Achievable Rate Regions for Bluetooth Piconets in Fading Channels

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Abstract—In this work we investigate the stability of a Bluetooth piconet operating in fading (Rice, Rayleigh or Nakagami) channels. By taking into account many protocol details, such as modulation scheme and packet formats, we provide an analytical characterization of the achievable rate regions, which, for a network of N nodes, turn out to be the interior of a convex polyhedron in a (2N-2)-dimensional space. We show also that some previous analysis of Bluetooth link performance, by neglecting the impact of the multiple access scheme, may lead to erroneous conclusions in terms of optimal packet format.

I. INTRODUCTION

The Bluetooth technology, promoted by the Bluetooth Special Interest Group [1], aims at providing wireless connectivity to low–power low–cost devices. A primary interest, approaching a new technology, is on performance aspects. In the specific case of Bluetooth, the standard provides up to six packet formats for asynchronous data traffic that differ for time duration, data capacity and error-protection. Unprotected and long packet formats show high payload capacity but are sensitive to payload errors. On the contrary, short and protected formats are less subject to payload errors to the detriment of a lower capacity. Therefore, the performance yielded by such different packet formats may show a tradeoff at varying of radio channel conditions. Furthermore, such performance may, as we will show, strongly interacts with the MAC layer mechanisms. In the paper, we first derive a detailed mathematical characterization of the Bluetooth baseband packet reception mechanism that takes into consideration the characteristics of the received packet format and the effects of radio propagation. The mathematical model is, hence, used to investigate the link performance in a wide range of channel conditions, including AWGN, Rayleigh, Rice and Nakagami fading. The obtained results, in terms of packet error probabilities, are then employed to study the stability of the limited–1 polling scheme commonly deployed in available Bluetooth implementations. In this way, we are able to characterize the limiting performance, in terms of sustainable offered traffic, for the basic Bluetooth network, the so–called piconet, under fading channel conditions. The analysis provides evidence that the MAC mechanisms influence the packet formats which are able to approach the boundary of the stability region. Such an influence should be taken into account in order to gain insight into the packet format to be used in given channel conditions. In particular, we will show that, in general, for homogeneous channel conditions (i.e. same packet error rate on all links) the packet format approaching the achievable rate region boundaries does not coincide with the one achieving the higher throughput over a single Bluetooth link operating on the same channel.

The paper is organized as follows: in Sec. II we describe the Bluetooth technology and point out some related works. Sec. III is devoted to the computation of the packet error rate for the various packet formats encompassed by the standard in Rice and Rayleigh fading channels. In Sec. IV the stability of a piconet operating in fading conditions is addressed. Sec. V presents some analytical results and points out some remarks on the optimal packet format. Sec. VI concludes the paper remarking some open issues.

II. BACKGROUND

A. The Bluetooth Technology

The basic network configuration, in the Bluetooth world, is the so–called piconet, a cluster of no more than eight devices sharing a common frequency–hopping radio channel. The access to the shared medium is regulated by one of the units, called master, which polls the others devices (slaves) to ask for data transmission. Full–duplex is achieved by means of a time division duplexing (TDD) mechanism. Note, that, as a results, only master–to–slave and slave–to–master communications are allowed. A slave–to–slave communication has to undergo a two hops path, passing necessarily through the master unit. The standard does not specify the polling scheme to be adopted; even if offering poor performance, limited–1 polling is the current choice, due to its simplicity and low implementation cost. The Bluetooth standard provides two types of links, asynchronous connectionless links (ACL), aimed at the transport of elastic data traffic, and synchronous connection–oriented links (SCO), explicitly designed for voice applications. In our work, we focus on data communications only, and thus consider ACL links only. To introduce flexibility in bandwidth allocation, the standard encompasses for ACL links the use of packets of three different lengths, namely 1, 3 and 5 slots.
long. Furthermore, to cope with bad channel conditions, an optional (15, 10) shortened Hamming code is provided. The main characteristics of the six different ACL packet formats provided by Bluetooth are summarized in Tab. I.

At the link layer, an automatic repeat request (ARQ) mechanism is provided to ensure correct end-to-end transmissions in lossy time-varying channels. In particular, the standard says that a packet not retransmitted in the subsequent reverse transmission has to be retransmitted “at the next available slot”. However, it is not clear whether packet retransmissions should take place immediately after the failed transmission or at the subsequent polling cycle. We will refer to these two possibilities as immediate and not-immediate ARQ, respectively. Both schemes present advantages and drawbacks. In particular, the immediate ARQ is more suitable for delay-sensitive applications, since it does not require to wait for a whole cycle time. On the other hand, a single host in deep fade may block the operations of the whole network, arising starvation problems. In this sense, not-immediate ARQ provides better resource management, since it does not devote more time (and hence bandwidth) to link with high error probabilities. Furthermore, it is apparent that not-immediate ARQ provides a higher degree of fairness, since, in the long term, the number of transmission attempts of any host will converge to the same value. For such reasons, not-immediate ARQ seems a reasonable choice for data-oriented networks. Thus, in the following, we will limit ourselves to the analysis of such a scheme.

B. Related works

The Bluetooth MAC has been extensively analyzed by means of numerical simulations, but only a very few papers deal with it from an analytical point of view. Although the TDD nature of Bluetooth communications puts it aside from classical polling system, it has been recently shown by Zussman et al. [2], [3] that, by using so-called “virtual” packets, the system may be reduced to a classical gated 1-limited polling system with non-zero switchover times. The basic idea is to consider the first slot of a packet transmission as switchover. Then, we will end up with virtual packets of length, respectively, 0, 2 and 4 slots. Even if this leads to nice results in case of symmetric traffic, the model has to be somewhat changed to accommodate one-way only communications. It is worth noting that these complications may be avoided by simply taking the last slot of a packet transmission as switchover time. In this case we face a simple 1-limited polling system, which may be analyzed by using the well-known classical approximate methods [4], [5], [6]. The Bluetooth link performance in error-prone channels has been first analyzed in [7], where, however, an optimistic full response FSK decoder is considered. The relationship between the BER and the signal-to-noise ratio (SNR), for a common receiver implementation, is widely discussed in [8].

III. LINK PERFORMANCE IN FADEING CHANNELS

The Bluetooth link performance in fading channels has been first analyzed by Valenti et al. [7]. Our analysis follows along similar lines; due to lack of space we omit the derivation and provide directly the final results.

Let $R_h$ be the average signal to noise ratio (SNR) at the $h$ node. In the typical scenario defined for Bluetooth system, the fading process can be assumed flat on the 1 MHz bandwidth and constant over the reception of an whole data packet [12]. Hence, the SNR during the reception of a packet can be expressed as $\gamma_h = \bar{R}_h^\frac{1}{2}$, where the pdf of $\gamma_h$ is given by:

$$f_{\gamma}(u) = 2u (1 + K) e^{-u^2(1+K)} - K \cdot I_0 (2u\sqrt{K(1+K)}) 1(u) ;$$

$$f_{\gamma}(u) = 2ue^{-u^2} 1(u) ;$$

$$f_{\gamma}(u) = 2um (u^2)^{m-1} e^{-mu^2} 1(u);$$

for Rice, Rayleigh and $m-$Nakagami fading, respectively. Let $f$ be a flag which indicates whether a packet is protected by forward error correction ($f = 1$) or not ($f = 0$). Thus, the probability for a $l$-slot long packet to be successfully received by a node $h$ is given by $P_{R_h}(\gamma_h;l,f) = REC_h(\gamma_h) \cdot CRC_h(\gamma_h;l,f)$, where

$$REC_h(\gamma_h) = ((1-\varepsilon(\gamma_h))^3 + 3\varepsilon(\gamma_h)(1-\varepsilon(\gamma_h))^2);$$

$$\sum_{k=0}^{s} \binom{64}{k} \varepsilon(\gamma_h)^k (1-\varepsilon(\gamma_h))^{64-k};$$

$$CRC_h(\gamma_h;l,f) = (1 - \zeta(\gamma_h)^{L_{l,f}}) ;$$

$$L_{l,0} = [DM1 PAYL length/15] ;$$

$$L_{l,1} = [DFl PAYL length/15] ;$$

$$\zeta(\gamma_h) = 1 - (1-\varepsilon(\gamma_h))^{15} - 15\varepsilon(\gamma_h)(1-\varepsilon(\gamma_h)^{14});$$

where $\varepsilon(\gamma)$ gives the BER as a function of the SNR $\gamma$, as specified in [8], while $S$ is the receivercorrelator margin as defined in [13]. (Throughout the following, we assume $S = 0$.) The retransmission probability for a $DFl$ packet transmitted by node $i$ to node $j$ (where either $i$ or $j$ is the master node) is, then, given by $P_{R_{i,j}}(\gamma_{i,j};l,f) = 1 - P_{R_{i,j}}(\gamma_{i,j};l,f) \cdot REC(\gamma_i)$. Assuming the WSSUS (Wide-Sense Stationary Uncorrelated Scattering) model holds, different carrier frequencies experiment independent fading statistics[14]. Hence, by virtue of the frequency hopping (FH) mechanism, we can assume that successive packet transmissions experience independent fading. Averaging over the fading distribution, we

<table>
<thead>
<tr>
<th>Type</th>
<th>Slot occupancy</th>
<th>Max. payload length (bytes)</th>
<th>FEC rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>DM1</td>
<td>1</td>
<td>17</td>
<td>2/3</td>
</tr>
<tr>
<td>DM3</td>
<td>3</td>
<td>121</td>
<td>2/3</td>
</tr>
<tr>
<td>DM5</td>
<td>5</td>
<td>224</td>
<td>2/3</td>
</tr>
<tr>
<td>DH1</td>
<td>1</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>DH3</td>
<td>3</td>
<td>183</td>
<td></td>
</tr>
<tr>
<td>DH5</td>
<td>5</td>
<td>339</td>
<td></td>
</tr>
</tbody>
</table>

TABLE I: Packet characteristics for ACL links
get the average Dfl packet retransmission probability along the \((i, j)\) link:
\[
    r_{i,j}(l, f) = \int_{0}^{\infty} \frac{\alpha_{i}^{2} \Gamma_{1}(\alpha_{i})}{\alpha_{j}^{2} \Gamma_{2}(\alpha_{j})} f_{a}(\alpha_{i}) f_{a}(\alpha_{j}) d\alpha_{i} d\alpha_{j}.
\]

In Fig.1, the goodput versus average SNR for AWGN and Rayleigh fading are plotted for the different packet formats.

Fig. 1. Goodput \((G)\) versus average SNR \((T)\) for AWGN (continuous lines) and Rayleigh (dotted lines) channel, with \(S = 0\).

**IV. STABILITY ANALYSIS**

**A. System Model**

A piconet consisting of \(N\) nodes may be modelled as a system of \(2(N - 1)\) interacting queues. Let us enumerate units in a piconet from 0 to \(N - 1\). For each \(i, j \in \{0, \ldots, N - 1\}\), we use the suffix \((i, j)\) to denote the link between node \(i\) and \(j\). The arrivals at the various queues are assumed to constitute \(2(N - 1)\) independent ergodic stationary processes, with average interarrival period (called also rate in the following) denoted by \(\delta_{i,j}\), expressed in packets/slot. The probabilities of packets generated at node \(i\) to node \(j\) being one, three and five slots long, are, respectively, \(p_{i,j}(1, f), p_{i,j}(3, f)\) and \(p_{i,j}(5, f)\), where \(f\) is the flag defined in the previous section. Note that, trivially, \(\sum_{i=1,3,5} \sum_{f=0,1} p_{i,j}(l, f) = 1\). The information on the traffic characteristics may then be enclosed in the virtual traffic matrix:
\[
    \Delta(s, x)_{i,j} = \delta_{i,j} \sum_{l=1,3,5} \sum_{f=0,1} p_{i,j}(l, f) e^{-slx}.
\]

Note that, \(\Delta\) includes direct slave-to-slave communications, and hence does not reflect the actual traffic pattern. Following the approach of [15], we may then compute the effective traffic matrix, which takes into account for the centralized architecture of the Bluetooth piconet. More in details, let \(k \in \{0, \ldots, N - 1\}\) be the master ID. Let us denote by \(Q_k\) the matrix defined as:
\[
    Q_k = \begin{pmatrix}
        0 & 0 & \ldots & 0 \\
        \vdots & \vdots & & \vdots \\
        1 & 1 & \ldots & 1 \\
        0 & 0 & \ldots & 0 \\
        \vdots & \vdots & & \vdots
    \end{pmatrix}
\]

where the \(k\)-th row is the only non-zero one, and by \(U_k\) the all-zeros matrix presenting a single one in position \((k, k)\). Then, we may write the effective traffic matrix as:
\[
    \Lambda(s, x, k) = Q_k \cdot \Delta(s, x) \cdot (I - U_k) + \Delta(s, x) \cdot Q_k' \cdot (I - U_k),
\]

where \(I\) is the identity matrix. The generic element of the matrix will be written as:
\[
    (\Lambda(s, x, k))_{i,j} = \sum_{l=1,3,5} \sum_{f=0,1} \lambda_{i,j} \pi_{i,j}(l, f) e^{-slx}.
\]

**B. Achievable Rate Regions**

We denote by \(\Psi_{i,j}(l, f)\) the number of transmission attempts necessary to successfully transmit a packet of type \((l, f)\) over the \((i, j)\) link. Due to the frequency hopping schemes provided at the physical layer, and assuming a low–to–medium mobility scenario, packet errors in successive transmission will be uncorrelated. Thus, given the average packet retransmission probability \(r_{i,j}(l, f)\) for the packet type \(Dfl\) over the link \((i, j)\), the number of transmissions for successful reception is a modified geometric random variable, having pmf:
\[
    P[\Psi_{i,j}(l, f) = t] = (1 - r_{i,j}(l, f)) \cdot r_{i,j}(l, f)^{t-1}, \quad t = 1, 2, \ldots
\]

Denoting by \(E[\cdot]\) the statistical expectation operator, we thus have
\[
    \psi_{i,j}(l, f) = E[\Psi_{i,j}(l, f)] = \frac{1}{1 - r_{i,j}(l, f)}.
\]

To consider the average number of retransmission for the link \((i, j)\), we may apply the total expectation theorem, obtaining:
\[
    \psi_{i,j} = \sum_{l=1,3,5} \sum_{f=0,1} \psi_{i,j}(l, f) \pi_{i,j}(l, f).
\]

Since the various packet types present different robustness against link failures, it is apparent that the statistics of head-of-queue packets will not reflect those represented by \(\pi_{i,j}(l, f)\). To make the analysis tractable, we assume that the packets of a single queue are served in random order. Although this assumption may seem odd, it allows us to neglect the dependency between the packets to be served at subsequent cycles due to the ARQ mechanism. Furthermore, from the
point of view of stability analysis, our assumption does not change the system behavior, since it is a well–known fact that stability issues do not depend on the service policy, as long as the latter is work–conserving. Thus, we assume that the probability for the packet served at queue \((i, j)\) to be of type \(DfI\) is given by:

\[
\pi_{i,j}(l, f) = \frac{\psi_{i,j}(l, f)\pi_{i,j}(l, f)}{\psi_{i,j}}, \quad (5)
\]

We will refer to these probabilities as modified packet probabilities.

We define the cycle time \(T_C\) as the time between two successive polls of a single queue; furthermore we define the supercycle time \(\hat{T}_{C_{i,j}}\) as the time between two successive successful transmissions on the link \((i, j)\). It is clear that the latter may be written as the sum of consecutive cycle times:

\[
\hat{T}_{C_{i,j}} = \sum_{k=1}^{\psi_{i,j}} T_C(k).
\]

Exploiting the independency between successive cycles, we take expectation of both members, obtaining:

\[
i_{C_{i,j}} = \psi_{i,j}C. \quad (6)
\]

The cycle time \(T_C\) may be written as the sum of the periods spent exchanging data on the \((i,j)\)-th link, denoted by \(B_{i,j}(\rho_{i,j})\), where \(\rho_{i,j} = \lambda_{i,j} \cdot i_{C_{i,j}}\) is the equivalent load factor of the \((i,j)\)-th queue. The random variable \(B(\cdot)\) has the following probability mass distribution:

\[
P[B_{i,j}(\rho_{i,j}) = k] = \begin{cases} 
\sum_{f=0}^{\rho_{i,j}} \rho_{i,j} \pi_{i,j}(f, k) + 1 - \rho_{i,j} & k = 1, \\
\sum_{f=0}^{\rho_{i,j}} \rho_{i,j} \pi_{i,j}(3, f) & k = 3, \\
\sum_{f=0}^{\rho_{i,j}} \rho_{i,j} \pi_{i,j}(5, f) & k = 5, \\
0 & \text{otherwise}.
\end{cases}
\]

Hence, taking expectation, we get:

\[
b_{i,j}(\rho_{i,j}) = \rho_{i,j} \sum_{f=0}^{\rho_{i,j}} [2\pi_{i,j}(3, f) + 4\pi_{i,j}(5, f)] + 1,
\]

which represents a system of \(2(N-1)\) equations in \(\rho_{i,j}\).

The system may be easily solved, leading to:

\[
\rho_{i,j} = \frac{2N\lambda_{i,j} \psi_{i,j}}{1 - \sum_{r,s=0,...,N-1} \lambda_{r,s} \psi_{r,s} \sum_{f=0}^{\rho_{i,j}} [2\pi_{r,s}(3, f) + 4\pi_{r,s}(5, f)]}. \quad (7)
\]

The stability condition for a 1–limited polling is given by the set of inequalities: \(\rho_{i,j} < 1\) for \(i, j \in \{0, 1, ..., N-1\}\). Referring the reader to [9] for more details on the issue, we remind that such a system of inequalities provides necessary and sufficient conditions for the network to be stable in a stationary ergodic framework. This, of course, holds if the routing matrix does not present a pathological pattern. In our system, for example, the routing matrix prevents packets to undergo more than a two–hop path (in other words, loops are avoided), so that the well–known Baccelli–Foss saturation rule [11] may be applied to justify the above stability conditions. Accordingly, the system of inequalities above leads to a stable system regardless of the statistics of the arrival processes at the various queues.

The stability equations may be more conveniently written in terms of the mean arrival rates:

\[
\lambda_{i,j} < \frac{1 - \sum_{(r,s) \neq (i,j)} \lambda_{r,s} \psi_{r,s} \sum_{f=0}^{1} [2\pi_{r,s}(3, f) + 4\pi_{r,s}(5, f)]}{2N\psi_{i,j} + \psi_{i,j} \sum_{f=0}^{1} [2\pi_{i,j}(3, f) + 4\pi_{i,j}(5, f)]}.
\]

Note that the above system defines a convex polyhedron in a \(2N–2\)-dimensional space. The above systems may be easily translated into an equivalent one for the transmission rates (i.e. goodput) by using the values reported in Tab.I. However, note that in principle the resulting polyhedron in that space could not be convex. Experimentally, we noted that the vertices of the resulting polyhedron are achieved for “pure” policies (i.e. \(\pi_{i,j}(l, f) = 0\) or 1 \forall i, j). It is thus clear that by allowing for “mixed” SAR policies on the various links the resulting polyhedron turns out to be convex.

V. NUMERICAL RESULTS

In this section we will provide some numerical examples of achievable rate regions for some special cases. It is clear that the stability region may be depicted only for the case \(N = 2\), i.e. a single Bluetooth link. Some examples are reported in Fig.2, where the impact of the channel model and of the average SNR are reported. The analysis of such a case, although of limited practical interest, still allows us to get some insight into the impact of some system design parameters. A question of interest, for example, is the impact of the FEC–protection in packets payload field. Fig.3 shows the results obtained for an AWGN channel with an average SNR of 18 dB. We can see that the rate-region achieved by \(DMn\) packets contains the one obtained by unprotected packets. Nevertheless, a larger stability region is reached by using a mix of protected and unprotected packet formats. Another interesting observation is that, in most cases, the boundary of the achievable rate regions may be approached employing 1- and 5-slot packets only, and this may raise questions on the effective utility of \(Dx3\) packets. An exception occurs for SNR values around 18 dB, where the rate region achievable by a mix of \(Dx1\) and \(Dx3\) packets may be not contained in the region achievable by using \(Dx1\) and \(Dx5\) packets. An example of such an "anomalous" behavior is shown in Fig.4 for AWGN channel with 18 and 18.28 dB of average SNR. (Lower values of SNR are of low interest due to the limited performance of the Bluetooth radio system in such
Fig. 5. Achievable rate regions, \( N = 4 \), \( SNR = 18 \) dB in fading channel conditions. We showed how, given channel conditions and packet formats, the problem of defining the achievable rate region may be reduced to the stability analysis of the particular polling mechanism employed, in our case the widely used limited–1 policy. For a piconet of \( N \) nodes, the achievable rate regions turns out to be the interior of a convex polyhedron in a \( 2N - 2 \) dimensional space. The results obtained holds for general stationary ergodic traffic patterns in presence of customer routing. Numerical results were presented, which suggested how the MAC mechanisms might influence the optimal packet format for given channel conditions.

Two research directions seem of interest to enlarge the presented results, namely the extension to a more complex scatternet structure (for which even the case of ideal channel condition seems non–trivial) and to more general polling policies (e.g., exhaustive), where the TDD scheme implemented by Bluetooth does not allow for an immediate application of the results already present in the literature.

VI. CONCLUSIONS

In this paper we studied the performance limits, in terms of achievable rate region, for a Bluetooth piconet operating conditions. We studied also the stability region for the case \( N = 4 \). By considering downlink traffic only and, for the sake of simplicity, equal SNRs on the various links, the achievable rate region takes the shape depicted in Fig.5. Generally, a phenomenon we observed through our analysis is that the boundaries of the stability region may be achieved by employing packet formats which not necessarily are the optimal in the sense of maximizing the link throughput. We believe that our results can be used to improve algorithms such those presented in [17], where the optimization is carried out considering only the aggregated throughput.

REFERENCES


