and \( L_u \leq L \), from (2.6) and (2.8) we obtain
\[ J_{MMSE} = E \left[ \sum_{\ell=0}^{L_u} s(kP + n - \ell)u_\ell + \sum_{\ell=1}^{N_{FB}} s(kP + n - \ell)g_{FB,\ell} + \tilde{w}_{kP+n} - a(kM + n)^2 \right], \quad n = 0, 1, \ldots, M - 1. \] (2.17)

Now, we rewrite (2.18) in the frequency domain. Firstly, we introduce the \( P \)-size DFT of the FB filter
\[ G_{FB,p} = \sum_{\ell=1}^{N_{FB}} g_{FB,\ell} e^{-j2\pi \ell p/P}, \quad p = 0, 1, \ldots, P - 1. \] (2.18)

Moreover, from (2.3) the gain of the useful data at the decision point can be written as
\[ u_0 = \sum_{p=0}^{P-1} \mathcal{H}_p G_{FF,p}. \] (2.19)
Hence, from (2.18), (2.18) and (2.19), according to the minimum mean square error criterion, the functional to be minimized is

$$J_{\text{MMSE}} = \frac{1}{P} \sum_{p=0}^{P-1} \left[ \sigma_w^2 |G_{\text{FF},p}|^2 + \sigma_a^2 \left| 1 - (G_{\text{FF},p} \mathcal{H}_p + G_{\text{FB},p}) \right|^2 \right], \quad (2.20)$$

where $\sigma_a^2$ is the power of the signal $\{a(n)\}$.

Due to the $PN$-extension structure, the FB filter is not able to cancel more than $L$ interferers, hence, we must impose that $N_{\text{FB}} \leq L$. Here we consider the case $N_{\text{FB}} = L$. In order to compute the design of the FF and FB filters, we write the functional $J_{\text{MMSE}}$ only as a function of $\{G_{\text{FB},p}\}$. In particular, we observe that, given the feedback filter, by applying the gradient method to (2.20), the feedforward filter is given by

$$G_{\text{FF},p} = \frac{\mathcal{H}_p^\ast(1 - G_{\text{FB},p})}{|\mathcal{H}_p|^2 + \sigma_a^2/\sigma_w^2}, \quad p = 0, 1, \ldots, P - 1, \quad (2.21)$$

where $(\cdot)^\ast$ indicates the complex conjugate. Inserting now (2.21) in (2.20) we obtain

$$J_{\text{MMSE}} = \frac{\sigma_w^2}{P} \sum_{p=0}^{P-1} \left| 1 - G_{\text{FB},p} \right|^2 \frac{1}{|\mathcal{H}_p|^2 + \sigma_a^2/\sigma_w^2} \left| 1 - \sum_{\ell=1}^{L} g_{\text{FB},\ell} e^{-j2\pi \ell p/P} \right|^2 \quad (2.22)$$

using (2.18).

We define $g_{\text{FB}} = [g_{\text{FB},1}, g_{\text{FB},2}, \ldots, g_{\text{FB},L}]^T$ and the following matrix $\mathbf{A}_{\text{MMSE}}$ and column vector $\mathbf{b}_{\text{MMSE}}$ as

$$[\mathbf{A}_{\text{MMSE}}]_{m,\ell} = \sum_{n=0}^{P-1} \frac{e^{-j2\pi n(\ell-m)/P}}{|\mathcal{H}_n|^2 + \sigma_a^2/\sigma_w^2}, \quad 1 \leq m, \ell \leq L, \quad (2.23)$$

$$[\mathbf{b}_{\text{MMSE}}]_m = \sum_{n=0}^{P-1} \frac{e^{j2\pi nm/P}}{|\mathcal{H}_n|^2 + \sigma_a^2/\sigma_w^2}, \quad 1 \leq m \leq L. \quad (2.24)$$

As in the ZF design, by applying the gradient method to minimize $J_{\text{MMSE}}$, we obtain the linear system of $L$ equations with $L$ unknowns $\mathbf{A}_{\text{MMSE}} g_{\text{FB}} = \mathbf{b}_{\text{MMSE}}$. We note that the complexity of this method is similar to the ZF. Once the FB filter is determined, the FF filter is given by (2.21) and (2.18).

Note that the MMSE solution will reduce to the ZF solution when $\sigma_w^2 \to 0$.

### 2.3 Channel loading and coding

To compare the performance of SC and OFDM systems, we consider two scenarios.
In the first scenario, the transmitter knows the channel impulse response and maximizes the capacity of the system by selecting the constellation for each subcarrier of the OFDM and for the unique channel of the SC modulation. This operation is named channel loading [36].

In some situations channel loading is not possible or unsuitable. For example, it can not be used in a broadcast transmission and it is not suitable (especially for OFDM) when it requires a considerable amount of information to be sent back to the transmitter. Note also that in a wireless mobile scenario the channel loading must be updated whenever the transmission channel changes significantly. Therefore we considered also a second scenario where channel loading is not performed and the constellations are fixed. Now the performance is driven by modulation and coding structure. The symbol interleaver plays also a significant rule for OFDM [8], since it reduces burst errors due to adjacent subcarriers experiencing similar fading. We compare the performance of coded OFDM and SC systems in terms of frame error rate (FER).

### 2.3.1 Channel loading and coding scenario

In the first scenario, performance of OFDM and SC systems are compared under the hypothesis of perfect channel loading, with a continuous varying constellation size and an optimum coding. Channel loading is widely used in wired transmissions [34]. More recently it has also been considered for wireless applications since it can ease the problem of slowly time-varying channels by exploiting the variation of the signal quality [36]. Hence, we evaluate here the performance of the proposed systems using as performance measure the achievable bit rate (ABR), [34]. For the OFDM system, let us define the signal to distortion plus noise ratio at the decision device of the $m$th subcarrier,

$$\gamma_m = \frac{\sigma_a^2 |H_m|^2}{\sigma_w^2},$$

where \( \{H_m\} \) is the $M$-size DFT of \( h_\ell \). The $ABR$ is defined as

$$ABR_{OFDM} = \frac{1}{T} \sum_{m=0}^{M-1} \log_2(1 + \gamma_m),$$

where $1/T$ is the symbol rate.

For SC modulation, by assuming $N_{FB} \leq L$ and $L_u \leq L$, we define the signal to distortion plus noise ratio at the decision device as

$$\gamma = \frac{\sigma_a^2 |u_0|^2}{J},$$

where $J$ is given by (2.12) or (2.18) and the value of $u_0$ varies according to the equalizer: for ZF FD-DFE $u_0 = 1$ (see (2.9)), while for MMSE FD-DFE $u_0$ is given by (2.19). Note that we are neglecting here the error propagation phenomena due to the DFE since we do not simulate the channel loading and therefore we do not know the constellation and the code of the transmitted signal.
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For SC systems, loading is performed by choosing only one constellation instead of $M$ as for the OFDM, hence the $ABR$ is given by

$$ABR_{FD-DFE} = \frac{1}{T_d} \log_2(1 + \gamma),$$  \hspace{1cm} (2.28)

where $T_d = T/M$.

In the Appendix we show analytically that assuming the number of the feedback coefficients $N_{FB} = P$, then the $ABR$ of a ZF FD-DFE is very close to that of an OFDM. By simulations we verify that the ABR of MMSE FD-DFE is very close to that of an OFDM even when $N_{FB} = L$.

2.3.2 Coding scenario

In the coding scenario, we consider a convolutional encoder with rate $R$, free distance $\delta_f$ and BPSK constellation. Let’s indicate with $\{q_k\}$ the number of codewords with distance $\delta_f + k$, and with $Z(\gamma) = e^{-\frac{\gamma}{2}}$ the Bhattacharyya parameter [6] for an AWGN channel with BPSK modulation.

For an SC system, to partially consider the problem of error propagation in the DFE, under the assumptions $L_u \leq L$ and $N_{FB} \leq L$, we introduce

$$\hat{J} = E[|\hat{a}(n) - a(n)|^2],$$  \hspace{1cm} (2.29)

where $\hat{a}(n)$ is the detected symbol in correspondence of the equalizer output. Indeed, $\hat{J}$ is similar to $J_{ZF}$ and $J_{MMSE}$ given by (2.12) and (2.18), respectively, with the addition that now the error propagation phenomena of the DFE are considered. The signal to distortion plus noise ratio at the decision device is defined as

$$\hat{\gamma} = \frac{\sigma_a^2|u_0|^2}{\hat{J}},$$  \hspace{1cm} (2.30)

where $\hat{\gamma}$ is estimated by simulations.

A good approximation of the $FER$ with frames of length $N$ is given by [37]

$$FER \approx P(\hat{\gamma}) = \frac{\sum_{k=0}^{K_d} \frac{N}{2^k} q_k Z^{\delta_f + k}(\hat{\gamma})}{1 + \sum_{k=0}^{K_d} \frac{N}{2^k} q_k Z^{\delta_f + k}(\hat{\gamma})},$$  \hspace{1cm} (2.31)

where $\beta$ is a correction factor to fit simulated data, $K_d$ is given by (2.30) and $K_d = 16$ for the HIPERLAN-2 channel code, [1].

We observe that even with the new definition (2.29), in the analytical results of (2.31), we are neglecting the correlation among errors at the input of the decoder, due to the error-propagation phenomenon of the DFE. In any case, as analyzed in [35, pp. 335–370], the output error of the DFE is approximately white, and in our setup an interleaver is inserted before decoding
(see Section 2.4) to further decorrelate errors. For these reasons, at detector input the resulting distortion can be approximated as white noise.

By simulations we first derived values of FER for lower SNRs. The value of the $\beta$ parameter in (2.31) was then derived by fitting the curve. Lastly, the curves were extended to higher SNRs using (2.31).

For an OFDM system we use the analytical method proposed by Nanda and Rege [37] in a different system. Let’s indicate with $D$ the length of the shortest error event of the code. Let’s define the partial terms

$$
\gamma^{(n)}_{\text{eff}} = \frac{1}{D} \sum_{\ell=0}^{D-1} \gamma^{(n+\ell \mod M)}
$$

where $n = 1, 2, \ldots, N - D + 1$. A good estimate of the FER can be obtained by

$$
FER = P\left(\min_{n=1,\ldots,N-D+1}\left\{\gamma^{(n)}_{\text{eff}}\right\}\right),
$$

where the function $P(\cdot)$ is defined in (2.31).

In our analysis we assumed a frame length of $N = M$.

### 2.4 Performance comparison

We have considered a HIPERLAN-2 scenario with a symbol rate of $1/T_d = 20$ MHz. The systems’ performance was obtained by simulation on a typical environment considered for HIPERLAN-2, i.e. a no line of sight multipath channel. The channel has a Rayleigh characteristic with an exponential decaying power profile with an rms delay spread of 100ns, according to channel model $B$ in [38]. The data block size is $M = 128$ and $L = 16$, (option II of model $B$ [38]). The interleaver of the HIPERLAN-2 standard [1] is included. Frequency and time synchronization are assumed perfect. The channel impulse response is assumed known at the receiver and time invariant over the transmission of a block. Schemes considered are OFDM, FD-LE (as described in [10]) and FD-DFE designed both with a ZF and with an MMSE method. $N_{FB} = L$ for all DFE schemes.

#### 2.4.1 Channel loading and coding scenario

With reference to the scenario with channel loading, we assume that at the input of the feedback past decisions are correct, since we do not simulate the channel loading and therefore do not know the constellation.

The complementary cumulative distribution function (cCDF) of the $ABR$ is used to compare the different systems in Fig. 2.2. The SNR between the average power of the useful received signal (with respect to the channel variations) and the power of the noise, is assumed
to be 12dB. It has already been shown that when channel loading is performed, the OFDM exhibits a sharp increase of performance over FD-LE [9]. This is confirmed by Fig. 2.2. However, we observe that OFDM and MMSE FD-DFE yield a very close performance (see also the Appendix). The MMSE FD-DFE instead has better performance than the ZF FD-LE, for the usual ZF problem of neglecting the presence of noise in the design.

For the various systems, the mean ABR vs. SNR is shown in Fig. 2.3. The same observations made for Fig. 2.2 can be repeated here. In particular we note that MMSE FD-DFE and OFDM yield a very close performance. Additionally, observe that with respect to FD-LE, the feedback part of the DFE becomes more relevant as the SNR increases.

2.4.2 Coding scenario

With reference to the coding scenario, the BPSK modulation is considered and the standard (133s, 171s) HIPERLAN-2 convolutional code [1] is used for all systems. The analytical evaluation of FER for both OFDM and FD-DFE fitted the simulation results for \( \beta = 1.2 \). For the DFEs in the presence of coding, the hard-detected coded data are used as input to the feedback filter. Hence, error propagation phenomena are considered.

Fig. 2.4 shows the cCDF of the FER for the different coded systems. We observe that the SC system with an MMSE FD-DFE performs similarly to OFDM. Again, MMSE FD-DFE outperforms the MMSE FD-LE.
2.4. PERFORMANCE COMPARISON

Figure 2.3: Mean $ABR$ for different equalizer structures as a function of the $SNR$.

Figure 2.4: Complementary CDF of the frame error rate, in the presence of coding, for different equalizer structures. $SNR = 12$dB.
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The mean FER for various values of SNR is shown in Fig. 2.5. The MMSE FD-DFE has a better performance than ZF FD-DFE at low SNRs. For SNR \( \geq 9 \) dB we see that OFDM outperforms MMSE FD-DFE by about 0.5dB.

Finally, the performance of uncoded systems is shown in Fig. 2.6, in terms of averaged FER for various values of SNR. In this case, the performance of FD-LE is much better than OFDM, while the MMSE FD-DFE gains about 3dB over FD-LE.

2.4.3 Computational complexity

Computational complexity of the system, in terms of the number of complex multiplications per output sample, is reported in Table 2.1 for OFDM and SC FD-DFE. We also included the complexity of the block DFE [33], denoted as B-DFE. We have considered that a \( P \)-size DFT requires \( \frac{P}{2} \log_2(P) - P \) complex multiplications. For the OFDM system we have also included the IDFT of the transmitter. The channel estimation was not considered because both OFDM and FD-DFE need the same estimate of the channel frequency domain response which can be obtained by known techniques (see [22] and references therein). We see that, when compared to OFDM, FD-DFE has an increased complexity due to the additional FB filter. Moreover, as already mentioned, the B-DFE has a significantly higher complexity than both OFDM and FD-DFE.
2.4. PERFORMANCE COMPARISON

Figure 2.6: Mean frame error rate, in the absence of coding, for different equalizer structures as a function of the SNR.

Table 2.1: Computational complexity of the system, in terms of number of complex multiplications per output sample, of OFDM and SC systems with block and FD equalization.

<table>
<thead>
<tr>
<th>Structure</th>
<th>Computational complexity of the system</th>
</tr>
</thead>
<tbody>
<tr>
<td>OFDM</td>
<td>( \log_2 M )</td>
</tr>
<tr>
<td>SC</td>
<td>( \frac{P}{M} \log_2 P + N_{FB} )</td>
</tr>
<tr>
<td></td>
<td>( 2P )</td>
</tr>
<tr>
<td></td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>256</td>
</tr>
</tbody>
</table>
Table 2.2: Computational complexity, in terms of numbers of complex multiplications, due to the equalizer design.

<table>
<thead>
<tr>
<th>Structure</th>
<th>Computational complexity of the equalizer design</th>
<th>HIPERLAN-2 scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>OFDM</td>
<td>$\mathcal{O}(M)$</td>
<td>128</td>
</tr>
<tr>
<td>SC ZF FD-DFE</td>
<td>$\mathcal{O}(L^2 + \frac{L}{2} \log_2 L + \frac{P}{2} \log_2 P)$</td>
<td>804</td>
</tr>
<tr>
<td>MMSE FD-DFE</td>
<td>$\mathcal{O}(L^2 + \frac{P}{2} \log_2 L + \frac{P}{2} \log_2 P)$</td>
<td>804</td>
</tr>
</tbody>
</table>

Table 2.2, instead, compares the computational complexity of the equalizer design. Given the channel frequency response, the equalizer design for OFDM is carried out by one complex division per subcarrier. For the FD-DFE, firstly, matrix $A_{ZF}$ (2.14) and vector $b_{ZF}$ (2.15) or matrix $A_{MMSE}$ (2.23) and vector $b_{MMSE}$ (2.24) can be computed through DFTs. Then a $L_u$-size linear system of equations must be solved, whose complexity is on the order of $\mathcal{O}(L_u^2)$ multiplications.

Indeed, the filter design for OFDM is much less complex than that for FD systems. However, when operating in a variable rate mode with channel loading, other factors as the number of parameters that must be fed back to the transmitter should be considered. In fact, while for the FD-DFE only one constellation size must be fed back to the transmitter, for OFDM $M$ constellation sizes, one for each subcarrier, are needed. In this case, complexity of channel loading must be inserted.
Appendix

Zervos and Kalet [39] have already proven that for an unconstrained length ZF DFE and high SNRs, OFDM and SC FD-DFE have the same capacity. Here we show that if $N_{FB} = P$, the ABRs of ZF FD-DFE and of OFDM are very close. The same conclusion is valid for MMSE FD-DFE, because of its intrinsic superior performance over ZF FD-DFE.

Now we show that $ABR_{OFDM} \approx ABR_{FD-DFE}$ by proving that $ABR_{FD-DFE} \leq ABR_{OFDM}$ and at the same time $ABR_{FD-DFE} \gtrsim ABR_{OFDM}$.

As observed by Bingham [7], comparing (2.26) to the water pouring formulas, we conclude that if the number of subcarriers $M$ is sufficiently high, the $ABR_{OFDM}$ is the highest possible $ABR$ for any system with the same bandwidth. Therefore we conclude that

$$ABR_{FD-DFE} \leq ABR_{OFDM}.$$  \hfill (2.34)

Now, we show that under the condition $N_{FB} = P$ then

$$ABR_{FD-DFE} \gtrsim ABR_{OFDM}.$$  \hfill (2.35)

From (2.26) and (2.28) we obtain

$$2^{T_dABR_{OFDM}} = \left[ \prod_{m=0}^{M-1} \left( 1 + \frac{\sigma_a^2}{\sigma_w^2} |H_m|^2 \right) \right]^{1/M},$$  \hfill (2.36)

and since $N_{FB} > L$, (2.8) becomes

$$\hat{a}(kM + n) = y_{kP+n} + \sum_{\ell=1}^{\min(N_{FB},L+n)} g_{FB,\ell} \hat{s}(kP + n - \ell),$$  \hfill (2.39)

because in the first $N_{FB} - L$ symbols at most $L + n$ interferers can be subtracted. In this case the mean square error

$$J_n = E[|\hat{a}(n) - a(n)|^2],$$  \hfill (2.40)

depends on the symbol index $n$ as $\gamma_n = \sigma_a^2 |u_0|^2 / J_n$. In particular, from (2.38) we can infer that $\gamma_{n+1} \geq \gamma_n$. When $n = M$ all interferers can be canceled and a ZF solution exists.
Now we will compute the value of $\gamma_M$ that will give an upper bound on the SC FD-DFE performance.

From (2.12) we minimize

$$J_{ZF} = \frac{\sigma_w^2}{P} \sum_{p=0}^{P-1} \frac{|1 - G_{FB,p}|^2}{|H_p|^2}$$

only under the constraint $g_{FB,0} = 0$, i.e. $\sum_{p=0}^{P-1} G_{FB,p} = 0$.

The functional to be minimized becomes

$$\frac{\sigma_w^2}{P} \sum_{p=0}^{P-1} \frac{|1 - G_{FB,p}|^2}{|H_p|^2} + \lambda \sum_{p=0}^{P-1} G_{FB,p}, \tag{2.41}$$

where $\lambda$ is the Lagrange multiplier. Applying the gradient method to (2.41) we obtain

$$G_{FB,p} = 1 - \frac{|H_p|^2}{\frac{1}{P} \sum_{p=0}^{P-1} |H_p|^2} \tag{2.42}.$$

Therefore (2.37) yields

$$2T_{d ABR_{FD-DFE}} \approx 1 + \frac{\sigma_a^2}{\sigma_w^2} \left( \frac{1}{P} \sum_{p=0}^{P-1} |H_p|^2 \right). \tag{2.43}$$

Since $\frac{1}{P} \sum_{p=0}^{P-1} |H_p|^2 = \frac{1}{M} \sum_{m=0}^{M-1} |H_m|^2$, we obtain

$$2T_{d ABR_{FD-DFE}} \approx 1 + \frac{\sigma_a^2}{\sigma_w^2} \left( \frac{1}{M} \sum_{m=0}^{M-1} |H_m|^2 \right). \tag{2.44}$$

Now, the arithmetic mean is lower bounded by the geometric mean [40] and it holds true

$$2T_{d ABR_{FD-DFE}} \geq \left[ \prod_{m=0}^{M-1} \left( \frac{\sigma_a^2}{\sigma_w^2} |H_m|^2 + 1 \right) \right]^{1/M} \tag{2.45}.$$

Therefore, we conclude that (2.35) holds true.
Chapter 3

Signal Processing for Multicarrier Mobile Communications

In the transmission of OFDM signals on a time-varying channel, orthogonality among different subcarriers is disrupted and interchannel interference (ICI) arises. This affects both data detection and channel estimation [41–43].

A decision feedback ICI cancellation scheme (DF-ICI) has been recently proposed [26] which performs equalization in two stages. Initially, a standard one-tap per subcarrier equalizer is applied before data detection. Then, the detected data are used to generate the interference which is then cancelled from the received signal. As the initial decision on data is affected by interference, this scheme is not suited for high-speed transmissions with dense constellation, because error propagation strongly limits performance.

In this chapter we consider a wireless broadband OFDM transmission using long OFDM blocks with dense constellations, without a training sequence. The reference scenario is the mobile reception of the DVB-T modes [44] conveying the highest rates. The objective is the design of a receiver that allows reception of the DVB-T signal at a speed higher than 100 km/h, while using reduced-complexity techniques. In this respect we propose an iterative ICI cancellation scheme where the error propagation phenomenon is mitigated by deleting progressively the interference and increasing gradually the reliability of the data estimate. An extension of the new scheme to a two-antenna architecture is also considered, where signals are appropriately combined in order to exploit the space diversity.

3.1 Iterative ICI cancellation

According to the model of Section 1.3.2, the received signal vector \( \mathbf{R}(k) \) is described as the contribution of \( \mathcal{P} \) derivatives of the channel. Since in this chapter we consider operations on a single OFDM block, we drop the index \( (k) \). Indeed, we recall that from (1.25) the leakage matrices \( \Xi^{(p)} \) are not diagonal, hence the channel variations yield ICI on the receive signal. The
use of a linear equalizer to compensate the ICI, as proposed in [14], has a significant complexity since the equalization matrix must be adapted to the channel variations. A decision feedback equalizer (DFE) is considered here instead, which performs the cancellation of the ICI term.

In order to obtain a simple scheme, no feedforward operation is applied. Moreover, since the cancellation operated by the feedback requires the knowledge of the transmitted data and error propagation phenomena may significantly limit the performance of the DFE, we consider an iterative DFE. At each iteration the feedback filters are adapted to the reliability of the past detected and decoded data, in order to progressively cancel the interference, while reducing the effects of error propagation. At the \( n \)th iteration, the detected data in the \((n-1)\)th iteration \( \hat{D}^{(n-1)} \) are used to partially cancel the interference through the matrix filter \( B^{(n)} \), to yield

\[
\dot{R}^{(n)} = R - B^{(n)} \hat{D}^{(n-1)}, \quad n = 1, 2, \ldots, N_I,
\]

where \((N_I - 1)\) is the total number of iterations. The initial conditions are \( B^{(1)} = 0 \) and \( \hat{D}^{(0)} = 0 \). The resulting iterative scheme is shown in Fig. 3.1. Note that when \( N_I = 2 \) and

\[
B^{(2)} = \sum_{p=1}^{P-1} \Xi^{(p)} \text{diag}\{H^{(p)}\},
\]

the scheme is reduced to the DF-ICI of [26].

The vector \( R^{(n)} \) is affected by the residual ICI and by the errors on both past decisions \( \hat{D}^{(n-1)} \) on the derivative estimation. The strong correlation of the global error both with the data and with the channel parameters, may yield a significant power of the residual interference. Moreover, because of the correlation of the channel parameters on adjacent subcarriers, errors are usually grouped in clusters of subcarriers. In order to decorrelate the errors due to data and
parameter estimate, coding of the transmitted data can be used. In particular, once the DFE has been applied, data are de-interleaved and then re-encoded and re-interleaved. This process helps in reducing errors on data and in spreading them across the subcarriers.

### 3.1.1 Filter design for the iterative scheme

The design of the feedback filters $B^{(n)}$ is performed with the objective of maximizing the signal to noise plus interference power ratio (SNIR) before detection/decoding of the data. This SNIR depends on errors of the data used to produce the feedback signal.

If coding and interleaving are present, errors on the data are equally distributed among the various subcarriers and independent of the data symbols. The column vector of the errors at the $n$th iteration

$$e^{(n)} = \hat{D}^{(n)} - D$$

is a Gaussian random vector with zero mean and uncorrelated elements.

The average SNIR is defined as

$$Z^{(n)} = \frac{\sigma^2_a ||H^{(0)}||^2}{E[||\sum_{p=1}^{P} \Xi^{(p)} \text{diag}\{H^{(p)}\} + B^{(n)}D + B^{(n)}e^{(n-1)}||^2] + \sigma^2_w}.$$  (3.4)

Since $Z^{(n)}$ includes the contribution of the error propagation due to erroneous past decisions $e^{(n-1)}$, it is useful to define the average SNIR after decoding, at the $n$th iteration, as

$$\Gamma_n = \frac{E[||D||^2]}{E[||e^{(n)}||^2]}.$$  (3.5)

By minimizing (3.4), as shown in Appendix, we obtain the feedback filter

$$B^{(n)} = -\frac{1}{1 + 1/\Gamma_{n-1}} \sum_{p=1}^{P} \Xi^{(p)} \text{diag}\{H^{(p)}\}.$$  (3.6)

For the computation of $\Gamma_n$ we observe that the most probable errors occur when a constellation point is mistaken for one of its nearest points. By indicating with $d_{\text{min}}$ the minimum distance between any two constellation points, and with $P_e^{(n)}$ the symbol error probability at the $n$th iteration, the power of the error will be $P_e^{(n)} d_{\text{min}}^2$ and an approximated expression of $\Gamma_n$ is given by

$$\Gamma_n \approx \frac{\sigma^2_a}{P_e^{(n)} d_{\text{min}}^2}.$$  (3.7)

In Appendix an expression of the error probability $P_e^{(n)}$ as a function of the SNIR is derived. The resulting iterative algorithm for the filter design is shown in Table 3.1.
Table 3.1: Iterative ICI Cancellation Algorithm

1. Set $n = 1$.
2. Detect the data. Perform decoding and re-encoding.
3. Evaluate the error probability $P_{e}^{(n)}$ by (3.24).
4. Compute $B^{(n)}$ according to (3.6).
5. Increment $n$ by 1.
6. Goto point 2.

3.2 Multiple receive antennae

When multiple antennae are used at the receiver, a diversity gain can be achieved by properly combining the signals of the different antennae. For a rich environment and properly spaced $N_R$ antennae, the channels are uncorrelated. In this case, after the ICI cancellation has been applied on each branch, the signals are combined before detection. Let’s indicate with $R_{m,n}^{(k)}$ the signal after ICI cancellation at the $n$th iteration and on the $m$th subcarrier for the $k$th antenna. We indicate with $\gamma_m^{(k)}$ the combining coefficient for the $m$th subcarrier on the $k$th antenna. Then the signal after MRC is

$$R_{MRC,m} = \sum_{k=1}^{N_R} R_{m,n}^{(k)} \gamma_m^{(k)}, \quad m = 0, 1, \ldots, M - 1.$$  (3.8)

By assuming independent receive channels, the highest SNIR after the combining is achieved by using the maximum ratio combining (MRC) [28], i.e. $\gamma_m^{(k)} = \bar{H}_{m}^{(k)\ast}$. When the channels of the various antennae are correlated, the ICI is also correlated and the coefficients $\gamma_m^{(k)}$ that maximize the SNIR of the initial data estimate are given by the optimum combining (OC) [19]. From Fig. 3.1 we observe that the MRC is performed twice: firstly, after the estimation of the zero derivative of the channel, in order to improve the first tentative decision, and then after the ICI cancellation, before the final decision on the data is taken.

There are two main benefits in using $N_R$ antennae at the receiver. The first benefit is due to the averaging of the noise and the residual interference. This gives an average gain of $N_R$ in terms of SNIR. The latter benefit is a frequency diversity gain, i.e. a reduced variance of the gain among different subcarriers. This flattening of the gains yields that the worst case situations are less likely to happen, and it represent an extra coding gain.
3.3. NUMERICAL RESULTS

The performance evaluation of the new iterative interference cancellation and channel estimation techniques has been performed on the DVB-T system, both for the 2k and the 8k mode, having $M = 2048$ and $M = 8192$ subcarriers, respectively [44]. Various lengths of the cyclic prefix, constellation sizes and code rates were considered. We used a Rayleigh fading channel with exponential decaying power profile, having a root mean square delay spread of $\tau = 1.1\mu s$ and a maximum delay of $7\mu s$. These parameters are in accordance with the Typically Urban (TU6) model defined by the COST 207 project for GSM [45], which was shown to give an accurate description of the DVB-T mobile radio channel [12]. All simulations were performed on channel CH40 (626 MHz) with a bandwidth of 8 MHz. Note that analog TV channels span the frequencies between 400 and 790 MHz, so that the considered carrier is roughly in the middle of the spectrum.

In order to compare the performance of the various techniques presented in this chapter, we have estimated two parameters: the bit error rate (BER) at the output of the ICI cancellation scheme and the mean carrier over noise ratio (C/N) after the ICI cancellation, before the last decoding, i.e.

$$C/N = \frac{||H^{(0)}||^2}{\text{E}[||R^{(N_I)} - H^{(0)}D||^2]}$$

(3.9)

and it collects both the residual ICI and the channel estimation mismatches.

**Iterative ICI cancellation.** Fig. 3.2 shows the C/N and BER as a function of the speed for various numbers of iterations of the iterative ICI cancellation system. The channel parameters are assumed to be known at the receiver and the number of derivatives is $P = 1$. With reference
to a system without ICI cancellation ($N_I = 1$), we observe that there is a significant improvement when $N_I = 2$ and $N_I = 3$. Instead, when more iterations are considered the additional improvement is negligible.

### 3.3.1 Maximum achievable speed

A relevant measure of performance of a mobile receiver is the maximum speed at which the quality of service required by the application is satisfied. For the DVB-T reception, laboratory experiments have been conducted in the frame of the Motivate project [12] and for each FEC code a target bit error rate can be obtained for the bit stream at the input of the Viterbi decoder, as shown in Table 3.2. By setting these BERs as thresholds for correct reception of the DVB-T stream, we obtained the maximum speed for the following receiver architectures:

- **Standard receiver** (STD). No ICI cancellation is performed and the channel estimation is performed by time (2 pilots) and frequency (8 pilots) linear interpolation.

- Two antenna standard receiver (TSTD). Multi antenna receiver with advanced channel estimation described in [12]. In this scheme time interpolation is performed on 16 pilots and MRC is used to combine the signals before decoding.

- **Iterative ICI cancellation scheme with multistage estimator and iterative zero-derivative estimator** (IMI). For this scheme $N_I = 4$ and other settings are as for MI and initial estimate performed by the time-frequency interpolator of the STD scheme and iterative zero derivative estimator (see Section 5.2.4) with 2 iterations and prediction.

- **ICI cancellation scheme with ML estimator** (ML). For this scheme $N_I = 1$ and the joint ML estimator described in [26] is used.

Fig. 3.3 offers a synoptic comparison of the performance of these architectures. The maximum speed for various constellation sizes, code rates and FFT sizes are shown. We observe that for the 2k mode as well as for lower constellation sizes and the 8k mode, the standard receiver yields a good performance. The 8k mode with 64-QAM gives the maximum bit rate (150 Mbit/s) and it requires more sophisticated ICI compensation techniques to achieve speeds beyond 100 km/h. We note that IMI yields a performance similar to the ML estimator.

<table>
<thead>
<tr>
<th>Code Rate</th>
<th>Target BER</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>$7 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>2/3</td>
<td>$4 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>3/4</td>
<td>$2 \cdot 10^{-3}$</td>
</tr>
</tbody>
</table>
Figure 3.3: Maximum achievable speed.
### 3.3.2 Implementation complexity

For the evaluation of the implementation complexity we considered both the required storage (in terms of number of OFDM blocks) and the computational complexity (in terms of number of equivalent FFTs). In order to have a single measure of the system requirements, we evaluated the silicon area required for the implementation of the algorithms. The unit measure is the silicon area required for the storage of one OFDM symbol (1 SA), while for the computation of one FFT, an area of 2.5 SA was considered.

Other complex multiplications required by the schemes have been accounted under the assumption that one 8k FFT requires $M \log_2(M)$ complex multiplications. For the inversion of a $L \times L$ matrix we assumed a complexity of $L^3$. Decoding and re-encoding has been accounted for one FFT.

The implementation complexity of various ICI cancellation and channel estimation schemes is shown in Table 3.3. We observe that by employing the multistage estimator scheme, a significant drop in complexity is achieved, with respect to the ML estimator. From Fig. 3.3 and Table 3.3 we observe that the systems with low complexity do not guarantee the appropriate QoS for speeds above 30 km/h. Therefore they are not suitable for the reception of the DVB-T signal on motor ways outside the city center. In order to achieve higher speeds, the iterative ICI cancellation schemes or multiple antennae receiver should be used. In the combination of multiple antennae and advanced channel estimation the main contribution to the silicon area is due to the extra storage needed for the longer time interpolation. On the other hand, for the ICI cancellation scheme the complexity is mainly due to the extra signal processing of the channel estimation and the ICI generation.
**APPENDIX**

**Appendix**

For the $m$th subcarrier, the SNIR before decoding is defined as

$$Z_m^{(n)} = \frac{\sigma_a^2 |H_m^{(0)}|^2}{E[||\sum_{p=1}^{P} \Xi^{(p)} \text{diag}\{H^{(p)}\} + B^{(n)}m D + B_m^{(n)}e^{(n-1)}||^2] + \sigma_w^2},$$

(3.10)

while the average SNIR $Z^{(n)}$ is defined in (3.4). Since $Z^{(n)}$ and $Z_m^{(n)}$ include the contribution of the error propagation due to erroneous past decisions $e^{(n-1)}$, it is useful to define the average SNIR after decoding, at the $n$th iteration, as

$$\Gamma_n = \frac{E[||D||^2]}{E[||e^{(n)}||^2]}.$$

(3.11)

Here we propose the design of the feedforward and feedback matrix filters with the objective of maximizing $Z^{(n)}$. By observing that the denominator of (3.4) may be written as

$$\Delta^{(n)} = E[||(\sum_{p=1}^{P} \Xi^{(p)} \text{diag}\{H^{(p)}\} + B^{(n)}D + B^{(n)}e^{(n)}||^2] + \sigma_w^2$$

$$= \sum_{p=1}^{P} \left| \sum_{k=0}^{M-1} \Xi^{(p)} \text{diag}\{H^{(p)}\} \right| \sigma_a^2 + \left| \sum_{k=0}^{M-1} B_k^{(n)} \right|^2 \sigma_a^2 / \Gamma_{n-1} + \sigma_w^2,$$

(3.12)

By setting the gradient of $\Delta^{(n)}$ to zero, we obtain the feedback filter (3.6) that maximizes $Z^{(n)}$.

From (3.10) and (3.6) we obtain that the SNIR of the subcarrier $m$ is

$$Z_m^{(n)} = \frac{\sigma_a^2 |H_m^{(0)}|^2}{||\sum_{p=1}^{P} \Xi^{(p)} \text{diag}\{H^{(p)}\}||^2(\sigma_a^2 + E[||e^{(n-1)}||^2] + \sigma_w^2)}.$$

(3.13)

From (1.45) and (3.4), we observe that the denominator of $Z^{(n)}$ includes the average power of the ICI

$$g = \frac{E_s}{P - 1} \sum_{p=1}^{P-1} \sigma^{(p)}.$$

(3.14)

By inserting (3.6) in (3.10) and using (1.45), we obtain

$$Z_m^{(n)} = \frac{E_s |H_m^{(0)}|^2}{g(1 + 1/\Gamma_{n-1}) + N_0} = \phi_n |H_m^{(0)}|^2,$$

(3.15)

where

$$\phi_n = \frac{E_s}{g(1 + 1/\Gamma_{n-1}) + N_0}.$$

(3.16)
Lastly, we compute $\Gamma_n$ in two steps. The first step relates $\Gamma_n$ to the symbol error probability, while the latter relates the error probability to $\phi_n$.

In order to find the relation between $\Gamma_n$ and the symbol error probability $P_e^{(n)}$ at the $n$th iteration, we observe that the most probable errors occur when a constellation point is mistaken for one of its nearest points. An expression of $\Gamma_n$ as a function of the symbol error probability at the $n$th iteration, is given by (3.7). Here we derive an expression of the error probability $P_e^{(n)}$ as a function of the SNIR.

In this analysis we assume that the error probability is dominated by the most probable error events, i.e., as a common assumption, we restrict the analysis on the error events with the free distance $d_f$ of the code. Let’s also indicate with $\nu_f$ the number of error events with distance $d_f$.

Let $\{I_\ell\}$ be the sets of $d_f$ indexes $\{k_1, k_2, \ldots, k_{d_f}\}$ of the symbols (subcarriers) involved in the $\ell$th error event with free distance $d_f$. Under the assumption of using ML decoding, the bit error probability may be approximated as

$$P_e^{(n)} \approx \nu_f \mathbb{E}_\{I_\ell\} \left\{ \exp \left[ -\frac{d_{\text{min}}^2}{4} \sum_{k \in I_\ell} Z_k^{(n-1)} \right] \right\}, \quad (3.17)$$

where expectation is taken on the sets $I_\ell$ and $d_{\text{min}}$ is the minimum distance between any two constellation points. From (3.15) we obtain

$$P_e^{(n)} \approx \nu_f \mathbb{E}_\{I_\ell\} \left\{ \exp \left[ -\frac{d_{\text{min}}^2}{4} \phi_{n-1} \sum_{k \in I_\ell} |H_k^{(0)}|^2 \right] \right\}. \quad (3.18)$$

We indicate with $F_\ell$ the column vector containing $\{H_k^{(0)}\}$, $k \in I_\ell$, and we indicate with $\lambda_k(\ell)$ the eigenvalues of the correlation matrix of $F_\ell$

$$R_{I_\ell} = \mathbb{E}[F_\ell F_\ell^H]. \quad (3.19)$$

The sum of stochastic terms in (3.18) may be rewritten as

$$\zeta_\ell = \sum_{k \in I_\ell} |H_k^{(0)}|^2 = \sum_{k=0}^{d_f-1} \lambda_k(\ell) |x_k(\ell)|^2, \quad (3.20)$$

where $x_k(\ell)$, $k = 0, 1, \ldots, d_f - 1$ are independent zero-mean circulant Gaussian variables. A close form expression of the p.d.f. of $\zeta_\ell$ is available for the case of $K$ distinct $\lambda_k$, $k = 0, 1, \ldots, K-1$,

$$p_\ell(\zeta) = \sum_{k=0}^{K-1} \exp[-\lambda_k(\ell)\zeta] \lambda_k(\ell)^{-1} \prod_{i \neq k} [1 - \lambda_k^{-1}(\ell)\lambda_i^{-1}(\ell)]^{-1}. \quad (3.21)$$

However, since in general $x_{k_0}(\ell_0)$ and $x_{k_1}(\ell_1)$ are correlated for $\ell_0 \neq \ell_1$ the expectation (3.18) is not easily computed. Instead, we consider here the worst case scenario, where a particular
combination of interleaving and error event corresponding to the set $\bar{I}$ of subcarriers yields the dominant term in the sum of (3.18). In this case, $P_e^{(n)}$ may be approximated as

$$P_e^{(n)} \approx \nu_f \exp \left[ -\frac{d_{\text{min}}^2}{4} \phi_{n-1} \sum_{k=0}^{d_f-1} \lambda_k |x_k|^2 \right],$$

(3.22)

where $\{\lambda_k\}$ are the eigenvalues of $R_f$. Note that $P_e^{(n)}$ depends on the particular realization of the channel (defined by $\{x_i\}$).

From pdf (3.21), an outage value of $\bar{\zeta}$ may be obtained, i.e.

$$\zeta_\epsilon = \sup \{x : P[\bar{\zeta} \leq x] < \epsilon\},$$

(3.23)

which is defined with respect to a certain (small) outage rate $\epsilon$. Similarly, an upper bound on the error probability can be used, i.e.

$$P_e^{(n)} = \nu_f \exp \left[ -\frac{d_{\text{min}}^2}{4} \phi_{n-1} \zeta_\epsilon \right].$$

(3.24)

Lastly, from (3.20) we observe that as the number of non-zero eigenvalues $\lambda_k(\ell)$ increases, by the central limit theorem, $\zeta_\epsilon$ converges to one, while reducing the number of non-zero eigenvalues increases its variance. On the other hand, a larger variance yields, from (3.21), a smaller $\zeta_\epsilon$. Hence, the worst case scenario corresponds to the minimum number of eigenvalues $\lambda_k(\ell)$ i.e., maximally correlated subcarriers amplitudes.

**Rayleigh fading channel**

For a Rayleigh fading channel having independent taps, and a transmission employing an interleaver with depth $D$, the correlation matrix $R_f$ for the worst case scenario is

$$R_f = \bar{F} \Lambda \bar{F}^H,$$

(3.25)

where $\Lambda$ is a diagonal matrix containing the power profile of the channel and $\bar{F}$ is the reduced DFT matrix with elements

$$[\bar{F}]_{r,c} = e^{-j2\pi c (rD \mod M)/M}, \quad r, c = 0, 1, \ldots, M - 1.$$  

(3.26)
CHAPTER 3. SIGNAL PROCESSING FOR MC MOBILE COMMUNICATIONS
Chapter 4

Multiantenna OFDM with Co-channel Interference

Besides the interference generated by the dispersive channel, another limiting factor for broadband communications is the co-channel interference generated by other devices that are using the same radio bandwidth at the same time. In order to cope with this phenomenon, multiple antennae can be used both at the transmitter and the receiver. In particular, in systems with no co-channel interference, transmit adaptation has been considered in [16–18], while in the presence of co-channel interference, receive equalizers only have been considered in [19, 20].

Here we present various design techniques to optimize a multiantenna transmitter with OFDM. Two coding schemes are considered: the first is a simple repetition code, while the latter is a space-time block code. To limit complexity, the receiver adopts maximum ratio combining whose optimization depends only on the channel and not on the interference. For both schemes we derive the transmit coding gains that maximize the signal to interference power ratio. As suboptimal schemes we also consider cases where gains have the same phase or the same amplitude. Furthermore, as an upper bound on the system performance, we derive a novel expression of the capacity for a system with two receive antennae and adaptive transmit gains. In fact, previous results are limited to the system where each transmitted signal is the linear combination of all space-coded data [21].

4.1 Multiple antennae with co-channel interference

In this chapter we consider an OFDM system where both the transmitter and the receiver are equipped with multiple antennae. Moreover, the frequency-response of the channel from antenna \( n_T \) to antenna \( n_R \) on the \( m \)th subcarrier is denoted as \( H_{m}^{(n_T,n_R)} \). At the transmitter, each antenna is equipped with a transmit gain per subcarrier, i.e. the signal transmitted on antenna \( n_T \) at time \( k \) on the \( m \)th subcarrier is multiplied by the complex gain \( \alpha_{m}^{(n_T)}(k) \). Since the adaptation
of the transmitter will be performed on a per-subcarrier basis, we drop in this chapter the index \( m \). When not needed, we also drop the index \( k \) and we define

\[
\alpha_{nT} = \alpha_m^{(nT)}(k). \tag{4.1}
\]

A source of disturbance is also present, and the frequency response of the channel between the source of disturbance and the \( n_R \)th receiver antenna is denoted as \( G_{nR} \), and the interference signal on the \( n_R \)th antenna at time \( k \) is denoted as \( i^{(nR)}(k) \), while the transmitted interference signal is denoted as \( I(k) \).

We assume that both useful and interference data signals have unitary power, while, to set a constraint on the transmit total power, it must be

\[
\sum_{m=0}^{M-1} \sum_{t=1}^{N_T} |\alpha_m^{(t)}(k)|^2 = 1. \tag{4.2}
\]

In order to simplify the solution, we consider a separate adaptation for each subcarrier and we consider the constraint

\[
\sum_{t=1}^{N_T} |\alpha_t|^2 = 1. \tag{4.3}
\]

### 4.2 Repetition code transmission

We first consider the repetition code (RC) scheme. In this approach the same signal is sent to all the transmit antennae, after being multiplied by suitable transmit gains. We denote by \( q(k) = [D(k) I(k)]^T \) the vector containing the useful and interference data signals at the transmitters. In this scheme the same data signal is sent simultaneously on the two transmit antennae. Defined the matrix

\[
C = \begin{bmatrix}
\alpha_1 H^{(1,1)} + \alpha_2 H^{(2,1)} & G_1 \\
\alpha_1 H^{(1,2)} + \alpha_2 H^{(2,2)} & G_2
\end{bmatrix}, \tag{4.4}
\]

the signal vector after demodulation (\( m \)th tone) in correspondence to the two receive antennae, can be written as

\[
\bar{R}(k) = [R^{(1)}(k) R^{(2)}(k)] = Cq(k) + w(k), \tag{4.5}
\]

where \( w(k) = [W^{(1)}(k) W^{(2)}(k)]^T \) is the received noise vector whose components are AWGN with variance \( \sigma_w^2 \). We assume that data and interference signals \( D(k) \) and \( I(k) \) have unitary power. To recover the transmitted symbol, a combining with gains \( \gamma_1 \) and \( \gamma_2 \) is performed to yield

\[
v(k) = \gamma_1 R^{(1)}(k) + \gamma_2 R^{(2)}(k). \tag{4.6}
\]
4.3 Transmit gain selection for RC transmission

For the RC scheme, the expression of the signal to interference plus noise ratio at the receiver \((\Gamma_{RC})\) is now derived. From \((4.5)\) at the receiver the useful signal can be written as
\[
\tilde{D}(k) = [\gamma_1(\alpha_1 H^{(1,1)} + \alpha_2 H^{(2,1)}) + \gamma_2(\alpha_1 H^{(1,2)} + \alpha_2 H^{(2,2)})]D(k), \tag{4.7}
\]
while the noise plus interference signal is
\[
Z(k) = (\gamma_1 G_1 + \gamma_2 G_2)I(k) + \gamma_1 W^{(1)}(k) + \gamma_2 W^{(2)}(k). \tag{4.8}
\]
Hence, \(\Gamma_{RC}\) is given by
\[
\Gamma_{RC} = \frac{|\gamma_1(\alpha_1 H^{(1,1)} + \alpha_2 H^{(2,1)}) + \gamma_2(\alpha_1 H^{(1,2)} + \alpha_2 H^{(2,2)})|^2}{|\gamma_1 G_1 + \gamma_2 G_2|^2 + (|\gamma_1|^2 + |\gamma_2|^2)\sigma_w^2}. \tag{4.9}
\]
As \(\Gamma_{RC}\) depends only on the ratio between \(\gamma_1\) and \(\gamma_2\), let
\[
\phi = \gamma_2/\gamma_1, \tag{4.10}
\]
\((4.9)\) can be simplified as
\[
\Gamma_{RC} = \frac{|\alpha_1 H^{(1,1)} + \alpha_2 H^{(2,1)} + \phi(\alpha_1 H^{(1,2)} + \alpha_2 H^{(2,2)})|^2}{|G_1 + \phi G_2|^2 + (1 + |\phi|^2)\sigma_w^2}. \tag{4.11}
\]

We illustrate now three methods, with increasing computational complexity, to determine the coefficients \(\alpha_1, \alpha_2, \) and \(\phi\) in \((4.11)\).

4.3.1 Interference suppression (RC-IS)

Obviously, when interference is much greater than the noise, usually a good strategy is to select \(\phi\) to completely cancel interference at the receiver (RC-IS). In this case, from the denominator of \((4.11)\) it must be
\[
\phi = -\frac{G_1}{G_2}. \tag{4.12}
\]
Now the values of \(\alpha_1\) and \(\alpha_2\) maximizing \(\Gamma_{RC}\) are derived. Defined \(\bar{H}_1 = H^{(1,1)} + \phi H^{(1,2)}\) and \(\bar{H}_2 = H^{(2,1)} + \phi H^{(2,2)}\), \((4.11)\) becomes
\[
\Gamma_{RC} = \frac{|\alpha_1 \bar{H}_1 + \alpha_2 \bar{H}_2|^2}{\sigma_w^2(1 + |\phi|^2)}. \tag{4.13}
\]
Hence \(\Gamma_{RC}\) is maximized by maximizing
\[
|\alpha_1 \bar{H}_1 + \alpha_2 \bar{H}_2|^2, \tag{4.14}
\]
under the constraint \((4.3)\). By applying the Lagrange multipliers method, the maximization of \((4.14)\) is obtained for
\[
\alpha_2 = \frac{\bar{H}_2^*}{\sqrt{|\bar{H}_1|^2 + |\bar{H}_2|^2}}, \quad \alpha_1 = \frac{\bar{H}_1^*}{\sqrt{|\bar{H}_1|^2 + |\bar{H}_2|^2}}. \tag{4.15}
\]
CHAPTER 4. MULTIANTENNA OFDM WITH CO-CHANNEL INTERFERENCE

Table 4.1: An iterative algorithm to maximize $\Gamma_{RC}$.

1. Set $\alpha_1 = 1/\sqrt{2}$ and $\alpha_2 = 1/\sqrt{2}$. Set $\Gamma_{ex} = \infty$.
2. Compute $\gamma_1$ and $\gamma_2$ from (4.20).
3. Compute $\alpha_1$ and $\alpha_2$ from (4.16) and (4.17).
4. Compute $\Gamma_{RC}$ using (4.9).
5. If $\Gamma_{RC} = \Gamma_{ex}$
   - then: stop.
   - else: set $\Gamma_{ex} = \Gamma_{RC}$ and goto point 2.

4.3.2 Transmit gains with a constant amplitude (RC-CA)

The phase differences between the useful signals received by the two antennae and between the useful signal and interference play an important rule in the system performance. As a first step toward the more general solution, we restrict $\phi$ to have a constant amplitude (RC-CA) and a phase $\theta$, namely

$$\alpha_1 = \frac{1}{\sqrt{2}}, \quad \alpha_2 = \frac{e^{j\theta}}{\sqrt{2}}.$$  (4.16)

Unfortunately, even in this case we are not able to determine a closed form solution and we resort to the algorithm of Table 4.1 where the receive and transmit gains are optimized iteratively. Note that at each step $\Gamma_{RC}$ is not increasing, hence the algorithm converges. We illustrate now the procedure.

From (4.9), once $\gamma_1$ and $\gamma_2$ are given, the values of $\theta$ which maximizes $\Gamma_{RC}$ turns out to be

$$\theta = \angle \left( \frac{\gamma_1 H^{(1,1)} + \gamma_2 H^{(1,2)}}{\gamma_1 H^{(2,1)} + \gamma_2 H^{(2,2)}} \right).$$  (4.17)

In turn, once $\theta$ is given, the optimum value of $\gamma_1$ and $\gamma_2$ can be computed as follows. Defined $\bar{H}_1 = (H^{(1,1)} + e^{j\theta} H^{(2,1)})/\sqrt{2}$ and $\bar{H}_2 = (H^{(1,2)} + e^{j\theta} H^{(2,2)})/\sqrt{2}$, (4.9) becomes

$$\Gamma_{RC} = \frac{|\gamma_1 \bar{H}_1 + \gamma_2 \bar{H}_2|^2}{|\gamma_1 G_1 + \gamma_2 G_2|^2 + \sigma_w^2 (|\gamma_1|^2 + |\gamma_2|^2)}.$$  (4.18)

Now, since the same data signal is sent simultaneously through the two transmit antennae, the system can be seen as a one-transmitter and two receiver antennae system, with the two paths gains given by $\bar{H}_1$ and $\bar{H}_2$, respectively. Hence, $\Gamma_{RC}$ is maximized by using the optimum combining (OC) gains.
In particular, as derived in [19], by indicating with
\[ \Delta = (|G_1|^2 + \sigma_w^2)(|G_2|^2 + \sigma_w^2) - |G_1 G_2|^2, \] \hfill (4.19)
the optimum transmit coefficients are given by
\[ \gamma_1 = \frac{1}{\Delta} |G_2|^2 (\alpha_1 H^{(1,1)} + \alpha_2 H^{(2,1)})^* \]
\[ - G_1 G_2^* (\alpha_1 H^{(1,2)} + \alpha_2 H^{(2,2)})^*, \] \hfill (4.20a)
\[ \gamma_2 = \frac{1}{\Delta} |G_1|^2 (\alpha_1 H^{(1,2)} + \alpha_2 H^{(2,2)})^* \]
\[ - G_1 G_2^* (\alpha_1 H^{(1,1)} + \alpha_2 H^{(2,1)})^*. \] \hfill (4.20b)

4.3.3 General solution (RC-GEN)

A more general iterative (optimum) design technique (RC-GEN) seeks the transmit and receive

\[ \text{gains that maximize } \Gamma_{\text{RC}}, \] \hfill (4.15)
under the constraint (4.3). In this case, an iterative algorithm similar
to that of Table 4.1 is proposed where at each iteration, \( \gamma_1 \) and \( \gamma_2 \) are computed by (4.20), while \( \alpha_1 \) and \( \alpha_2 \) are computed by (4.15). The corresponding value of \( \Gamma_{\text{RC}} \) is given by (4.9). However,

\[ \text{there is no guarantee that at convergence this solution is optimum.} \]

4.4 Space-time block coded transmission

In the space-time block coded scheme, data are encoded using the block codes of [46, 47]. At
the receiver maximum ratio combining (MRC) of the received signals is applied, according to
the channel coefficients and the transmit gains. In particular, by indicating with \( R^{(q)}(t) \) the
received signal at time \( t \) on the antenna \( q \), the \( k \)th transmitted signal of the \( s \)th block is obtained

\[ \tilde{R}^{(s)}(k) = \sum_{t=1}^{N_T} \sum_{q=1}^{N_R} H^{(\epsilon_q(k),q)} \alpha_{\epsilon_q(k)}^* \rho_t(k) R^{(q)}(s + t), \] \hfill (4.21)
where for each \( k \), \( \epsilon_q(k) \) is a permutation function of the indexes \( \{1, 2, \ldots, N_R\} \) and \( \{\rho_q(k)\} \)
depend on the code. For example, for orthogonal design codes \( \rho_q(k) \in \{-1, +1\}, \) [47]. In the
following, without loss of generality we will assume \( s = 0 \) and we will drop the indexes \( (s) \) and \( (k) \). After the MRC, from (4.21) the power of the useful signal is

\[ \sigma_R^2 = \left( \sum_{t=1}^{N_T} |\alpha_t|^2 \sum_{r=1}^{N_R} |H^{(t,r)}|^2 \right)^2, \] \hfill (4.22)
while the power of the residual interference is

$$\sigma_i^2 = E \left[ \sum_{t=1}^{N_T} \sum_{r=1}^{N_R} i_t^{(r)} \alpha_{\epsilon_t}^* H^{(\epsilon_t,r)*} \rho_t \right]^2. \tag{4.23}$$

The signal to noise plus interference ratio (SNIR) is given by

$$\Gamma = \frac{\sigma_R^2}{\sigma_w^2 \sigma_R^2 + E \left[ \sum_{t=1}^{N_T} \sum_{r=1}^{N_R} i_t^{(r)} \alpha_{\epsilon_t}^* H^{(\epsilon_t,r)*} \rho_t \right]^2}. \tag{4.24}$$

### 4.5 Transmit gain selection for ST transmission

According to the information available at the transmitter and the overall complexity of the device, different criteria for the choice of the transmit gains may be considered.

As a first option we investigate the minimization of the interference (MI), regardless of the noise. However, this choice may decrease the power of the useful signal at the detection point. Hence we consider as cost function the maximization of the SNIR (MSNIR).

As a reduced complexity solution we consider also the choice of transmit gains with equal amplitude (EA) or equal phase (or power adaptation, EP). For both cases we adopt the MSNIR criterion.

#### 4.5.1 Minimum interference (MI)

If the interference is the limiting factor for the communication, a reasonable target for the choice of the transmit gains is the minimization of the residual interference. In order to minimize (4.23) under constraint (4.3), we apply the Lagrange multiplier method. Let’s indicate with $f_m$ the inverse function of $\epsilon_t$, i.e.

$$\epsilon_{f_m} = m. \tag{4.25}$$

By defining the matrix $B$ with entries

$$[B]_{\ell,m} = \sum_{r=1}^{N_R} \sum_{q=1}^{N_R} E \left[ i_{f_{\ell}}^{(r)} i_{f_{m}}^{(q)} \right] H^{(m,r)*} \rho_{f_m} H^{(\ell,q)} \rho_{f_{\ell}}, \tag{4.26}$$

and the vector $\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_{N_T}]$ collecting the $N_T$ transmit gains, the interference power (4.24) can be written in the quadratic form

$$E \left[ \sum_{t=1}^{N_T} \sum_{r=1}^{N_R} i_t^{(r)} \alpha_{\epsilon_t}^* H^{(\epsilon_t,r)*} \rho_t \right]^2 = \alpha^* B \alpha. \tag{4.27}$$
4.5. TRANSMIT GAIN SELECTION FOR ST TRANSMISSION

Then the minimization problem is solved by the following linear system of equations

\[ \mathbf{B}\alpha + \lambda \alpha = 0, \] (4.28)

under the constraint (4.3). From (4.28) we conclude that the minimization of the interference is achieved when \( \alpha \) is the eigenvector of \( \mathbf{B} \) corresponding to the minimum eigenvalue of \( \mathbf{B} \).

Note that if the minimum eigenvalue of \( \mathbf{B} \) is zero, then the interference can be completely canceled.

4.5.2 Maximum signal to noise plus interference ratio (MSNIR)

The minimization of the interference can lead to poor performance when the interference has a similar propagation characteristic of the useful channel, since the resulting received useful signal may also be partially canceled. Hence we consider here the more general target of maximizing the SNIR \( \Gamma \) under the constraint (4.3).

By applying the Lagrange multiplier method to (4.24) under the constraint (4.3) a non-linear system of equations is obtained. In order to find a solution we observe that by multiplying all transmit gains by a constant real positive value \( c^2 \), \( \Gamma \) is multiplied by \( c \). Hence, in order to find the solution under the constraint (4.3) first a set of transmit gains \( \{\tilde{\alpha}_t\} \) which maximize \( \Gamma \) is found and then (4.3) is satisfied by setting

\[ \alpha_t = \frac{\tilde{\alpha}_t}{\sum_{t=1}^{N_T} |\tilde{\alpha}_t|^2}. \] (4.29)

In order to maximize (4.24) we minimize its denominator

\[
\sigma_w^2 \left( \sum_{t=1}^{N_T} |\tilde{\alpha}_t|^2 \sum_{r=1}^{N_R} |H(t,r)|^2 \right) + \mathbb{E} \left[ \sum_{t=1}^{N_T} \sum_{r=1}^{N_R} i_t^{(r)} \alpha_t^* H_t^{(r)} \rho_t \right]^2
\]

under the constraint that the numerator is a constant, i.e.

\[ \sum_{t=1}^{N_T} |\tilde{\alpha}_t|^2 \sum_{r=1}^{N_R} |H(t,r)|^2 = 1. \] (4.30)

Now, by defining the vector \( \beta = [\beta_1, \beta_2, \ldots, \beta_{N_T}] \) with entries

\[ \beta_n = \frac{\tilde{\alpha}_n}{\sum_{r=1}^{N_R} |H(n,r)|^2}. \] (4.31)
and the matrix $A$ with entries

$$[A]_{\ell,m} = \frac{[B]_{\ell,m}}{\sqrt{\sum_{r=1}^{N_R} |H(\ell,r)|^2}},$$

the Lagrange multiplier method yields the following system of equations

$$A\beta + \lambda\beta = 0, \quad (4.33a)$$
$$\sum_{t=1}^{N_T} |\beta_t|^2 = 1. \quad (4.33b)$$

Hence, first we need to find the eigenvector $\beta$ corresponding to the minimum eigenvalue of $A$, then the coefficients $\{\tilde{\alpha}_n\}$ can be computed by (4.31). Lastly, in order to satisfy the constraint (4.3), we normalize $\{\tilde{\alpha}_n\}$ by (4.29).

Note that if the minimum eigenvalue is null, then there is no interference at the decision point and the MSNIR criterion is equivalent to the maximization of $\sigma_{\tilde{R}}$ as given by (4.24). In this case, $\Gamma$ is maximized by allocating all the power to the transmit antenna $t$ with the maximum value of

$$\sum_{r=1}^{N_R} |H(q,r)|^2, \quad q = 1, 2, \ldots, N_T. \quad (4.34)$$

We examine now two particular cases for the transmit gains.

### 4.5.3 Equal phase (EP)

When only the gain amplitude adaptation is considered, this is equivalent to assume that $\{\alpha_t\}$ are real numbers. In this case, we maximize (4.24) under the constraint (4.3) and we consider only the real solution for the transmit gains. Hence, the transmit gains that solves the problem are the solution of the linear system of equations

$$\text{Re}[A]\beta + \lambda\beta = 0, \quad (4.35)$$

where $A$ and $\beta$ are defined by (4.32) and (4.31), respectively. The linear system (4.35) must be solved under the constraint (4.3). In this case, the solution $\beta$ is the eigenvector corresponding to the minimum eigenvalue of $\text{Re}[A]$.

### 4.5.4 Equal amplitude (EA)

We consider here the adaptation of only the phase of the transmit gains, i.e.

$$\alpha_t = \frac{e^{j\theta_t}}{\sqrt{N_T}}, \quad t = 1, 2, \ldots, N_T. \quad (4.36)$$
4.6. CAPACITY CONSIDERATIONS

From (4.22) we note that by forming an equal gain amplitude, the power of the received user signal is independent of the transmit gains and the MI and the MSNIR criteria yield the same solution. Additionally, from (4.27) we have that it is not restrictive to set $\theta_1 = 0$.

Now, by imposing the constraint (4.36) to (4.27), we obtain a problem which in general does not have a closed form solution, to the author’s knowledge. However, a close form solution for the case $N_T = 2$ is straightforward. From (4.27), the interference power is minimized by minimizing the cost function

\begin{equation}
([B]_{1,1} + [B]_{2,2}) + 2|[B]_{1,2}| \cos(\theta_1 + \angle[B]_{1,2}).
\end{equation}

(4.37)

Hence the solution is

\begin{equation}
\theta_1 = \cos^{-1} \left( \frac{2|[B]_{1,2}|}{[B]_{1,1} + [B]_{2,2}} \right) - \angle[B]_{1,2}.
\end{equation}

(4.38)

4.6 Capacity considerations

As an upper bound on the performance of a STBC with adaptive transmit gains, we give the capacity that can be achieved by a multi antenna system with adaptive transmit gains in the presence of interference.

Let’s define the matrix $H$ having as entries \{ $H^{(k,n)}$ \} for $k = 1, 2, \ldots, N_R$, $n = 1, 2, \ldots, N_T$, and let’s denote with $R_i$ the $N_R \times N_R$ interference autocorrelation matrix. Let’s also indicate with $T$ the $N_T \times N_T$ diagonal matrix having as entries \{ $\alpha_n$ \}.

From [48], the capacity of the considered multi antenna system is given by

\begin{equation}
C = \log_2 \frac{\det[\Gamma R_i + I_{N_R} + \Gamma HTT^H H^H]}{\det[\Gamma R_i + I_{N_R}]}.
\end{equation}

(4.39)

Since the denominator of $C$ in (4.39) does not depend on $T$, the maximization of $C$ with respect to $T$ yields the following problem

\begin{equation}
\max_T \log_2 \{\det[I_{N_R} + \Gamma (R_i + HTT^H H^H)]\} \quad \text{(4.40a)}
\end{equation}

\begin{equation}
\text{trace} T T^H = 1. \quad \text{(4.40b)}
\end{equation}

In [21] Farrokhi et al. computed the matrix $T$ that solve the above problem in the case $T$ is not constrained to be diagonal. In this general case, (4.40) can be rewritten as

\begin{equation}
\max_T \log_2 \{\det[I_{N_R} + \Gamma HTT^H \bar{H}^H]\}
\end{equation}

(4.41)

and the solution is attained by diagonalizing $\bar{H} T T^H \bar{H}^H$. Hence, by indicating with $\bar{H} = VWU$ the SVD of $H$, the optimum transmit matrix that maximizes the capacity is $T = U^H \Psi$ where $\Psi$ is a diagonal matrix with entries computed according to the water-filling principle [21].
Unfortunately, when we force $T$ to be diagonal, the matrix $\tilde{H}T^H\tilde{H}^H$ cannot be diagonalized and for the a system with any number of transmit antennae there is no a close solution to the problem, to the author’s knowledge. However, for the interesting case of $N_T = 2$ and a general number of receive antennae, we derive the transmit gains that maximizes the capacity.

By using the property $\det[I + AB] = \det[I + BA]$, the equation (4.41) can be rewritten as

$$\max_T \log_2 \{\det[I_2 + QTT^H]\}, \quad (4.42)$$

where $Q = \tilde{H}^H\tilde{H}$ is a $2 \times 2$ matrix with entries $[Q]_{n,m}, m, n = 1, 2$. By applying the Lagrange multiplier method to (4.42) under the constraint (4.3), we obtain the system of equations

$$\begin{align*}
[Q]_{1,1} \alpha_1^* + \det[Q] |\alpha_2|^2 \alpha_1^* + \lambda \alpha_1^* &= 0, \\
[Q]_{2,2} \alpha_2^* + \det[Q] |\alpha_1|^2 \alpha_2^* + \lambda \alpha_2^* &= 0,
\end{align*} \quad (4.43a)$$

$$\frac{|[Q]_{1,1} - [Q]_{2,2}|}{\det[Q]} \leq 1 \quad (4.44)$$

the transmit gains that maximize the capacity are given by

$$|\alpha_1|^2 = \frac{1}{2} + \frac{[Q]_{1,1} - [Q]_{2,2}}{2 \det[Q]} \quad (4.45a)$$

$$|\alpha_2|^2 = \frac{1}{2} + \frac{[Q]_{2,2} - [Q]_{1,1}}{2 \det[Q]} \quad (4.45b)$$

If (4.44) is not satisfied, by indicating with $k = \arg\max_{\mu} \{|[Q]_{\mu,\mu}|\}$ we set $\alpha_k = 1$, while the other gain is zero.

Note that, since only $TT^H$ is present in the capacity expression (4.39), the phases of the transmit gains do not affect the capacity.

### 4.7 Performance comparison

For the performance comparison we consider the channel model Wind-Flex, see Section 1.2.1. An OFDM system with 64 subcarriers and a cyclic prefix of length 8 was simulated on a line of sight channel, with an average SNR at the channel output of 10 dB. As a performance measure we use the bit rate that can be achieved by the system, assuming perfect channel loading and coding, namely

$$ABR = \frac{1}{T} \sum_{m=0}^{M-1} \log_2(1 + \Gamma_m), \quad (4.46)$$

where $\Gamma_m$ is the SNIR after the combining at the receiver on the $m$th OFDM subcarrier. We considered a system with $N_T = N_R = 2$. 
4.7. PERFORMANCE COMPARISON

Figure 4.1: Achievable bit rate as a function of the signal to interference ratio (SIR), for different transmit selection schemes. The average SNR at the channel output is 10dB.

In the figures we indicate with SIR the signal to interference ratio at the transmitter, i.e. the ratio between the power transmitted by the useful device and the overall power transmitted by the interfering devices, while the transmission channel is assumed to have unitary gain on average.

Fig. 4.1 shows the ABR as a function of the SIR for the ST-coded systems. For reference, we also plot the performance of the system with fixed transmit gains, $\alpha_1 = \alpha_2 = 1/\sqrt{2}$, indicated with the label Fixed Tx gains. From the figure we observe that for a SIR of 10 dB both the EA and the EP solutions outperform by about 3 dB the Fixed Tx gains technique, while being only 1 dB poorer than the optimum MSNIR solution.

Fig. 4.3 shows the ABR as a function of the average signal to interference (SIR) ratio. For reference, we also plot the performance of the two systems with fixed transmit gains, $\alpha_1 = \alpha_2 = 1/\sqrt{2}$, indicated with the labels RC and ST, using the OC [17] and the MRC receiver [46], respectively. Indeed, the greater flexibility of the RC scheme allows for a better interference suppression. Moreover, when interference is the predominant effect (i.e. for lower values of SIR), the simpler IS method yields a performance very close to the more complex GEN method.

Fig. 4.4 shows the complementary cumulative distribution function of the ABR for both the RC and the ST schemes, in a scenario with a SIR of 5dB.
Figure 4.2: Complementary cdf of the achievable bit rate for different transmit gains selection schemes. The average $SNR$ is 10dB, while the average $SIR$ is 5dB.

Figure 4.3: Achievable bit rate as a function of the signal to interference ratio (SIR), for different transmit selection schemes. The average SNR at the channel output is 10dB.
Figure 4.4: Complementary cdf of the achievable bit rate for different transmit gains selection schemes. The average $SNR$ is 10dB, while the average $SIR$ is 5dB.
Chapter 5

Efficient Channel Estimation for OFDM Systems

In OFDM systems, a good performance is achieved when an accurate estimate of the channel frequency response is available. In order to perform the estimation, either training symbols or pilot tones are inserted in the transmitted signal. In the first case, an entire OFDM symbol is known at receiver, and linear estimators are applied, [22–25]. For time-varying channels, instead, some carriers (pilot tones) are reserved for the transmission of training symbols, [23, 49]. In both cases, linear minimum mean square error (LMMSE) estimators have been derived, both for static [22] and for time-varying channels [26]. In particular, in the time-varying case, higher order derivatives are estimated according to the model of Section 1.3.2.

However, for long OFDM symbols, these estimators turn out to be exceedingly complex. For the static case, suboptimal solutions with a reduced complexity based on the time-domain properties of the channel have been considered [23, 24]; we mention in particular the scheme by Yang et al., [25].

As a contribution toward estimators with a further reduced complexity, in this chapter we present two channel estimation methods for the static channel. The first one is a reduced complexity implementation of the SVD estimator and has indeed the same performance. The second method is a least square (LS) polynomial interpolation of the system output. While giving slightly worse performance than SVD, it has a much reduced complexity.

For a mobile channel, we analyze the mean square error of the pilot-based estimation with interpolation and we derive a new decision-driven multistage estimator. In this scheme, by progressively deleting the contribution of the lower-order derivatives from the received signal, derivatives are estimated singularly. As a result, the joint estimation of the derivatives is reduced to a set of smaller problems, each involving a zero-derivative estimate, for which the algorithms derived for the static channels can be used.
5.1 Pilot-based channel estimators

In order to estimate the channel frequency response, some of the OFDM carriers may be used to transmit fixed symbols which are already known at the receiver. These carriers are named pilot tones. There are many possible arrangements of the pilot symbols in the frequency-time domain plane. A good introduction to the advantages and drawbacks of each allocation is provided in [50]. The two mainly used configurations are the block-type pilot arrangement and the comb-type pilot arrangement, which are shown in Fig. 5.1. Another interesting configuration is the drifting-type arrangement.

In the block-type arrangement the pilot signal is assigned to a particular OFDM block, which is sent periodically in time-domain. In the comb-type pilot arrangement, the pilot signals are uniformly distributed within each OFDM block. Assuming that the payloads of pilot signals of the two arrangements are the same, the comb-type arrangement provides a better resolution of the channel changes in the time domain and therefore is more suitable to fast fading channels. On the other hand, for slow fading channels the block-type arrangement provides a faster resolution in the frequency domain.

For the block-type arrangement, channel is estimated on regular intervals on the basis of the pilot symbols and the estimation is held until a new pilot block is received. For the comb-type arrangement, interpolation is required both in time and in frequency domain in order both to obtain an estimate also for carriers without pilot tones and to update the estimation in the time domain.

In the drifting-type (or scattered type) arrangement each symbol includes equally-spaced pilot tones, but their position varies from symbol to symbol. In particular, pilots are shifted by one carrier per OFDM symbol. For the drifting-type arrangement, interpolation is required both in time and in frequency domain. Fig. 5.2 shows a drifting-type arrangement of the pilots.

A complete analysis of an OFDM system with pilot-tones is provided in [49]. It includes
5.1. PILOT-BASED CHANNEL ESTIMATORS

5.1.1 Evaluation of MSE of interpolated pilot estimate

Let’s indicate with $T(k)$ the set of pilot subcarriers for the $k$th OFDM symbol, and let us denote with $\{p_m(k)\}$ the set of pilot symbols on the frequencies $m \in T(k)$. An estimate of the channel frequency response on the subcarriers $m \in T(k)$ is easily obtained at the receiver as

$$\hat{H}_m(k) = \frac{R_m}{p_m(k)}$$

(5.1)

Then, by interpolation on both time and frequency, the estimate is extended to the other subcarriers. The time interpolation provides an estimate on all the available pilot tones, while frequency interpolation extends the estimate to the remaining subcarriers.

When the channel is time-varying, the estimate (5.1) is affected by additional ICI generated by the data subcarriers. This interference can be modeled as white Gaussian noise, and, as it will be seen in Chapter 3, we obtain the variance of the interference from (3.15) as

$$\Delta = g(1 + \frac{1}{\Gamma_0}).$$

(5.2)

Moreover, the time domain interpolation suffers from the time-variation of the channel.
Let’s indicate with $H_m(t)$ the channel frequency response at the $m$th subcarrier and at time $t \in \mathbb{R}$. By assuming that all the frequencies have the same statistical properties, we can restrict the analysis to one of the subcarriers, whose time-varying complex value is indicated with $H(t)$.

By indicating with $T_p$ the time between two pilots on the same subcarrier, the corresponding received signal after division by the pilot symbol is

$$G(kT_p) = H(kT_p) + V(kT_p),$$

where $V$ is the ICI term. In order to derive the frequency response at time $T' = T(M + L)$, an interpolating filter with frequency response $Q(f)$ is applied to obtain $\tilde{H}(k'T')$.

The power of the residual error is

$$\sigma_i^2 = E[|\tilde{H}(k'T') - H(k'T')|^2] = E[|(Q \otimes V)(k'T')|^2] + E[|Q \otimes (H - H)(k'T')|^2],$$

where $\otimes$ denotes convolution. As shown in Appendix B, by indicating with $Y(f)$ the spectrum of $H(k'T')$, (5.4) can be rewritten as

$$\sigma_i^2 = \int_0^{1/T'} |Q(f)|^2 \sigma_e^2 + [Y(f) - 1]Y(f) + Q(f) \sum_{\ell=1}^{T_p/T'-1} Y(f - \ell(1/T_p))^2 df,$$

where $\sigma_e^2$ is given by (1.33). The interpolation error has two contributions: an ICI contribution and an insufficient sampling contribution (the spectrum $Y(f)$ extends beyond $1/T_p$). In Appendix B the expressions of $Y(f)$ and $\sigma_i^2$ are derived.

Fig. 5.3 shows the mean square error of the pilot-based estimate for a DVB-T system [44] operating with $M = 8192$, as a function of the mobile speed, for a 64-QAM transmission and a SNR of 30 dB. The channel model is a typical DVB-T scenario, as described in more details in Section 3.3. From the figure, we note that the MSE of the pilot-based estimate increases 3dB every 60 km/h. Hence, for an 8k transmission at the speed of 100km/h a MSE of 18dB is obtained.

### 5.1.2 Predictive estimator

In order to reduce the MSE of the channel estimate, when pilot symbols are present, we propose a predictive method. In particular, we assume that a reliable estimate of the channel and the parameters of the time variations according to the model of Section 1.3.2 are available and we try to predict the estimate for the next symbols. For methods to improve the reliability of the channel estimate, see Section 5.2.4.

The prediction is performed using the model of Section 1.3.2, which describes the time-variations of the channel. Using the notation of Chapter 3 for the derivatives, the first two estimated derivatives at time $k$ we obtain the estimate at time $(k + 1)$ of the zero-derivative as

$$\tilde{H}^{(0)}(k + 1) = \tilde{H}^{(0)}(k) + (1 + N + L)\tilde{H}^{(1)}(k).$$

5.2. SYMBOL-BASED CHANNEL ESTIMATORS

In order to reduce the estimation error, the pilot-based estimate $\hat{H}^{(0)'}(k)$ and the predictive estimate are averaged with a weighting factor $\kappa_k$, i.e.

$$\hat{H}^{(0)}(k) = \kappa_k \hat{H}^{(0)'}(k) + (1 - \kappa_k) \bar{H}^{(0)}(k),$$  \hspace{1cm} (5.7)

where initially, $\kappa_0 = 1$.

The coefficients $\kappa_k$ are chosen in order to minimize the MSE of the estimate. In particular, we indicate with $\gamma_0^2$ and $\gamma_k^2$ the MSE of the estimate $\hat{H}^{(0)}(k)$ and $\bar{H}^{(0)}(k)$, respectively. The variance of the error of the average estimate of the zero derivative at time $k$, is then

$$\sigma_k^2 = \mathbb{E}[||\hat{H}^{(0)}(k) - \mathbf{H}^{(0)}||^2] = (1 - \kappa_k)^2 \gamma_k^2 + \kappa_k^2 \gamma_0^2,$$  \hspace{1cm} (5.8)

and the minimization of $\sigma_k^2$ with respect to $\kappa_k$ yields

$$\kappa_k = \frac{\gamma_k^2}{\gamma_0^2 + \gamma_k^2}. \hspace{1cm} (5.9)$$

5.2 Symbol-based channel estimators

We consider a time-invariant channel with $N_h$ coefficients and estimation techniques which use only one OFDM symbol, i.e., we assume that the entire OFDM symbol is known at the receiver.
To simplify the notation, the time index indicating the OFDM symbol number is omitted from all signals.

The $M$-size DFT of the channel is indicated with

$$
\mathbf{H} = [H_0, H_1, \ldots, H_{M-1}]^T.
$$

(5.10)

A first estimate of the frequency response of the channel is immediately obtained at the receiver by dividing the received signal by the training symbol, i.e.

$$
G_{ls,m} = R_m/D_m, \quad m = 0, 1, \ldots, M - 1.
$$

(5.11)

This estimate is denoted as least square (LS) estimate [22]. Note that however, this is a very noisy estimate that does not use the correlation among the different frequency of the channel. In this Section we will first revise the SVD method that performs the MMSE estimate of the channel, and then we will introduce two new reduced-complexity channel estimation techniques.

### 5.2.1 The O-SVD method

Here we briefly recall the optimum SVD (O-SVD) estimation method [22]. The $M \times M$ autocorrelation matrix $R_{\mathbf{H}}$ of $\mathbf{H}$, is decomposed by SVD into

$$
R_{\mathbf{H}} = \mathbf{U} \Lambda \mathbf{U}^H,
$$

(5.12)

where $\mathbf{U}$ is a unitary matrix containing the singular vectors, and $\Lambda$ is a diagonal matrix containing the singular values $\{\Lambda_m\}$. Let’s define the diagonal matrix $\Delta$ with elements

$$
\Delta_m = \frac{\Lambda_m}{\Lambda_m + \eta/\Gamma},
$$

(5.13)

where $\eta$ is a constant depending on the symbols constellation and $\Gamma$ is the signal to noise ratio. We recall that for channels with a finite length impulse response of length $N_h$

$$
\Delta_m = 0, \quad \text{for } m \geq N_h.
$$

(5.14)

The O-SVD channel estimator yields

$$
\mathbf{G}_{svd} = \mathbf{U} \Delta \mathbf{U}^H \mathbf{G}_{ls},
$$

(5.15)

where $\mathbf{G}_{ls}$ is a $M$-size vector with elements $G_{ls,m}$.

Among the various simplified versions of (5.15) we mention the reduced rank-$r$ estimator, where $\Delta_m = 0$ for $m = r, r+1, \ldots, M - 1$, [22]. A further simplification is derived when $h_n$ are independent zero mean random variables, which yields

$$
[R_{\mathbf{H}}]_{m,n} = \mathbb{E} \left[ \frac{1}{M} \sum_{\ell=0}^{M-1} e^{-j2\pi \frac{m}{N_h} h_\ell} \sum_{k=0}^{N_h-1} e^{j2\pi \frac{km}{M} h_k^*} \right]
$$

$$
= \frac{1}{M} \sum_{\ell=0}^{M-1} e^{-j2\pi \frac{\ell(n-m)}{M}} \mathbb{E}[|h_\ell|^2],
$$

(5.16)
and matrix $U$ coincides with the DFT matrix $F_M$ with entries $[F_M]_{\ell,n} = \frac{1}{\sqrt{M}} e^{-j \frac{2\pi \ell n}{M}}$.

As shown by Li et al. [23], by setting $U = F_M$, $\Delta_m = 1$, for $m = 0, 1, \ldots, r - 1$ and $\Delta_m = 0$, for $m = r, r + 1, \ldots, M - 1$, a robust estimator is obtained independently of the channel statistics. This method will be denoted as R-SVD.

Even in the simplified form (5.16), the SVD estimator may be expensive for systems with many tones operating in very dispersive channels, i.e. when $M$ and/or $N_h$ are large. Here we investigate reduced complexity techniques with the aim of developing estimators which operate mostly on subsets of tones, in such a way to divide the original problem into simpler problems.

### 5.2.2 Comb tones partitioning

A reduced complexity SVD method, denoted as CTP-SVD, is now derived. The basic idea comes by observing that the channel frequency response is fully described by $N_h$ coefficients. Hence we reduce the original $M$-size LS estimate into one of size $N_h$ to which the O-SVD method is applied.

To derive the CTP-SVD, we divide $G_{ls}$ into $M/N_h$ sub-blocks of length $N_h$, by comb tones partitioning (CTP),

$$G_{CTP,u} = [G_{ls,u}, G_{ls,M/N_h+u}, \ldots, G_{ls,M/(N_h-1)+u}]^T,$$  \hspace{1cm} (5.17)

where $u = 0, 1, \ldots, M/N_h - 1$. We transform each vector $G_{CTP,u}$ into time domain by a $N_h$-size IFFT to obtain $g_{CTP,u}$. Then, the $m$th element of $g_{CTP,u}$ is multiplied by $e^{j2\pi \frac{mu}{M}}$, to remove the frequency shift. By taking the mean of these estimates we obtain the vector $\bar{g}_{CTP}$,

$$\bar{g}_{CTP,m} = \frac{N_h}{M} \sum_{u=0}^{M/N_h-1} g_{CTP,u,m},$$  \hspace{1cm} (5.18)

where the original noise power present in (5.17) is reduced by a factor $M/N_h$. Lastly, a $N_h$-size FFT is performed on $g_{CTP}$ to yield $\bar{G}_{CTP} = F_{N_h} \bar{g}_{CTP}$.

As shown in Fig. 5.4, the SVD method is now applied to $\bar{G}_{CTP}$, and

$$\bar{G}_{svd} = U_{CTP} \Delta_{CTP} U_{CTP}^H F_{N_h} \bar{g}_{CTP},$$  \hspace{1cm} (5.19)

where, with regard to (5.15), $\Delta_{CTP}$ is the $N_h \times N_h$ upper-left block of $\Delta$ and $U_{CTP}$ is a $N_h \times N_h$ unitary matrix. Note that $U_{CTP}^H F_{N_h}$ are computed once, thus reducing complexity.

From (5.19), we obtain the desired $M$-size channel frequency response, by first taking the IFFT

$$\bar{g}_{svd} = F_{N_h}^H \bar{G}_{svd},$$  \hspace{1cm} (5.20)

and then performing $M/N_h$ separate FFTs

$$G_{svd,\ell M/N_h+u} = \sqrt{\frac{N_h}{M}} \frac{1}{\sqrt{N_h}} \sum_{m=0}^{N_h-1} e^{-j2\pi \frac{\ell m}{N_h}} e^{-j2\pi \frac{um}{M}} \bar{g}_{svd,m},$$  \hspace{1cm} (5.21)
Figure 5.4: SVD estimator scheme with comb tones partitioning.

where \( u = 0, 1, \ldots, M/N_h - 1 \) and \( \ell = 0, 1, \ldots, N_h - 1 \). We have used the same notation \( G_{svd} \) for both O-SVD and CTP-SVD because, as shown in Appendix A, they yield the same MSE.

Analogously to R-SVD, for \( U_{CTP} = F_{N_h} \) and \( \Delta_{CTP,m} = 1 \) we will have a new robust method, here denoted as CTP-R-SVD. Note that in this case \( U_{CTP}^H F_{N_h} \) are identity matrices and from (5.19) and (5.20) we have a further simplification, since \( \bar{g}_{svd,m} = \Delta_{CTP,m} \bar{g}_{CTP,m} \).

### 5.2.3 Adjacent tones partitioning

The adjacent tones partitioning (ATP) method divides the vector \( G_{ls} \) into sub-vectors of \( L \) adjacent samples,

\[
G_{ATP,u} = [G_{ls,uL}, G_{ls,uL+1}, \ldots, G_{ls,uL+L-1}]^T,
\]

where \( u = 0, 1, \ldots, M/L - 1 \). For time- and band-limited signals, reducing the number of tones, implies a reduction of the essential base vectors, hence fewer coefficients are needed to
describe the channel.

To reduce the rank of the estimator from \( r \) to \( N = rL/M + 1 \), Edfors et al. [22] proposed the SVD estimation method for each block \( G_{\text{ATP},u} \) (ATP-SVD method). However, the computational complexity is still considerable.

Instead of using the base vectors given by the SVD of \( R_{\hat{\Sigma}} \), here we propose a polynomial base. Indeed, polynomials are already used in channel estimation with pilot tones, see for example [53] and references therein. However, while in that case a polynomial interpolation is performed on sampled frequency channel estimate to obtain an estimate for all the tones, here the polynomial base is to fit the LS frequency channel estimates. This method will be denoted as ATP-POL.

In particular, \( G_{\text{ATP},u} \) is described by a \( N \)-degree polynomial, whose \((N + 1)\) coefficients are denoted by the vector \( v_u \). Let’s define the \( L \times (N + 1) \) polynomial matrix \( V \) with entries:

\[ V[l,n] = l^n, \quad l = 0, 1, \ldots, L - 1, \quad n = 0, 1, \ldots, N. \]

We look for the vector of coefficients \( v_u \) which minimizes the LS error \( ||G_{\text{ATP},u} - Vv_u||^2 \). In turn, the estimated channel frequency response is derived as \( G_{\text{pol},u} = Vv_u \). By writing \( G_{\text{pol},u} \) as a function of \( G_{\text{ATP},u} \), it holds true

\[
G_{\text{pol},u} = PG_{\text{ATP},u} = V \left[ \left( V^T V \right)^{-1} V^T G_{\text{ATP},u} \right], \quad u = 0, 1, \ldots, M/L - 1. \tag{5.23}
\]

The matrix multiplication by \( P \) in (5.23) can be better seen as the multiplication of two rectangular matrices, as indicated by the square parenthesis. Additionally, we observe that the entries of matrices \( V \) and \( P \) are real numbers and multiplications by \( G_{\text{ATP},u} \) have a reduced computational complexity, at least 1/2, in terms of complex multiplications.

From (5.23) we obtain that the polynomial estimate is the vector \( G_{\text{pol}} \) with entries

\[
G_{\text{pol},m} = G_{\text{pol},\left[ \frac{m}{L} \right] \mod L}. \tag{5.24}
\]

The resulting MSE of the estimate is

\[
\gamma_{\text{ATP-POL}} = \sum_{m=0}^{M-1} E \left[ |H_m - G_{\text{pol},m}|^2 \right]. \tag{5.25}
\]

From (5.11) and (5.22), the MSE can be rewritten as

\[
\gamma_{\text{ATP-POL}} = \sum_{m=0}^{M-1} E \left[ |H_m - \sum_{\ell=0}^{L-1} [P]_{(m \mod L), \ell} G_{\text{ATP},\left[ \frac{m}{L} \right] \ell}|^2 \right] \]

\[
= \sum_{m=0}^{M-1} E \left[ |H_m - \sum_{\ell=0}^{L-1} [P]_{(m \mod L), \ell} H_{\left[ \frac{m}{L} \right] L+\ell}|^2 \right] \tag{5.26}
\]

\[
+ \frac{1}{\Gamma} \sum_{\ell=0}^{L-1} |[P]_{(m \mod L), \ell}|^2.
\]
5.2.4 Iterative decision-directed estimator

When a training symbol is not available, but only pilot-symbols are available, a refinement of the estimation of the channel can be obtained by a decision-directed estimator. In particular, an initial decision is taken on the data $\hat{D}^{(0)}$ using the pilot-based estimation. Then, the decided data are used to perform a new symbol-based estimation of the channel. For this purpose, any of the methods presented in the previous section can be considered.

With the new estimate of the channel, data are detected again to obtain, at the $n$th iteration, the vector $\hat{D}^{(n)}$, which is used for a new channel estimation. At each iteration, the reliability of the estimate as well as the data is increased. The resulting scheme is shown in Fig. 5.5.

5.2.5 Time-variant channel estimation

When the channel is time-varying, also the parameters that describe the time variations must be estimated. With reference to the model introduced in Section 1.3.2, for the estimation of the derivatives $\{H^{(p)}\}$ that determines the ICI, the ML estimator has been introduced in [26]. Since the matrices $\Xi^{(p)}$ are not orthogonal, the ML estimator can not be divided into $P$ parallel ML estimators, one for each derivative, but it operates jointly on all of them. In particular, by defining the matrix

$$\Xi = [\Xi^{(0)} \text{diag}\{D\} F, \ldots, \Xi^{(P-1)} \text{diag}\{D\} F]$$

the ML estimator for the reduced set of parameters is

$$\hat{H} = \frac{\sigma_a^2}{\sigma_w^2} \left( \frac{\sigma_a^2}{\sigma_w^2} \Xi^H \Xi + (R_c^{-1} \otimes R_f^{-1}) \right)^{-1} \Xi^H R,$$

where $R_f$ collects the $M$ principal components of $R_f$. In order to compute $\Xi^H \Xi$, a matrix of size $PL \times M$ should be multiplied by its hermitian, with a complexity of $O(M^2MP)$. If a
training sequence is available, the multiplication is performed off-line and the resulting matrix is stored. When no training sequence is available and the number of subcarriers is high (such as for DVB-T), the multiplication should be performed on the received data and the required operations are exceedingly expensive for consumer electronics products.

In order to simplify the estimate of the ICI derivatives, we consider here a suboptimal approach, where the derivatives are estimated in a multistage fashion. We can order the derivatives with decreasing average power. For the Rayleigh channel the derivatives are naturally ordered, since from (1.45) we note that the power of the derivatives is super-exponentially decaying. In the multistage scheme, we estimate first the derivatives with higher power, by modeling the contribution of lower-power derivatives as white noise. At each step the contribution of the previously estimated derivatives is deleted from the received signal and the next order of derivatives is estimated. Let’s consider for simplicity the Rayleigh fading model, being straightforward the extension to the general case.

The estimator works as follows. At the $p$th stage we compute the vector

$$R^{(p)} = R - \sum_{k=0}^{p-1} \Xi^{(k)} \{\hat{H}^{(k)}\} D = \Xi^{(p)} \{\hat{H}^{(p)}\} D + \mathbf{W}^{(p)}, \quad (5.29)$$

where $\mathbf{W}^{(p)}$ includes the residual interference and noise. By performing a MMSE inversion of $\Xi^{(p)}$ we obtain

$$\left(\Xi^{(p)} + \tilde{\Gamma}^{(p)} \mathbf{I}\right)^{-1} \Xi^{(p)} H R^{(p)} = \left(\Xi^{(p)} + \tilde{\Gamma}^{(p)} \mathbf{I}\right)^{-1} \Xi^{(p)} H \Xi^{(p)} \{\hat{H}^{(p)}\} D + q^{(p)}, \quad (5.30)$$

where $q^{(p)} = (\Xi^{(p)} + \tilde{\Gamma}^{(p)} \mathbf{I}^{-1}) \Xi^{(p)} H \mathbf{W}^{(p)}$ and $\tilde{\Gamma}^{(p)}$ is the SNIR factor that includes the ICI of the derivatives higher than $p$, i.e. from (1.45)

$$\tilde{\Gamma}^{(p)} = \frac{\sigma^{(p)2}}{\sigma_w^2 + \sum_{k>p} \sigma^{(k)2}}. \quad (5.31)$$

An estimate of $\hat{H}^{(p)}$ is then carried out under the assumption that the vector $q^{(p)}$ is zero-mean Gaussian distributed. The procedure allows the reduction of the problem of estimating jointly $P$ derivatives, to the problem of estimating a set of zero derivatives separately.

When the multistage channel estimator is included into the iterative scheme, the estimate of the derivatives of the previous iterations can be used to delete its contribution before a new estimate is performed. In particular, by indicating with $\hat{H}^{(p)}_q$ the estimate of the $p$th derivative at the $q$th iteration, the estimation at the iteration $q + 1$ of the $k$th derivative is performed on the signal

$$R - \left[\sum_{p=0}^{k-1} \Xi^{(p)} \{\hat{H}^{(p)}_q\} - \sum_{p=k+1}^{P-1} \Xi^{(p)} \{\hat{H}^{(p)}_q\}\right] \hat{D}^{(q)}. \quad (5.32)$$

When compared with (5.29), we observe that (5.32) includes the cancellation of higher order derivatives.
CHAPTER 5. EFFICIENT CHANNEL ESTIMATION FOR OFDM SYSTEMS

5.3 Performance comparison

In this section we show some performance comparison both for the static channel estimators and for the time-varying channel estimators.

5.3.1 Static channel estimator

For the performance comparison of the static channel estimator we considered Wind-Flex model of Section 1.2.1. An OFDM system with $M = 128$ tones and a CP of length $N_h = 8$ was simulated on a line of sight channel, with SNR $\Gamma = 15$ dB. As a performance measure we used the mean bits per tone that the system is eligible to deliver, assuming perfect channel loading and coding. Under the hypothesis of using a zero-forcing equalizer, the sample at the input of the decision element is

$$\tilde{D}_m(k) = R_m(k)/G_m = D_m(k)H_m/G_m + W_m(k)/G_m,$$

(5.33)

where $G_m$ is the generic channel estimate. The resulting signal to disturbance ratio on the $m$th tone, comprehensive of the contribution of noise and channel estimate error is therefore

$$\Gamma_{eq,m} = \frac{|H_m|^2}{|G_m - H_m|^2 + 1/\Gamma} = \frac{|H_m|^2}{\left(\frac{1}{\Gamma} + \gamma_m\right)^{-1}},$$

(5.34)

where $\gamma_m$ is the MSE of the channel estimate for the $m$th tone. Hence, the mean bits per tone of the systems is

$$b = \frac{1}{M} \sum_{m=0}^{M-1} \log_2 \left(1 + \Gamma_{eq,m}\right).$$

(5.35)

The ATP methods assumed $L = 16$ and $N = 3$.

Fig. 5.6 shows the complementary cumulative distribution function (cdf) of $b$ when the estimators are designed with the perfect knowledge of the channel statistics. As already noticed, O-SVD is the best performing method, which can be efficiently implemented by the CTP-SVD. When sub-optimal methods are considered, the ATP-SVD has a capacity about 2% lower than O-SVD and another 2% is lost by ATP-POL with respect to ATP-SVD.

Tab. 5.1 shows the computational complexity, in terms of number of complex multiplications, for various estimation algorithms. Note the decrease of complexity for both the CTP-SVD and the ATP-POL.

5.3.2 Mobile channel estimator

For the estimation of the mobile channel estimators we considered a DVB-T transmission [44]. Two transmission modes are defined: 2k mode, with $M = 2048$ and 8k mode, with $M = 8192$. 
5.3. PERFORMANCE COMPARISON

Figure 5.6: Mean bits per tone of various channel estimation methods in a Wind-Flex line of sight channel, with $\Gamma = 15$ dB.

For simulation we considered the $2k$ mode, which allows faster simulations, and the resulting maximum speeds are around 800 km/h. As shown in Fig. 3.3, going from the $2k$ to the $8k$ mode yields four-impact of speed. Hence, we can obtain an estimate of the maximum achievable speed for the $8k$ mode by dividing by a factor of 4 the maximum speed of the $2k$ mode.

**Zero derivative estimation.** Fig. 5.7 shows the C/N and BER as a function of the parameter $\beta$ for the averaging of the prediction estimate, at speed $v = 480$ km/h, with $N_I = 1$. We note that the highest C/N ratio is obtained for $\beta = 0.6$. The dashed lines show the performance for a fixed value of $\kappa_k = 0.6$. We conclude that a fixed value of $\kappa$ can be a good approximation for the averaging.

Fig. 5.8 shows the C/N and BER as a function of the SNR for various techniques of estimation of the channel parameters. The DVB-T mode is $8k$, the code rate is $1/2$ and the constellation is 64-QAM and the speed is 120 km/h. For this scheme (denoted IMI in the Figure), iterative ICI cancellation scheme is applied with multistage estimator and iterative estimator of the zero order derivative. The initial estimate is performed by the time-frequency interpolator of the STD scheme and iterative zero derivative estimator is then applied (see Section 5.2.4) with 2 iterations and prediction. The curves show the performance for $N_I = 2, 3$ and 4 and $\beta = 0.6$. Also in this case we note the increased reliability of data, as the number of iteration increases.

**Higher order derivative estimation.** Fig. 5.9 shows the C/N and the BER as a function of the speed for a scheme using the ML estimator and the scheme used for Fig. 5.8 with $N_I = 1$ and
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Figure 5.7: C/N ratio as a function of the averaging parameter $\beta$ for an iterative ICI cancellation scheme with averaging of the predicted zero-derivative estimation. $N_I = 1$.

Figure 5.8: BER and C/N ratio as a function of the speed for the IMI scheme.
5.3. PERFORMANCE COMPARISON

Table 5.1: Computational complexity in terms of complex multiplications of the estimation methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>Complexity</th>
<th>Wind-Flex</th>
</tr>
</thead>
<tbody>
<tr>
<td>O-SVD</td>
<td>$2MN_h$</td>
<td>2048</td>
</tr>
<tr>
<td>R-SVD</td>
<td>$2M \log_2 M + N_h$</td>
<td>1800</td>
</tr>
<tr>
<td>ATP-SVD</td>
<td>$2M(N + 1)$</td>
<td>1024</td>
</tr>
<tr>
<td>CTP-SVD</td>
<td>$2M \log_2 N_h + 2N_h^2$</td>
<td>896</td>
</tr>
<tr>
<td>CTP-R-SVD</td>
<td>$2M \log_2 N_h + N_h$</td>
<td>776</td>
</tr>
<tr>
<td>ATP-POL</td>
<td>$M(N + 1)/2$</td>
<td>512</td>
</tr>
</tbody>
</table>

2. We observe that the IMI scheme yields a reduction of speed of about 50 km/h, with respect to the performance of the ML scheme.
Figure 5.9: BER and C/N ratio as a function of the speed for two estimation methods of the higher derivatives parameters: joint ML estimation and multistage estimator, for $N_I = 1$ (solid lines) and $N_I = 2$ (dashed lines).
Appendix A

We show that estimation methods CTP-SVD and O-SVD yield the same MSE.

Let’s indicate with $\mathbf{R}_h^{(N_h)}$ and $\mathbf{R}_h^{(M)}$ the autocorrelation matrices, of size $N_h \times N_h$ and $M \times M$, respectively, of the impulse response $\{h_n\}$. Considering that $\{h_n\}$ has a length $N_h < M$, we can partition $\mathbf{R}_h^{(M)}$ as follows:

$$
\mathbf{R}_h^{(M)} = \begin{bmatrix}
\mathbf{R}_h^{(N_h)} & \mathbf{0}_{N_h \times (M-N_h)} \\
\mathbf{0}_{(M-N_h) \times N_h} & \mathbf{0}_{(M-N_h) \times (M-N_h)}
\end{bmatrix},
$$

(5.36)

where $\mathbf{0}_{m,\ell}$ is a $m \times \ell$ matrix with 0’s entries. If we define the SVD of $\mathbf{R}_h^{(N_h)}$ as

$$
\mathbf{R}_h^{(N_h)} = \mathbf{U}_{N_h} \mathbf{A}_{N_h} \mathbf{U}_{N_h}^H,
$$

(5.37)

from (5.36) the SVD of $\mathbf{R}_h^{(M)}$ is

$$
\mathbf{R}_h^{(M)} = \mathbf{U}_M \mathbf{A}_M \mathbf{U}_M^H,
$$

(5.38)

where

$$
\mathbf{U}_M = \begin{bmatrix}
\mathbf{U}_{N_h} & \mathbf{0}_{N_h \times (M-N_h)} \\
\mathbf{0}_{(M-N_h) \times N_h} & \mathbf{I}_{(M-N_h) \times (M-N_h)}
\end{bmatrix},
$$

(5.39)

where $\mathbf{I}_{m,\ell}$ is the $m \times \ell$ identity matrix and

$$
\mathbf{A}_M = \begin{bmatrix}
\mathbf{A}_{N_h} & \mathbf{0}_{N_h \times (M-N_h)} \\
\mathbf{0}_{(M-N_h) \times N_h} & \mathbf{0}_{(M-N_h) \times (M-N_h)}
\end{bmatrix}.
$$

(5.40)

In turn, the $M \times M$ autocorrelation matrix of $\mathbf{H}$ has entries

$$
[R_{\mathbf{H}}]_{\ell,m} = E[H_m H_\ell^*] = \frac{1}{M} \sum_{q=0}^{N_h-1} \sum_{p=0}^{N_h-1} e^{-j2\pi \frac{pm}{M}} e^{-j2\pi \frac{q(\ell-m)}{M}} E[h_p h_q^*],
$$

(5.41)

i.e. in matrix notation

$$
\mathbf{R}_{\mathbf{H}} = \mathbf{F}_M \mathbf{R}_h^{(M)} \mathbf{F}_M^H.
$$

(5.42)

To evaluate performance of the CTP-SVD method, we apply the CTP procedure to $\mathbf{H}$, obtaining, with reference to (5.19), $\mathbf{G}_{\text{CTP}}$. Similarly to (5.42), the $N_h \times N_h$ autocorrelation matrix of $\mathbf{G}_{\text{CTP}}$ turns out to be

$$
\mathbf{R}_{\mathbf{G}_{\text{CTP}}} = \mathbf{F}_{N_h} \mathbf{R}_h^{(N_h)} \mathbf{F}_{N_h}^H.
$$

(5.43)

Lastly, observing that $\mathbf{F}_M \mathbf{U}_M$ and $\mathbf{F}_{N_h} \mathbf{U}_{N_h}$ are unitary matrices, we conclude that the SVDs of $\mathbf{R}_{\mathbf{H}}$ and $\mathbf{R}_{\mathbf{G}_{\text{CTP}}}$, are, respectively

$$
\mathbf{R}_{\mathbf{G}_{\text{CTP}}} = (\mathbf{F}_{N_h} \mathbf{U}_{N_h}) \mathbf{A}_{N_h} (\mathbf{F}_{N_h} \mathbf{U}_{N_h})^H = \mathbf{U}_{\text{CTP}} \mathbf{A}_{\text{CTP}} \mathbf{U}_{\text{CTP}}^H,
$$

(5.44)
CHAPTER 5. EFFICIENT CHANNEL ESTIMATION FOR OFDM SYSTEMS

\[ \hat{R}_G = (F_M U_M) \Lambda_M (F_M U_M)^H = U \Lambda U^H. \]  
(5.45)

Hence, from (5.40) we conclude that both SVDs have the same non-zero singular values, i.e.

\[ \Lambda_{CTP,m} = \Lambda_m, \quad m = 0, 1, \ldots, N_h - 1. \]  
(5.46)

Moreover from (5.13) we observe that \( \Delta_{CTP} \) is the upper-left block of \( \Delta \), i.e.

\[ \Delta_{CTP,m} = \Delta_m, \quad m = 0, 1, \ldots, N_h - 1. \]  
(5.47)

Using the results of [22, Appendix D], the MSE of the CTP-SVD estimate is

\[ \gamma_{CTP-SVD} = \frac{1}{M} \sum_{m=0}^{N_h-1} \left( \Lambda_{CTP,m} (1 - \Delta_{CTP,m})^2 + \frac{\eta}{\Gamma} \Delta_{CTP,m}^2 \right). \]  
(5.48)

On the other hand the MSE of the O-SVD estimate is

\[ \gamma_{O-SVD} = \frac{1}{M} \sum_{m=0}^{M-1} \left( \Lambda_m (1 - \Delta_m)^2 + \frac{\eta}{\Gamma} \Delta_m^2 \right), \]  
(5.49)

where the sum can be limited to the first \( N_h \) terms, since, from (5.46) and (5.47) \( \Lambda_m = \Delta_m = 0 \) for \( m = N_h, N_h + 1, \ldots, M - 1 \).

By substituting (5.46) and (5.47) in (5.48) we obtain

\[ \gamma_{CTP-SVD} = \gamma_{O-SVD}. \]  
(5.50)
Appendix B

We derive the spectrum of $H(k'T')$ and the power of the residual error after the interpolation of the pilot-based estimate.

By assuming that only one subcarrier is active, from (1.42) we compute the autocorrelation function of the received signal for the time delay $t$. i.e.

$$E[R_0(t-t_0)R_0^*(t_0)] = \sum_{\ell_1, \ell_2} Y_{\ell_1, \ell_2} E\left[\left(\sum_{j=0}^{M-1} e^{-2\pi jf_{\ell_1} T_i} \right) \left(\sum_{j=0}^{M-1} e^{-2\pi jf_{\ell_2} T_i} \right)^* e^{i2\pi [f_{\ell_1} t_0 - f_{\ell_2} (t-t_0)]} \right]$$

$$= \sum_{\ell} Y_{\ell, \ell} E\left[\left(\sum_{j=0}^{M-1} e^{-2\pi jf_{\ell} T_i} \right)^2 e^{-2\pi if_{\ell} t} \right] +$$

$$+ \sum_{\ell_1, \ell_2 \neq \ell} Y_{\ell_1, \ell_2} E\left[\left(\sum_{j=0}^{M-1} e^{-2\pi jf_{\ell_1} t_0} \sum_{j=0}^{M-1} e^{-2\pi jf_{\ell_2} T_i} \right) e^{-j2\pi f_{\ell_2} (t-t_0)} \sum_{j=0}^{M-1} e^{2\pi jf_{\ell_1} T_i} \right] \quad (5.51)$$

where

$$Y_{\ell_1, \ell_2} = \sigma_a^2 E[\tilde{H}_{\ell_1}^{(0)} \tilde{H}_{\ell_2}^{(0)*}] \quad (5.52)$$

By taking the expectations in (5.51) we obtain [40]

$$E[R_0(t-t_0)R_0^*(t_0)] = \sum_{\ell} Y_{\ell, \ell} \sum_{j_1=0}^{M-1} \sum_{j_2=0}^{M-1} J_0\{2\pi f_D[-t + (j_1 - j_2)T]\} +$$

$$\sum_{\ell_1, \ell_2 \neq \ell} Y_{\ell_1, \ell_2} \sum_{j_1=0}^{M-1} \sum_{j_2=0}^{M-1} J_0\{2\pi f_D(t_0 - Tj_1)\} \quad (5.53)$$

where $J_0(\cdot)$ is the Bessel function of the first kind of order zero.

Lastly, by taking the Fourier transform of (5.53) we obtain the spectrum of $H(t)$, which will be indicated as $\mathcal{H}(f)$. It turns out that for $|f| < f_D$

$$\mathcal{H}(f) = \sum_{\ell} Y_{\ell, \ell} \sum_{j_1=0}^{M-1} \sum_{j_2=0}^{M-1} \frac{e^{2\pi if(j_1 - j_2)T}}{2\pi \sqrt{f_D^2 - f^2}} +$$

$$+ \sum_{\ell_1, \ell_2 \neq \ell} Y_{\ell_1, \ell_2} \sum_{j_1=0}^{M-1} \sum_{j_2=0}^{M-1} J_0\{2\pi f_D(t_0 - Tj_1)\} \frac{e^{2\pi if(t_0 + j_2T)}}{2\pi \sqrt{f_D^2 - f^2}} \quad (5.54)$$
and $H(f) = 0$ for $|f| \geq f_D$. We obtain

$$
H|_{|f|<f_D}(f) = \frac{1}{2\pi\sqrt{f^2-f_D^2}} \left[ \sum_{\ell} Y_{\ell,\ell} \left| \sum_{j=0}^{M-1} e^{2\pi ji\ell T} \right|^2 + \sum_{\ell_1} \sum_{\ell_2 \neq \ell_1} Y_{\ell_1,\ell_2} \sum_{j_1=0}^{M-1} \sum_{j_2=0}^{M-1} J_0[2\pi f_D(t_0 - T j_1)] e^{2\pi i \left( t_0 + j_2 T \right)} \right].
$$

By rewriting (5.3) in the frequency domain we obtain that the spectrum of $G(kT_p)$ can be written as

$$
G(f) = H_{T_p}(f) + \sigma_e^2,
$$

where $G(f)$ is periodical of period $1/T_p$. After the interpolating filter has been applied, the spectrum of the resulting signal is periodical with period $1/T'$ and the power of the residual interpolation error (5.4) can be rewritten in the frequency domain as

$$
\sigma_t^2 = \sigma_e^2 \int_0^{1/T'} |Q(f)|^2 + |H(f) - Q(f)H_{T_p}(f)|^2 df.
$$
Conclusions

In this thesis we have addressed problems of equalization, transmitter adaptation and channel estimation for dispersive time-varying channels in the presence of co-channel interference. The distinctive point in the proposed solutions has been the study of efficient implementations based on FFTs of the involved signals.

After an overview of transmission over mobile broadband channels, Chapter 1 has introduced a simplified model for time-varying channels for OFDM systems and a new transmission format for SC transmissions. Among possible further research activities, we mention a generalization of the simplified model to other classes of channels, as well as an analysis of the properties of the various parameters involved, in order to find further simplifications. The new format for SC transmission has been devised for the development of a new block DFE, and possible research activities include efficient channel estimation based on the new format, as well as comparisons with other similar formats, such as cyclic extension and zero-padded transmission.

In Chapter 2 we have developed a new frequency-domain DFE, where the feedforward filtering is performed in the frequency domain. The new structure has a significant complexity advantage over existing DFE implementations, and it also yields a performance improvement, both over time-domain DFE and over frequency-domain linear equalizers. Further research topics include adaptation of the filters and integrated channel estimation techniques. Moreover, an interesting new area of research is the study of iterative DFE, with implementation in the frequency domain, on which the author has already started an investigation, [54].

Another contribution of this thesis has been the design of a iterative ICI cancellation scheme for mobile OFDM. In Chapter 3 the iterative scheme is introduced and some design criteria for the filters are proposed. A further investigation is required to assess the extension of the scheme to other scenarios, such as wireless LAN, as well as further simplifications of the architecture in order to reduce its complexity.

In Chapter 4 a broadband transmission system with co-channel interference has been considered. In a scenario where more terminals are transmitting at the same time on the same frequencies, the optimum adaptation of the transmit gains of a multicarrier system equipped with more antennae is considered. Reduced complexity solutions based on partial adaptation of phase or amplitudes of the transmit gains are also derived. Further investigations relate to the impact of lack of synchronization among the devices, as well as a possible frequency shift among different transmissions.
In Chapter 5 the problem of channel estimation for OFDM systems has been investigated. Both the time-invariant and the time-variant case have been considered and for both cases new parameters estimators have been introduced. Among possible future research topics, we mention the extension of these estimators to systems with multiple antennae, as well the use of these estimations in scenarios with co-channel interference.
Bibliography

[1] “Broadband radio access networks (BRAN); HIPERLAN Type 2; Physical (PHY) layer,” European Telecommunications Standards Institute, Dec. 2000.


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Conferences


