Mathematical Analysis of Bluetooth Energy Efficiency

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ABSTRACT

In this paper, we propose a mathematical framework for the analysis of Bluetooth systems energy efficiency. The dynamic of the system is modelled by means of a finite state Markov chain (FSMC). Hence, we resort to the renewal reward theory to derive an estimation of the average throughput and energy efficiency achieved by the different packet formats, both for AWGN and Rician fading radio channels. System behavior is investigated under a wide range of parameters, like receive– correlator margin, average signal to noise ratio and Rice factor. The analysis we present may provide precious guidelines for the design of energy–efficient Segmentation–and–Reassembly modules and baseband polling algorithms for Bluetooth piconets.

KEYWORDS

Bluetooth, energy efficiency, ad-hoc networks

I. INTRODUTION

Bluetooth [1], [2] is an emerging radio technology that is expected to play a leading role, in the near future, in the field of short–range personal communications. Although Bluetooth can hardly compete in terms of transmission speed with other existing radio technologies, like IEEE 802.11b, it is definitely competitive in terms of energy consumption. Bluetooth, indeed, was designed to be integrated in portable, battery driven electronic devices, for which energy saving is a key issue.

Although the reception mechanism is well defined by the Bluetooth standard, many aspects related to the energy efficiency achieved by the system still need to be investigated. One of these aspects is related to the impact on system performance of an important design parameter, namely the receive–correlator margin S, that, loosely speaking, determines the *selectivity* of the receiver with respect to packets containing errors. Moreover, it may be worth investigating the way units drain their energy depending on the network configuration, traffic pattern and channel conditions.

Such topics have been partially addressed by previous works in the literature (see, for instance, [3], [4]). To the authors knowledge, however, the literature still lacks in accurate performance analysis that takes into consideration, beside delay

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and throughput, also energy consumption in the specific case of the Bluetooth system.

In this paper, we investigate these topics by means of a simple mathematical model for the Bluetooth point-to-point connection. The analysis considers in details the reception mechanism defined by the Bluetooth standard and the characteristics of each packet type. We identify all the possible events that may occur during the reception of a downlink (master-toslave) packet and the corresponding uplink (slave-to-master) packet. For each of such events, we determine the amount of energy spent by the master and slave units, the quantity of useful data delivered in both directions and time elapsed. Then, we describe the dynamic of the system by means of a finite state Markov chain (FSMC) and we derive the state-transition probabilities of the FSMC referring to the before mentioned reception events. Hence, following the approach suggested in [5], we resort to the renewal reward analysis to compute the average throughput and energy performance achieved by the system.

The analysis is carried out in both AWGN and Rician fading radio channel. System behavior is, then, investigated under a wide range of parameters, like packet type used in downlink and uplink communications, receive–correlator margin and average signal to noise ratio. The results we obtain in terms of energy efficiency and average throughput may provide useful guidelines for the design of energy–aware algorithms for the piconet organization and management.

The remainder of this paper is organized as follows. Section II provides an overview of the Bluetooth radio system. In Section III, we derive the mathematic model used to describe the system dynamic. Section IV presents a detailed performance analysis, based on the provided mathematical model. Finally, Section V provides concluding remarks.

II. BLUETOOTH RADIO SYSTEM

This section shortly overviews the features of the Bluetooth standard that are more related to the topic treated in the paper. We refer to the literature (e.g., [1], [2]) for an extensive description of the standard.

A. Baseband

In order to communicate, Bluetooth units have to be time and frequency synchronized to a common Frequency Hopping channel, which characterizes a so-called *piconet*. A piconet can host up to eight active units, one of which assumes the role of master, while the others become slaves. Transmissions can directly occur between master and slaves only. Duplex communication is obtained by a slot–based Time Division Duplex scheme: time is divided into consecutive slots of $T_{\rm slot} = 625\,\mu s$ that are used for downlink (master-to-slave) and uplink (slave-to-master) transmissions, alternatively. Channel access is controlled by the master through a basic round–robin polling scheme. On the basis of this scheme, only the slave addressed by a downlink (DL) packet is allowed (and required) to transmit a packet to the master in the following uplink (UL) slot. The master can poll the slave implicitly, by using a useful data packet, or explicitly, with a short control packet (POLL) that does not contain the payload field. The recipient slave is required to reply immediately to the master by transmitting a data packet or a special control packet (NULL) with no payload.

B. Data packet formats & Reception Mechanism

Bluetooth supports both synchronous connection oriented (SCO) and asynchronous connectionless (ACL) links. In this paper, we focus on ACL links only, which are used for data applications. An ACL packet can extend over an odd number of consecutive slots, namely, one, three or five slots. Each baseband packet contains three main fields: Access Code (AC), Packet Header (HEAD) and, optionally, Payload (PAYL).

The AC field is used for synchronization and piconet identification and contains a synchronization word that assures a minimum Hamming distance of 14 between ACs of different piconets. The HEAD field, coded with a 1/3 forward error correction code (two-time repetition of every bit), contains link control information, including packet type, destination address, sequence number, acknowledgment flag (ARQN) and an Header Checksum field (HEC). Except for POLL and NULL, the other ACL data packet types are tailed by the PAYL field that can extend over one, three or five consecutive slots. The PAYL field can be optionally protected by a (15,10) shortened Hamming code, which is able to correct all single errors in each codeword. The payload field of each data packet also contains a cyclic redundancy code (CRC) that is used to check the integrity of the field.

Unprotected packet formats are usually denoted by DH5, DH3 and DH1, for the 5, 3 and 1-slot long types, respectively. Analogously, DM5, DM3 and DM1 are used to denote the corresponding protected formats.

C. Reception and retransmission mechanism

Energy-saving was a key feature in the design of the Bluetooth technology. On the basis of this perspective, a receiving unit stops reception and enters a low-power *sleep mode* as soon as it determines that the incoming packet is addressed to another unit or the signal strength is too low to guarantee a good reception (see [1], pg. 124).

At the beginning of each receive slot, the Bluetooth receiver correlates the incoming bit stream against the expected synchronization word. For an incoming packet to be recognized, the Hamming distance between expected and decoded sync words does not have to exceed the so-called *receive-correlator margin*, denoted by *S*, whose value is not specified by the standard. In case AC is not recognized, reception stops and the unit enters a low-power sleep mode until the following receive slot. On the contrary case, HEAD field is also received and

decoded. If the HEC test fails (or the packet was addressed to another unit), the unit enters the sleep mode until the following receive slot. Finally, in case the reception of both AC and HEAD succeeds, the PAYL field (if any) is decoded and checked by means of the CRC field.

Bluetooth provides a reliable data connection by using an Automatic Retransmission Query (ARQ) mechanism at the baseband layer. Each data packet is transmitted and retransmitted until acknowledgement (ACK) of a successful reception is returned by the destination. The acknowledgement information is carried in the HEAD field of the return packet (piggybacking). Negative acknowledgement is assumed by default. Hence, downlink retransmissions may also be triggered by AC or HEC errors in the uplink packets which piggy-back positive ACK. In this case, the slave keeps receiving duplicate packets (DUPCK), i.e., packets having same payload. To save energy, the slave does not decode the PAYL field of DUPCKs and simply replies to each recognized DUPCK by piggy-backing a positive acknowledgement. Note that slaves never transmit DUPCKs. Indeed, a slave is allowed to transmit an uplink packet only upon reception of a valid POLL that, in turn, carries also the ACK for the previous slave transmission. Thus, slave retransmissions occur only when needed, i.e., in case of reception of a negative ACK from the master.

III. MATHEMATICAL MODEL

A. Hypothesis & Notations

For the sake of simplicity, we limit the study to the case of a piconet with only two units: one master and one slave. (The extension of the analysis to the multi-slave case would complicate the exposition without adding any concept.) We consider a heavy traffic scenario, where master and slave have always packets waiting for transmission. We assume infinite retransmission timeout: packets are retransmitted over and over again until the sender receives a positive acknowledgement. In order to determine the performance achieved by the different packet formats, we consider a static Segmentation and Reassembly (SAR) policy, so that a unique packet type per connection is used. For the radio channel, we assume the classical WSSUS (Wide-Sense Stationary Uncorrelated Scattering) slow flat Rician fading model, so that, by virtue of the frequency hopping mechanism, each packet experiments an independent fading statistic. Finally, we assume nodes are able to sense the radio channel and identify possible packet transmissions during the sleep mode. Under this hypothesis, a slave node always waits until the end of the master transmission before attempting a new packet reception. Analogously, the master node always waits until the end of the slave transmission before attempting a new packet transmission. (A complete study that does not consider this hypothesis can be found in [6].)

Let AC_{er} , HEC_{er} and CRC_{er} denote the unsuccessful reception events for the AC, HEAD and PAYL fields of an incoming packet, respectively. Moreover, let PR_{ok} denote the successful packet reception events. Note that, the reception of each field is subordinated to the good recognition of the preceding fields. Consequently, HEC_{er} event implies the AC of the incoming packet was successfully received. Analogously, CRC_{er} event requires both AC and HEAD fields were successfully received. Hence, events AC_{er} , HEC_{er} , CRC_{er} and PR_{ok} are disjoint and their probabilities sum up to 1.



Fig. 1. Two-state Markov model of the system.

TABLE I PACKET RECEPTION EVENTS & MARKOV CHAIN STATE

| REI | DL ^(S) | UL ^(M) | MC state |
|-----|-------------------|-------------------|----------|
| 00 | PR. | PR. | N |
| 01 | PR. | CRC | N |
| 02 | PR. | HEC | D |
| 02 | PR | AC | |
| 23 | CBC | PB | N |
| 05 | CRC^{er} | CRC^{ok} | N |
| 06 | CRC^{er} | HEC^{er} | X |
| 07 | CRC^{er} | AC | X |
| 00 | HEC^{er} | | X |
| 09 | $AC_{}^{er}$ | _ | X |

When necessary, we use the superscript $^{(M)}$ and $^{(S)}$ to distinguish between master and slave units. For example, $AC_{er}^{(M)}$ denotes an AC error events occurred at the master side, i.e., an unsuccessful uplink packet reception.

B. Markovian Model

Under the hypothesis considered, the dynamic of the system can be captured by means of a Two–State Markov Chain (MC) with event space $\mathbf{E} = \{N, D\}$, as depicted in Fig. 1. State transitions correspond to a transmission-&-reception phase of the master unit. In *Normal state* (N), the master transmits new downlink packets or retransmit packets that have not been correctly received by the slave. *Duplicate state* (D) is entered when the master does not recognize an uplink packet carrying a positive acknowledgment. In state D, the master keeps retransmitting duplicate packets, i.e., packets correctly received by the slave unit but not yet acknowledged because of unrecoverable errors in the AC or HEAD fields of the return packets. State D is left when the master finally gets a positive acknowledgment from the slave.

Tab. I gives a schematic summary of all the possible events that may occur during the reception of a downlink (DL) packet and the corresponding uplink (UL) packet. (Note that DL packets are received by the slave unit, while UL packets are received by the master unit.) We identify 10 disjoint reception events that are indexed from ρ_0 to ρ_9 , as indicated in the first column of the table, under the label REI (Reception Event Index). The state of the MC entered after each reception event is indicated on the right–most column of the table ($X \Rightarrow$ state does not change).

Transitions from state N to D are determined by the occurrence of events ρ_2 or ρ_3 . Transitions from state D to N, instead, occur with events $\{\rho_0 \cup \rho_4\}$ or $\{\rho_1 \cup \rho_5\}$, where the symbol \cup denotes the union operator. We consider these compounded events because, in state D, the slave disregards the PAYL field of the incoming packets, since they are DUPCKs. Hence, the steady state probabilities π_N and π_D of the chain

being in states N and D, respectively, are given by

$$\pi_{\rm N} = \frac{P_{\rm ND}}{P_{\rm ND} + P_{\rm DN}}; \qquad (1)$$

$$\pi_{\rm D} = \frac{P_{\rm DN}}{P_{\rm ND} + P_{\rm DN}}; \qquad (2)$$

where:

$$P_{DN} = P(\varrho_2) + P(\varrho_3); \qquad (3)$$

$$P_{ND} = P(\varrho_0 \cup \varrho_4) + P(\varrho_1 \cup \varrho_5)$$

$$= P(\varrho_0) + P(\varrho_4) + P(\varrho_1) + P(\varrho_5). \quad (4)$$

C. Renewal Theory & Reward functions

Following the approach suggested in [5], Bluetooth performance can be investigated by resorting to the classical theory of renewal reward processes [7]. Consider two generic reward functions, $R^{(1)}$ and $R^{(2)}$, such that $R^{(1)}_i$ and $R^{(2)}_i$ are the average reward earned each time the Markov chain enters in state $j \in \mathbf{E}$. Furthermore, let $R^{(1)}(\tau)$ and $R^{(2)}(\tau)$ be the total reward earned through the system evolution in the interval $[0, \tau]$. Then, from renewal theory [7], we have:

$$\lim_{\tau \leftarrow \infty} \frac{R^{(1)}(\tau)}{R^{(2)}(\tau)} = \frac{\sum_{j \in \mathbf{E}} \pi_j R_j^{(1)}}{\sum_{j \in \mathbf{E}} \pi_j R_j^{(2)}} = \frac{\bar{R}^{(1)}}{\bar{R}^{(2)}}; \quad (5)$$

where π_j is the steady state probability of the chain being in state j, while $\bar{R}^{(1)}$ and $\bar{R}^{(2)}$ are the expected rewards per state transition.

A proper choice of the reward functions will allow us to derive a number of performance indexes. In particular, we consider the following functions:

- state transition time \overline{T} ;
- average number of successfully delivered data bits, \overline{D} ;
- amount of consumed energy, \overline{W} .

In order to derive the expected values of these reward functions, we need to introduce some further notations. Let p_i be the probability of the reception event ρ_i . Note that, for the frequency hopping mechanism, successive packets have mutually independent error probabilities. Therefore p_i can be factorized in the product of the probabilities of the corresponding reception events at slave and master units. For example, we have $p_0 = P(\rho_0) = P(PR_{ok}^{(S)}) \cdot P(PR_{ok}^{(M)})$, where notation is self-explaining. (For space constraints, we do not report the expressions of such probabilities, which can be found, for instance, in [8], [9].) Furthermore, let $w_{TX}(X)$, $w_{\scriptscriptstyle RX}(X)$ and $w_{\scriptscriptstyle SS}(X)$ be the amount of energy consumed by a unit for transmitting, receiving and sensing, respectively, the generic packet field X. Moreover, let $\mathbb{D}(x)$ be the number of data bits carried by the PAYL of packet type x. Finally, let Dxn and Dym, with $n, m \in \{1, 3, 5\}$ and $x, y \in \{H, M\}$, be the packet types used in downlink and uplink transmission, respectively.

State Transition Time

The transmission of a Dxn downlink packet takes n time slots. If the slave recognizes the AC and HEAD of the downlink packet, it replies with a Dym packet that lasts for m time slots. On the contrary case, the slave is not allowed to transmit and, hence, the uplink phase takes only one slot. Note that, the length of the incoming packet is written in the packet header. If an HEC_{er} event occurs, however, the receiving unit can determine that the packet transmission is over by sensing the radio channel. Consequently, reception events from ρ_0 to ρ_7 take n + m time slots, while events ρ_8 and ρ_9 , take n + 1 slots. The average reward earned per MC transition is, then, given by

$$\overline{T} = (n+m)(1-p_8-p_9) + (n+1)(p_8+p_9).$$
 (6)

Delivered Data

In state N, the master transmits useful packets that have never been correctly received by the slave. In state D, the master transmits DUPCKs that do not carry useful information. Thus, the average number of data bits successfully delivered by the master and slave units, respectively, is given by

$$\overline{D}^{(M)} = \mathbb{D}(Dxn) \pi_N(p_0 + p_1 + p_2 + p_3); \quad (7)$$

$$\overline{D}^{(3)} = \mathbb{D}(Dym)(p_0 + p_4).$$
(8)

Consumed Energy

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(S)

The computation of the energy spent by the master and slave units for each transition step of the MC, though cumbersome, is not complicate. We first focus on the master unit. At each step, the master spends $w_{TX}(Dxn)$ energy units by transmitting the Dxn downlink packet. The energy spent in reception depends on the reception status of the uplink packet fields. In case of events ρ_0 , ρ_1 , ρ_4 , ρ_5 , the master receives the entire uplink packet, consuming $w_{RX}(Dym)$ energy units. In case of events ρ_8 and ρ_9 , the slave does not return any uplink packet and, thus, the master turns off its receiver immediately after the failed reception of the AC, spending only $w_{RX}(AC)$ energy units. In the remaining cases, the master stops receiving after the first erroneous field, but keeps sensing the radio channel for the remaining of the uplink packet. Therefore, the average amount of energy spent by the master is given by

$$\overline{W}^{(M)} = w_{TX}(Dxn) + w_{RX}(Dym)(p_0 + p_1 + p_4 + p_5) +
+ w_{RX}(AC)(p_8 + p_9 + p_2 + p_6 + p_3 + p_7) +
+ (w_{RX}(HEAD) + w_{SS}(PAYL_{ym}))(p_2 + p_6) +
+ (w_{SS}(HEAD) + w_{SS}(PAYL_{ym}))(p_3 + p_7).$$
(9)

The energy spent by the slave unit depends also on the state of the system. Indeed, as explained in Sec. 2, the slave does not listen for the PAYL field of DUPCKs. Hence, if the system is in state D and the slave does recognize the HEAD field of the downlink packet, it enters sleep mode till the end of the incoming packet, saving energy. However, if the AC or HEAD fields are not recognized, the slave has to sense the channel to recognize the end of the downlink transmission. On the basis of the rationale discussed for the master case, it is easy to realize that the average amount of energy spent by the slave unit can be expressed, after some algebra, as follows

$$\overline{W}^{(2)} = (1 - p_8 - p_9)w_{TX}(Dym) + w_{RX}(AC)(p_8 + p_9) +
+ w_{RX}(HEAD)p_8 + w_{SS}(HEAD)p_9 +
+ w_{SS}(PAYL_{xn})(p_8 + p_9) + (10)
+ (1 - p_8 - p_9)(w_{RX}(Dxn) - w_{RX}(PAYL_{xn})\pi_D).$$
(10)



Fig. 2. Energy efficiency for AWGN and Rayleigh channel (S = 0).



Fig. 3. Energy efficiency variation due to master-slave swapping.

IV. PERFORMANCE ANALYSIS

In this section we analyze the performance achieved by various Bluetooth packet formats, in different cases. We assume the same average Signal to Noise Ratio (SNR) for both master and slave nodes. Furthermore, we assume that transmission and reception of L bits require L energy units, while channel sensing costs 1/10 of the energy. System performance is evaluated in terms *energy efficiency* (ξ), defined as the average amount of successfully delivered data bit per unit of energy [10]. The overall system efficiency is then given by

$$\xi = \frac{\bar{D}^{(M)} + \bar{D}^{(S)}}{\bar{W}^{(M)} + \bar{W}^{(S)}} .$$
(11)

Another important performance index is the *goodput*, defined as the average amount of successfully delivered data bits per unit of time. This metric can be easily derived from the mathematical framework presented [6]. However, space constraints do not allow us to present any goodput curve in the remaining of this section.

Performance of different packet types

In the following, we will focus on asymmetric connections only. We denote by $(M \succ S)$ the configuration with only downlink data traffic, and by $(S \succ M)$ the reverse configuration, with only uplink data traffic.

Fig. 2 shows the energy efficiency achieved by the six packet types against SNR for a $(M \succ S)$ connection, in both AWGN (left-hand side) and Rayleigh fading (right-hand side) channels. Curves have been obtained considering a correlator margin S = 0. As expected, system experiments a drastic

performance loss in a Rayleigh channel. DH_5 packet type achieves higher energy efficiency for almost all the SNR values, even though, for SNR< 16 dB, protected packet formats achieve slightly better performance, in particular for high values of the Rice factor K.

Swapping the master and slave roles of nodes, i.e., considering an $(S \succ M)$ asymmetric uplink connection, system performance shows some variation. In Fig. 3, we plot the energy efficiency ratio, given by:

$$\Delta \xi = \frac{\xi(S \succ M) - \xi(M \succ S)}{\xi(M \succ S)} \cdot 100 ; \qquad (12)$$

for Rayleigh fading channels. We can observe that $(S \succ M)$ configuration yields much higher performance than $(M \succ S)$ configuration for SNR ranging from 14 and 25 dB. Maximum performance improvement is obtained for protected packet formats. However, performance gain drastically reduces for increasing values of the Rice factor K.

The receive-correlator margin (S)

The receive–correlator margin S is an important design parameter that may strongly impact on system performance. Low Svalues determine higher selectivity on the packets containing bit errors and limit the energy wasted by receiving payload– corrupted packets. On the other hand, high S values may increase the reception probability for the HEAD field, thus preventing the transmission of DUPCKs and saving energy and capacity. Given that the minimum Hamming distance of two different sync words is of $d_{min} = 14$ bits, an upper bound for S may be set to $d_{min}/2 - 1 = 6$.

Fig. 4 shows energy efficiency curves versus S and SNR for AWGN (top) and Rayleigh (bottom) channels, respectively. Curves refer to an $(S \succ M)$ configuration using DH5 packets. Values are expressed in percentage of the performance achieved with S = 0. Fig.4 reveals that, for AWGN channels, performance achieved by the system improves S > 0, in particular for low SNR values. In case of Rayleigh fading channel, instead, performance get worse with S values greater than 0, except for very low values of SNR. Nevertheless, it may result convenient to set high values of S, since it may be seen that the energy efficiency loss is lower than the goodput gain. The impact of S on system performance, however, rapidly reduces for higher values of SNR.

V. CONCLUDING REMARKS

In this paper, we provided a simple mathematical model for the performance analysis of the Bluetooth system.

The model provides detailed description of the system behavior in different environmental conditions. The study has revealed that, in case of asymmetric client–server applications, better performance is achieved by configuring the unit that hosts the client as master. In this configuration, indeed, the server never retransmits packets that were already correctly received by the client, thus increasing performance. Moreover, the choice of S has shown to be critical, since it may significantly impact on performance achieved by short and protected packet types, although long and unprotected packet types show less dependence on this parameter.

Although our analysis was focused on a piconet with only two units, the mathematical model we propose can be easily extended to more complex piconet structures. The results



Fig. 4. Energy efficiency variation due to S (AWGN (top), Rayleigh channel (bottom), Dxn = POLL, Dym = DH5).

obtain in terms of energy efficiency and average goodput may, then, be exploited to design energy–efficient algorithms for the piconet management.

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