Performance Analysis of Limited–1 Polling in a Bluetooth Piconet

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Abstract

In this paper we present an analysis of the limited-1 polling policy in Bluetooth piconets employing multislot packets. We characterize the network performance in terms of achievable rate regions and packet delay distribution, where the latter is derived for the case of arrivals forming a marked Poisson point process. In a balanced scenario, our analysis provides exact expressions for the mean packet delay, improving existing results based on the theory of $M/G/1$ queues with vacations. A further approximation, based on a renewal argument, is then proposed. It is shown to successfully track the system behavior (even when the network approaches saturation) leading at the same time to a simple closed-form expression for the average packet delay.

Keywords: Bluetooth, polling, delay analysis, multislot packets

I. Introduction

Bluetooth [1] is a short–range low–power radio technology for ad–hoc communications. Although the delay performance of the basic Bluetooth network configuration (piconet) has been widely evaluated through numerical simulations, no satisfactory analytical framework has been yet proposed in the literature. Misic and Misic [2] presented a model, based on the theory of $M/G/1$ queues with vacations, which provides an approximate expression for the mean packet delay. In their model, however, some dependences among random variables are not taken into account, leading to an underestimation of the delay up to 40% for a heavily loaded piconet operating under the limited–1 regime. These problems have been pointed out in [3], where the authors propose a smart trick to analyze the system, reducing it to a classical gated limited–1 polling system with non–zero switch–over time. In [4] the authors presented another model, also based on $M/G/1$ queueing theory. Although that model slightly improves the one presented in [2], it also fails in tracking the delay behavior as the system approaches saturation.

In this paper we present a novel model for the limited–1 polling policy that, by taking into account some of the interdependencies neglected in the previous works, provides better performance estimation under a wide range of operating conditions. Furthermore, the model provides correct results in the two particular cases where exact expressions for the average delay have been derived in the literature.

We then present a further approximation, which leads to a closed–form expression for the estimation of the average delay. The paper is organized as follows: Sec. II provides a brief overview of the Bluetooth baseband layer characteristics and presents the system model we use for our analysis. In Sec. III we evaluate the network performance, first in terms of achievable rate regions and, then, in terms of packet delay distribution. Sec. IV provides numerical results, presenting comparison of the obtained results with those of [2] and [4]. Sec. V concludes the paper pointing out some open issues for future research.

II. Bluetooth Basis and System Model

A. The Bluetooth Technology

The basic network configuration, in the Bluetooth world, is the so–called piconet, a cluster of no more than eight devices sharing a common frequency–hopping radio channel. The access to the shared medium is regulated by one of the units, called master, which cyclically polls the other devices, named slaves. Full–duplex communication is achieved by means of a time–division duplex (TDD) mechanism. The standard does not specify the polling scheme to be adopted. Even if offering poor performance, limited–1 polling is the current choice, due to its simplicity and low implementation cost. The protocol encompasses two types of links. One, synchronous connection oriented (SCO), is aimed at the transport of real–time services (mainly voice), and is based upon a periodical reservation scheme. The other, asynchronous connectionless (ACL), is designed for the transport of elastic data traffic. In this paper we focus on ACL only. Further, the standard offers, for ACL links, six different packet formats, which differ in length (1, 3 of 5 time slots, where a slot duration is $T = 0.625$ ms) and in the presence/absence of FEC (a shortened $(15,20)$ Hamming code). In this paper we will limit our analysis to the case of error–free channels, so that we will deal only with unprotected packets. A resume of the packet characteristics is reported in Table I. It is worth stressing that in this paper we are interested in evaluating the performance of the MAC protocol only, so that the assumption of ideal channel conditions, while leading
to results which are not directly comparable with those of real world implementations, does not represent a limitation for our study.

B. System Model

Let us introduce some notation. The statistical expectation operator is denoted by $E[.]$. For any random variable $X$, we denote its mean by $x = E[X]$, its statistical power by $x^2 = E[X^2]$, its zeta–transform by $\Lambda(s) = E[z^X]$ and its Laplace–Stieltjes transform (LST) by $\Lambda^*(s) = E[e^{-sX}]$.

A piconet consisting of $N$ nodes may be modelled as a system of $(2N-2)$ interacting queues. Let us enumerate units in a piconet from $0$ to $N-1$. For each $i, j \in \{0, \ldots, N-1\}$, we use the suffix $(i, j)$ to denote the link between node $i$ and $j$. Arrivals to the system are modelled by means of a Poisson point process $\{n_t\}_{t \in \mathbb{N}}$, defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where $t_n$ represents the arrival epoch of the $n$–th packet [5]. We may then define the associated counting process, $N[0, t] = \sum_{n \in \mathbb{N}} \delta_{t_n}([0, t))$, where $\delta_{t_n}(\cdot)$ is the Dirac measure at $t_n$. We assume that the process has a finite non–zero intensity, such that:

$$0 < \lambda = E[N[0, T]) < +\infty,$$

where $T = 0.625$ ms is the slot length. To each point we then associate a mark $\sigma_n$, defined on the same probability space, of the form $(i, j, l)$, which takes value in $\{0, \ldots, N-1\} \times \{0, \ldots, N-1\} \times \{1,3,5\}$. The marks represent, respectively, the queue to which the packet arrives, the intended destination and the packet length. Further, we consider the marks to be independent identically distributed (i.i.d.). Then, we may define the arrival intensity at link $(i, j)$ as:

$$\lambda_{i,j} = \lambda \cdot P[\sigma_n \in \{(i, j, \{1,3,5\})\}], \quad i, j = 0, \ldots, N-1.$$

Note that the arrival processes at the various queues turn out to be independent Poisson processes with intensities $\lambda_{i,j}$. Similarly, we may define the packet length probability for the queue $(i, j)$:

$$p_{i,j}(l) = \frac{P[\sigma_n = (i, j, l)]}{P[\sigma_n \in \{(i, j, \{1,3,5\})\}]} \quad l = 1, 3, 5.$$

Clearly, we have $\sum_{l=1,3,5} p_{i,j}(l) = 1$ for any $(i, j)$. Furthermore, we assume that only one–hop communications take place, so that throughout the following, in writing $(i, j)$, it will be understood that either $i$ or $j$ is the master ID. (The analysis can be extended to take into account slave–to–slave communications as done in [4], where the classical tool of statistical routing is applied in order to get estimates of the mean packet delay.) The system may thus be completely described by means of the traffic matrix

$$A(s)_{i,j} = \lambda_{i,j} \sum_{l=1,3,5} \pi_{i,j}(l)e^{-sl}, \quad i, j \in \{0, \ldots, N-1\}. \quad (4)$$

Note that the LST of the transmission time $Z_{i,j}$ of a packet of queue $(i, j)$ is given by $Z_{i,j}^*(s) = \frac{A(s)_{i,j}}{\lambda_{i,j}}$.

III. Performance Analysis

A. Cycle time statistics

The cycle time $T_C$ is defined as the time interval between two successive polls of the same node. Let $B_{i,j}$ be the part of the cycle time dedicated to the queue $(i, j)$, and denote by $\rho_{i,j}$ the equivalent load factor of the $(i, j)$–th queue, defined as [4]:

$$\rho_{i,j} = \lambda_{i,j}T_C = \lambda_{i,j} \sum_{i,j=0,...,N-1} b_{i,j} \quad (5)$$

Considering that a 1–slot long POLL/NULL packet is sent whenever a queue is found empty, the probability mass distribution of the random variable (r.v.) $B_{i,j}$ is given by:

$$P[B_{i,j} = k] = \begin{cases} \rho_{i,j} \pi_{i,j}(1) + 1 - \rho_{i,j} & k = 1, \\ \rho_{i,j} \pi_{i,j}(3) & k = 3, \\ \rho_{i,j} \pi_{i,j}(5) & k = 5, \\ 0 & \text{otherwise}. \end{cases}$$

Taking expectation, we get:

$$b_{i,j} = \rho_{i,j} (2\pi_{i,j}(3) + 4\pi_{i,j}(5)) + 1.$$ Together with (5) this defines a system of $2N-2$ equations in $\rho_{i,j}$:

$$\rho_{i,j} = \lambda_{i,j} \sum_{l,m=0,...,N-1} [\rho_{l,m} (2\pi_{l,m}(3) + 4\pi_{l,m}(5)) + 1], \quad (6)$$

which (the computation is trivial) solves for

$$\rho_{i,j} = \frac{2N\lambda_{i,j}}{1 - \sum_{l,m} \lambda_{l,m} [2\pi_{l,m}(3) + 4\pi_{l,m}(5)]}. \quad (7)$$

We recall that the piconet is stable if and only if $\rho_{i,j} < 1$ for each $i, j$ [6]. Then, (7) defines the achievable rate regions of a Bluetooth piconet, providing a characterization of the limiting performance in terms of throughput (see [4] for some examples and [7] for an extension to fading channels). It is worth noting that the stability condition applies to a more general stationary ergodic framework, where the expectations are taken with respect to the measure induced by the corresponding Palm probability [8].

Then, in steady state, the average cycle time is given by:

$$T_C = \frac{2N}{1 - \sum_{i,j=0}^{N-1} \rho_{i,j} [2\pi_{i,j}(3) + 4\pi_{i,j}(5)]}. \quad (8)$$

### Table I

<table>
<thead>
<tr>
<th>Type</th>
<th>Slot occupancy</th>
<th>Max. payload length (bytes)</th>
<th>FEC rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>DM1</td>
<td>1</td>
<td>17</td>
<td>2/3</td>
</tr>
<tr>
<td>DM3</td>
<td>3</td>
<td>121</td>
<td>2/3</td>
</tr>
<tr>
<td>DM5</td>
<td>5</td>
<td>224</td>
<td>2/3</td>
</tr>
<tr>
<td>DH1</td>
<td>1</td>
<td>27</td>
<td>-</td>
</tr>
<tr>
<td>DH3</td>
<td>3</td>
<td>183</td>
<td>-</td>
</tr>
<tr>
<td>DH5</td>
<td>5</td>
<td>339</td>
<td>-</td>
</tr>
</tbody>
</table>

**Packet characteristics for ACL links**
The expression for the mean cycle time can be shown to hold in a more general stationary ergodic framework [9], and the same happens for the mean station times $b_{i,j}$, reported in (9), where, again, the expectations are taken with respect to the Palm probability $P^\rho$.

While no characterization of the complete cycle time statistics is possible in general, we may get an approximation by considering the $B_{i,j}$ to be independent random variables. In this way, we get for the cycle time LST:

$$T^*_C(s) = \prod_{i,j=0,\ldots,N-1} B^*_i(s). \quad (10)$$

For example, such an approximation would lead to the following (clearly optimistic) estimate of the cycle time variance

$$\sigma^2_{T_C} = \sum_{i,j=0,\ldots,N-1} \sigma^2_{B_{i,j}},$$

which could be used to obtain bounds on the cycle time distribution by means of the classical Chebyshev’s bound.

### B. Delay analysis

The access delay, $W$, is defined as the time between a packet arrival and its service time beginning, and may be thought as the sum of two terms. The first, $V$, denotes the time interval between the arrival of the packet to the queue and the time instant the queue gets the token. The second describes the time needed to serve all the packets found waiting in queue, whose instant the queue gets the token. The second describes the time between the arrival of the packet to the queue and the time providing (optimistic) estimates of the higher order moments. 

Transforming both members of (11) and applying relationship (12), we thus find

$$W^*_i,j(s) = V^*_i,j(s) \cdot W^*_i,j (\lambda_{i,j} - \lambda_{i,j}U^*_i,j(s)). \quad (13)$$

The packet delay is the sum of the access delay and the transmission delay. The two terms are independent and, thus:

$$D^*_i,j(s) = W^*_i,j(s) \cdot Z^*_i,j(s). \quad (14)$$

Note that, in general, (13) cannot be solved directly, but one should resort to numerical methods to obtain the access delay probability density function [11]. However, statistical moments of any order may be obtained by deriving both members, easily allowing one to get bounds on the delay distribution. To this end, we need to compute the LSTs of $V_{i,j}$ and $U_{i,j}$.

Although the inter-visit time $U_{i,j}$ is technically a polling cycle, its statistic is conditioned on the fact that there is at least one packet waiting at queue $(i,j)$ and, thus, the other queues experience a greater load. Therefore, we can write

$$U_{i,j} = Z_{i,j} + \sum_{(l,m) \neq (i,j)} B_{l,m,i,j}, \quad (15)$$

where $B_{l,m,i,j}$ is distributed like $B_{l,m}$ but with an equivalent load factor $\rho_{l,m,i,j}$. Clearly, in stationary regime, $\rho_{l,m,i,j} = \lambda_{l,m}u_{i,j}$. Therefore, we get a system of equations that, after some algebra, can be shown to solve for

$$\rho_{l,m,i,j} = \frac{\lambda_{l,m} [z_{i,j} + 2N - 1]}{1 - \sum_{(r,s) \neq (i,j)} \lambda_{r,s} [2\pi_{r,s}(3) + 4\pi_{r,s}(5)]}. \quad (16)$$

As for the cycle time, we consider the terms of (15) to be independent. Thus, we get:

$$U^*_i,j(s) = Z^*_i,j(s) \cdot \prod_{(l,m) \neq (i,j)} B^*_l,m,i,j(s). \quad (17)$$

For the computation of $V^*_i,j(s)$ we follow the footprints of [12], generalizing their results in terms of LST upon an independence approximation. Due to lack of space, we do not report the derivation procedure, and give directly the results:

$$V^*_i,j(s) = \sum_{(l,m)} \frac{1}{\lambda_{i,j}} \frac{1 - B^*_i,m(s)}{s} \cdot V^*_{l,m,i,j}(s), \quad (18)$$

$$V^*_{l,m,i,j}(s) = \prod_{(i,j) > (r,t) > (l,m)} B^*_{r,t,l,m}(s), \quad (19)$$

$$\hat{\rho}_{r,t,l,m} = \frac{\lambda_{r,t} (b_{r,t}^2 + 2N - 1)}{1 - \sum_{(u,v) \neq (l,m)} \lambda_{u,v} [2\pi_{u,v}(3) + 4\pi_{u,v}(5)]}, \quad (20)$$

where $B_{r,t,l,m}$ is distributed like $B_{r,t}$ but with an equivalent load factor $\rho_{r,t,l,m}$. Note that, in the productorial of (19) we have to consider all the queues that, in the polling cycle, come
\[
\tilde{w}_{i,j} = \frac{t^2_C}{2t_C \cdot \left( 1 - \lambda_{i,j} \right) \left( 2N + 2\pi_{i,j}(3) + 4\pi_{i,j}(5) + \sum_{l=1}^{N} \pi_l \cdot (2\pi_{l,m}(3) + 4\pi_{l,m}(5)) \right)}.
\]

It is apparent that the derivation of \( V^*_i(s) \) is lengthy and cumbersome. It seems then natural to look for a further approximation, which, on the one hand, is able to track the network behavior till high loads, while, on the other one, leads to a closed form expression for the mean packet delay, so that it can be easily used for dimensioning purposes.

The idea is to approximate the r.v. \( V \) as the residual life in a renewal process having renewal period equal to \( T_C \). Thus, we have [10]:

\[
\tilde{V}^*_i(s) = \frac{1 - T^*_C(s)}{sT_C},
\]

which turns out to be independent of the index \((i,j)\). For example, plugging (21) into (13), after some easy algebra we get a closed–form approximation, reported in (22), for the mean packet delay. We may also use such expression to provide some (rough) buffer dimensioning guidelines. For example, applying Little’s formula, we may get the approximate mean buffer length, \( \tilde{b}_{i,j} = \lambda_{i,j} \tilde{w}_{i,j} \). Then, using Markov bound, we get \( P[\tilde{b}_{i,j} \geq \alpha] \leq \frac{\alpha}{\lambda_{i,j}} \). A better estimate can be found by deriving the second order moments of \( \tilde{W}_{i,j} \) and applying Chebyshev’s bound.

IV. Numerical results

In this section, we present some numerical results which show the goodness of our approximations in two cases, for which the mean access delay may be exactly computed either by using the techniques proposed by [3] or by means of (13). It is worth recalling that, for a general limited–1 polling system with non–zero switchover times (as it is the case in Bluetooth, see [3]), exact expressions for the mean delay are known (for the case of Poisson arrivals) only for the balanced case, where all the queues experience the same load [6], [13].

Thus, we first consider balanced scenarios; in the first case, all queues are active, whereas, in the second one, only downlink traffic is present. We refer to these scenarios as symmetrical and asymmetrical, respectively. The resulting curves are plotted in Fig. 1 and Fig. 2 for \( N = 4, \pi(1) = \pi(3) = \frac{1}{5}, \pi(5) = \frac{7}{5} \). It is apparent that the approximation introduced in (22) leads to accurate results, clearly overcoming those obtained by using the techniques in [2] and [4].

V. Conclusion

In this paper, we have presented a novel mathematical model for performance evaluation of the 1–limited polling policy in a Bluetooth piconet. The proposed analysis improves the previous models in that it provides better performance estimations for a wider range of operating scenarios. Moreover, the model returns the exact average delay in two specific scenarios, for which the exact expression of the mean packet delay is known. We have then introduced a further simplification, deriving, upon a renewal approximation, a closed–form estimation of
the average packet delay that proved to provide fairly good results under a wide range of traffic loads.

Future research directions include, on the one hand, the inclusion of channel errors in the framework and, on the other one, the introduction of a more realistic traffic model (of particular interest is the case of batch arrivals, where a batch corresponds to the segmentation of a single L2CAP layer packet).

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