Capture Analysis in Wireless Radio Systems with Multi-Packet Reception Capabilities

Andrea Zanella
Dept. of Information Engineering
University of Padova (Italy)
Email: zanella@dei.unipd.it

Ramesh R. Rao
Dept. of ECE and Cal(IT)^2
UC San Diego (USA)
Email: rrao@ucsd.edu

Michele Zorzi
Dept. of Information Engineering
University of Padova (Italy)
Email: zorzi@dei.unipd.it

***Abstract*—In this paper, we address the problem of computing the probability that r out of n interfering signals can be correctly received in a random access wireless system with capture. We extend previous results on the capture probability computation, and provide an expression for the distribution of the number of captured packets that is scalable with n and r. We also provide an approximate expression, that is much easier to compute and provides good results for $r = 0$ and $r = n$. Finally, we study the dependence of the system throughput performance on the multi-packet reception capabilities of the receiver.**

I. Introduction

One of the main problems in radio systems consists in the interference produced by overlapping radio signals emitted by different transmitters. When the various signals involved are received with significantly different powers, the so-called *capture effect* may take place, i.e., the strongest signals may "capture" the receiver and survive the collision [1]. Multi-packet reception capabilities have been recently shown to be a key enabling factor for high-capacity wireless networks [2], [3]. In this context, it is of interest to better understand the capture behavior of the receiver, i.e., its ability to correctly decode one or more signals, as a function of its multi-packet reception capability as well as the statistics of the signal powers involved.

In the literature we find two different approaches for modeling the signal capture phenomena in radio systems, one based on the *protocol model* and the other on the *physical model*. The protocol model gives a geometric interpretation of the signal propagation according to which the capture of a signal only depends on the distance between the different transmitters and the common receiver. In [2], [3], in particular, it is assumed that the receiver captures all the signals transmitted within its reception range, provided that all other (interfering) transmitters are at a distance from the receiver larger than a given interference range. This approach makes it possible to carry out elegant performance analysis and to derive close-form bounds for the system capacity in different scenarios, but relies on an idealized and rather unrealistic model. On the other hand, the physical model, which we adopt in this paper, explicitly includes the physical propagation phenomena in the capture model, generally considering the random distribution of the signal powers at the receiver and introducing the Signal-to-Interference-plus-Noise-Ratio (SINR) criterion for determining the capture probability [4]–[6]. If P_j denotes the power of the j-th signal at the receiver, the SINR for that signal is defined as

$$\gamma_j = \frac{P_j}{\sum_{h \neq j} P_h + N_0} \tag{1}$$

where N_0 represents the background noise power. A signal j could be *captured*, i.e., correctly decoded despite the interference produced by the other overlapping signals, only if $\gamma_j > b$, with $b > 0$ representing the so-called *capture threshold* of the system. The capture threshold b is considered a system parameter, whose value depends on the structure of the receiver and, more generally, on the properties of the communication systems. In [5], [6], the authors only considered the case of narrowband systems with a single antenna, for which the capture threshold b is necessarily greater than one and, as a consequence, at most one signal at a time can be captured by the receiver. The authors of [7], [8] proposed a more general analysis of the capture probability, that holds also for $b < 1$, thereby including the case in which multiple signals can be simultaneously received, provided that all of them fulfill the SINR capture condition. In particular, in [7] the authors derive an expression for the probability that there is *at least* one signal above the capture threshold, which is significantly more difficult to compute than in the case $b > 1$.

In this work, we further generalized the results of [7] and provide the following original contributions. We derive an analytical expression of the *complete capture probability distribution*, i.e., we give the expression of the probability $C_n(r)$ that exactly r signals out of n are above the capture threshold for any $0 \leq r \leq n$. Moreover, the numerical evaluation of this expression is scalable with the values of both n and r, unlike the expression in [7] that involves n nested integrations, whose complexity is exponential in n. We also derive a simple approximate expression, based on the central limit theorem, for a lightweight computation of the capture probabilities, which only requires that the received power distribution possesses the first and second moments. Finally, we investigate the system throughput $S_n(k)$ when there is a limit k (called multi-packet reception capability) on the number of signals that can be simultaneously received.

II. Complete capture distribution analysis

In the analysis that follows, we focus on a scenario with n radio terminals, randomly scattered around a common receiver, that simultaneously transmit their signals with fixed transmission power. We assume that each signal is received with a power level in the range (P_m, P_M), with $0 \leq P_m \leq P_M \leq \infty$. The received powers P_j, with $j = 1, 2, \ldots, n$, are assumed to be independent identically distributed (iid) random variables, with Probability Density Function (PDF) $f_P(x)$ and Cumulative Distribution Function (CDF) $F_P(x)$, $x \in (P_m, P_M)$, that depend on the statistics of the stochastic propagation phenomena (fading, shadowing) as well as on the distribution of the distance between transmitter and receiver.

We define the aggregate received power Λ as

$$\Lambda = \sum_{j=1}^{n} P_j + N_0. \tag{2}$$

For the sake of simplicity, in the sequel we omit the noise term that is expected to be negligible with respect to the other terms.[1] The SINR of signal j can hence be rewritten as

$$\gamma_j = \frac{P_j}{\Lambda - P_j} \tag{3}$$

In this section, we will say that a signal is *captured* or *missed* when it experiences either the first or the second of the following conditions

$$P_j > \Lambda b' , \qquad P_j \le \Lambda b' , \tag{4}$$

respectively, with

$$b' = \frac{b}{b+1} \tag{5}$$

The coefficient b' is termed *modified capture threshold*, whereas $\Lambda b'$ gives the *absolute capture threshold* when the aggregate received power is Λ. We aim at determining the expression of the probability

$$C_n(r) = \Pr\left[r \text{ signals out of } n \text{ are captured}\right] \tag{6}$$

Due to the symmetry of the problem, the r captured signals can be arbitrarily chosen. Hence, without loss of generality, we have

$$C_n(r) = \binom{n}{r} c_n(r) \tag{7}$$

where $c_n(r)$ is the probability that signals $1, 2, \ldots, r$, are captured and signals $r+1, \ldots, n$ are missed. In formula:

$$c_n(r) = \Pr\left[P_{1:r} > \Lambda b', P_{r+1:n} \le \Lambda b'\right] \tag{8}$$

where, for brevity, we adopted the compact notation $\{P_{1:r} > \Lambda b'\}$ in place of $\{P_j > \Lambda b', j = 1, \ldots, r\}$ and similarly for the opposite inequalities. Applying the total law of probability on Λ, we get

$$c_n(r) = \int_0^{\infty} \Pr\left[P_{1:r} > xb', P_{r+1:n} \le xb' | \Lambda = x\right] f_\Lambda(x) dx \tag{9}$$

where $f_\Lambda(x)$ is the PDF of the aggregate received power Λ. Applying Bayes' rule we obtain

$$c_n(r) = \int_{nP_m}^{nP_M} f_{\Lambda_r}(x) (1 - F_P(xb'))^r F_P(xb')^{n-r} dx \tag{10}$$

where

$$f_{\Lambda_r}(x) = \lim_{\delta\to 0} \frac{\Pr\left[\sum_{j=1}^{n} P_j \in [x, x+\delta) \middle| P_{1:r} > xb', P_{r+1:n} \le xb'\right]}{\delta}$$

is the PDF of the *conditioned aggregate received power* Λ_r, defined as the aggregate power given that r signals have power above the threshold $\Lambda_r b'$, and $n-r$ have power below such a threshold. We now introduce the auxiliary random variable $\tilde{\Lambda}_r(u)$ defined as

$$\tilde{\Lambda}_r(u) = \sum_{h=1}^{r} \alpha_h(u) + \sum_{k=1}^{n-r} \beta_k(u) . \tag{11}$$

where, for any $u \in (P_m, P_M)$, the random variables $\alpha_h(u), h = 1, \ldots, r$, are independent and identically distributed (iid), with PDF

$$f_{\alpha(u)}(a) = \begin{cases} \dfrac{f_P(a)}{1 - F_P(u)} & \text{for} \quad a \ge u \\ 0 & \text{for} \quad a < u \end{cases} \tag{12}$$

whereas the random variables $\beta_k(u), k = 1, \ldots, n-r$, are iid, and also independent of the $\alpha_h(u)$'s, with PDF

$$f_{\beta(u)}(a) = \begin{cases} \dfrac{f_P(a)}{F_P(u)} & \text{for} \quad a < u \\ 0 & \text{for} \quad a \ge u \end{cases} \tag{13}$$

Accordingly, the PDF $f_{\tilde{\Lambda}_r(u)}(a)$ of $\tilde{\Lambda}_r(u)$ is given by the multifold convolution of $f_{\alpha(u)}(a)$ and $f_{\beta(u)}(a)$. In the frequency domain, the Fourier Transform (FT) $\Psi_{\tilde{\Lambda}_r(u)}(\xi)$ of $f_{\tilde{\Lambda}_r(u)}(a)$ becomes

$$\Psi_{\tilde{\Lambda}_r(u)}(\xi) = \left[\Psi_{\alpha(u)}(\xi)\right]^r \left[\Psi_{\beta(u)}(\xi)\right]^{n-r} \tag{14}$$

where $\Psi_{\alpha(u)}(\xi)$ and $\Psi_{\beta(u)}(\xi)$ are the FTs of $f_{\alpha(u)}(a)$ and $f_{\beta(u)}(a)$, respectively, which are given by

$$\Psi_{\alpha(u)}(\xi) = \int_u^{P_M} \frac{f_P(a)}{1 - F_P(u)} e^{-i2\pi\xi a} da \tag{15}$$

$$\Psi_{\beta(u)}(\xi) = \int_{P_m}^{u} \frac{f_P(a)}{F_P(u)} e^{-i2\pi\xi a} da \tag{16}$$

where $i = \sqrt{-1}$. The function $f_{\tilde{\Lambda}_r(u)}(x)$ can be obtained from (14) through inverse FT, that is

$$f_{\tilde{\Lambda}_r(u)}(x) = \int_{-\infty}^{\infty} \left[\Psi_{\alpha(u)}(\xi)\right]^r \left[\Psi_{\beta(u)}(\xi)\right]^{n-r} e^{i2\pi x\xi} \, d\xi \tag{17}$$

We now notice that, for any x, the function $f_{\tilde{\Lambda}_r(u)}(x)$ with $u = xb'$ is equal to $f_{\Lambda_r}(x)$. Hence, (10) can be expressed as

$$c_n(r) = \int_{nP_m}^{nP_M} f_{\tilde{\Lambda}_r(xb')}(x) \left(1 - F_P(b'x)\right)^r \; F_P(b'x)^{n-r} dx \tag{18}$$

Replacing (17) into (18) and the result into (7) we get

$$C_n(r) = \binom{n}{r} \int_{nP_m}^{nP_M} \left(1 - F_P(b'x)\right)^r \; F_P(b'x)^{n-r} \times \left(\int_{-\infty}^{\infty} \left[\Psi_{\alpha(xb')}(\xi)\right]^r \left[\Psi_{\beta(xb')}(\xi)\right]^{n-r} e^{i2\pi x\xi} \, d\xi\right) dx \tag{19}$$

that provides an exact expression for the probability of capturing r out of n packets, for any values $0 \le r \le n$. Note that this result is completely general and holds for any spatial distribution of the transmitters and any propagation model, provided that the received powers are iid. The actual evaluation of (19) might require numerical methods for the computation of

[1] The analysis can be extended to include the noise term, though at the cost of a more complex notation with no additional insight.

the two nested integrals and of the Fourier transforms (15) and (16), when they cannot be expressed in closed form. However, the computational complexity of (19) is limited for all the cases of interest and, most importantly, it is essentially independent of r and n, so that our method is very scalable. On the other hand, the expression provided in [7, Eq. (19)] only gives the probability of capturing *at least* one signal (which is equal to $1 - C_n(0)$), and involves the explicit computation of n nested integrals, whose complexity grows exponentially with n, and therefore cannot be used except for very small collision sizes.

A. Approximate capture probability distribution

Although in most cases the numerical solution of (19) is affordable, sometimes it might be preferable to resort to an approximate method that provides fairly good results at a much lower computational cost. In fact, for sufficiently large n, the distribution of $\tilde{\Lambda}_r(u)$ can be approximated by a normal distribution, with mean $r\, m_{\alpha(u)} + (n-r)\, m_{\beta(u)}$ and variance $r\sigma^2_{\alpha(u)} + (n-r)\,\sigma^2_{\beta(u)}$, where $m_{\alpha(u)}$, $\sigma^2_{\alpha(u)}$ and $m_{\beta(u)}$, $\sigma^2_{\beta(u)}$ are the mean and variance of $\alpha(u)$ and $\beta(u)$, respectively, provided they exist and are finite. Hence, according to (18), $C_n(r)$ can be approximated by

$$\tilde{C}_n(r) = \binom{n}{r} \int_{nP_m}^{nP_M} \frac{\exp\left(-\frac{\left(x - r m_{\alpha(xb')} - (n-r)\, m_{\beta(xb')}\right)^2}{2\left(r\sigma^2_{\alpha(xb')} + (n-r)\sigma^2_{\beta(xb')}\right)}\right)}{\sqrt{2\pi\left[r\sigma^2_{\alpha(xb')} + (n-r)\sigma^2_{\beta(xb')}\right]}} \cdot (1 - F_P(b'x)^r) F_P(b'x)^{n-r} dx \quad (20)$$

The numerical solution of (20) requires a single integration, which is generally much faster than the numerical solution of (19), and can therefore be used as a simple approximation. In particular, the approximation is excellent for $r = 0$, and $\tilde{C}_n(0)$ turns out to be very close to the correct value $C_n(0)$ already for $n > 4$. This result is of particular interest because it provides a very simple way to have an accurate estimate of the probability that *at least* one signal is captured, $1 - C_n(0)$, which is the performance metric considered in most of the previous literature on the subject [4], [5], [7], [8].

III. Throughput analysis

We now turn our attention to the *system throughput*, defined as the expected number of packets that can be successfully decoded in a slot in which n users transmit. This performance figure has been deeply analyzed in the previous literature, mainly for systems with single reception capability, i.e., able to decode only one packet even when multiple signals experience $SINR > b$. In [7], the analysis was extended to systems with full reception capability, i.e., having the ability of correctly receiving all the packets that satisfy the capture condition.

In this work, we generalize the analysis to systems that can actually decode no more than k simultaneous signals (e.g., due to hardware limitations), even when the number of captured signals is larger than k. We call k the *reception capability* of the system. Denoting by $S_n(k)$ the throughput of a system with reception capability $k \geq 1$, we have

$$S_n(k) = \sum_{r=1}^{k-1} r\, C_n(r) + k \sum_{r=k}^{n} C_n(r) = \sum_{r=1}^{k-1} r\, C_n(r) + kQ_n(k) \quad (21)$$

where $Q_n(k) = \sum_{r=k}^{n} C_n(r)$ denotes the probability that k or more signals are above the capture threshold. Using (19) into (21), we can compute the system throughput for any value of the reception capability k. In particular, the throughput of single reception systems ($k = 1$) is equal to $S_n(1) = Q_n(1) = 1 - C_n(0)$ and can be well approximated using (20), whereas the throughput of full reception systems ($k = \infty$) is $S_n(\infty) = E[r]$, where $E[r]$ denotes the expected value of the number of captured signals and can be computed as in [5].

IV. Case study

In this section, we analyze the performance of a multi-receiver system in three reference scenarios, namely simple Path Loss (PL), simple Rayleigh Fading (RF) and combined Path-Loss and Rayleigh Fading (PLRF).

A. Simple Path Loss model (PL)

For the sake of comparison with the previous literature, the first scenario included in our analysis refers to the case proposed in Section II.E of [7]. The scenario consists of n users uniformly distributed in a circle of radius R centered at the Base Station. The radio propagation is governed by a simple deterministic path-loss law, with neither fading nor shadowing, so that the received power at a distance r from the transmitter is equal to $P(r) = (1 + r)^{-\eta}$ where η is the path loss coefficient. Note that the unit term in the expression accounts for the non-ideality of the power attenuation law in the near field [7]. The net effect is that the maximum value of the received signal is limited to 1. Hence, the power P received from a generic node is a random variable that takes values in the interval $(P_m, P_M) = ((R+1)^{-\eta}, 1)$, with PDF given by

$$f_P(x) = \frac{2}{R^2\eta}\left(x^{-\frac{2}{\eta}-1} - x^{-\frac{1}{\eta}-1}\right) \quad (22)$$

for $(R+1)^{-\eta} \leq x \leq 1$ and zero otherwise. From (22), it is then easy to derive the PDF, CDF, mean and variance of the auxiliary random variables $\alpha(u)$ and $\beta(u)$, though we do not report here the expressions due to space constraints. Unfortunately, in this case the FTs of $\alpha(u)$ and $\beta(u)$ cannot be obtained in closed form, so that all integrals in (19) need to be evaluated through numerical methods. Nonetheless, our evaluations have shown that the numerical computation of (19) can be performed in just a few seconds on an average PC for any n and r, showing that in general our method is very efficient even in the worst case. For the special case of $r = 0$, an even faster evaluation is possible through Eq. (20).

B. Simple Rayleigh Fading model (RF)

In this scenario, all the transmitters are randomly distributed along the circle of unit radius centered at the receiver, so that $r_j = 1$ for all j. However, signals are affected by multi-path fading, which is represented by multiplicative coefficients ρ_j that are assumed to be iid random variables with Rayleigh distribution. It is then easy to realize that the received power

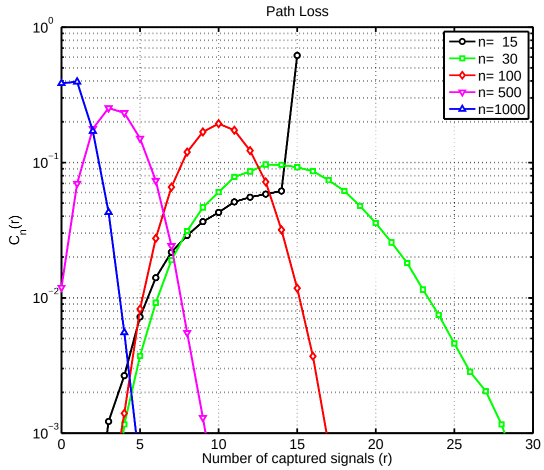

Figure 1. Capture probability distributions $C_n(r)$ vs r in PL scenario when varying the collision size n ($b = 0.02$, $\eta = 2$, $R = 10$).

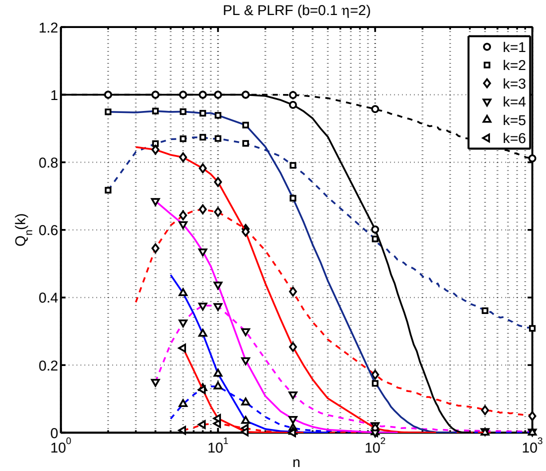

Figure 2. Complementary capture probability $Q_n(k)$ vs n, in PL (solid) and PLRF (dashed) scenarios ($b = 0.1$, $\eta = 2$, $R = 10$).

distribution is given by $f_P(a) = e^{-a}$, for $a \geq 0$ and 0 otherwise, whereas the FTs (15) and (16) are given by

$$\Psi_{\alpha(u)}(\xi) = \frac{e^{-i\,u\,2\pi\,\xi}}{1 + i\,2\pi\,\xi}\,; \quad \Psi_{\beta(u)}(\xi) = \frac{1 - e^{-u\,(1+i\,2\pi\,\xi)}}{(1 + i\,2\pi\,\xi)(1 - e^{-u})}$$

Furthermore, it is possible to determine the first and second order moments of $\alpha(u)$ and $\beta(u)$, which can be used in the approximate expression (20). If we limit our attention to $\tilde{C}_n(0)$, which provides a very accurate approximation of $C_n(0)$, we solely need the moments of $\beta(u)$, reported below

$$m_{\beta(u)} = \mathtt{E}\left[\beta(u)\right] = 1 - \frac{u\,e^{-u}}{1 - e^{-u}} \tag{23}$$

$$M_{\beta(u)} = \mathtt{E}\left[\beta^2(u)\right] = u^2 e^{-u} + 2 - \frac{2u\,e^{-u}}{1 - e^{-u}} \tag{24}$$

C. Combined Path Loss and Rayleigh Fading model (PLRF)

In the last scenario, we combine the effect of path loss and multipath fading. We hence consider n users uniformly scattered around a common receiver, within a disk of radius R. Each signal is affected by independent Rayleigh-distributed multipath fading. By considering a path loss coefficient $\eta = 2$, it is possible to express the PDF of the received power as

$$f_P(a) = \frac{1 - e^{-a\,R^2}(1 + a\,R^2)}{a^2 R^2}, \quad \text{for } a \geq 0 \tag{25}$$

whereas the computation of the FTs (15) and (16) requires numerical methods. Instead, mean and statistical power of $\beta(u)$ can be expressed as

$$\begin{aligned} m_{\beta}(u) &= \frac{\mathtt{Ein}(u\,R^2)}{R^2} - \frac{1 - e^{-uR^2}}{R^2} \\ M_{\beta(u)} &= \frac{-2 + uR^2 + e^{-uR^2}(2 + uR^2)}{R^4} \end{aligned} \tag{26}$$

respectively, where $\mathtt{Ein}(\mathtt{z}) = \int_0^z \frac{1-e^{-t}}{t}dt$ is the exponential integral function [9]. Note that both values in (26) grow indefinitely as u approaches infinity, so that the approximation provided by (20) is not formally valid. Nevertheless, we observed from Fig. 4 that, in all cases considered in this study, it still provides accurate results.

D. Performance analysis

Here we present only a selection of the results obtained in the three scenarios, with the purpose of illustrating how the method proposed in this paper can be used.

Fig. 1 shows the capture probability $C_n(r)$ for $0 \leq r \leq n$, when varying n. To reduce clutter, we plotted only the case PL with $b = 0.02$, $R = 10$, $\eta = 2$. When n is well below $1/b' = 51$, which gives an upper bound on the number of signals that can be potentially captured, then the curves present a spike in $r = n$ because the most likely event is that all the n signals are captured (full capture). When n increases, the full capture probability decreases and the distribution roughly assumes a bell-shaped form, with mean and variance that progressively decrease. Finally, for very large values of n, $C_n(0)$ tends to increase, and the system can capture fewer and fewer signals.

Fig. 2 shows $Q_n(k)$ vs n for different values of the reception capability parameter k, in PL (solid curves) and PLRF (dashed curves) scenarios, with $b = 0.1$, $R = 10$, $\eta = 2$. Although with $b = 0.1$ it would be theoretically possible to capture up to $1/b' = 11$ signals, we observed that for $k > 6$ the capture probability is practically negligible. We also note that the presence of Rayleigh fading augments the diversity of the received signal strength, thus increasing the capture probabilities for large values of n. This observation is confirmed by the throughput curves, obtained with the same settings and reported in Fig. 3. It is interesting to note that increasing the reception capability beyond a certain point yields diminishing returns. For example, in the case shown $k = 6$ already provides a throughput very close to the maximum possible. This result suggests that it is possible to design radio systems with partial reception capability that attain the same performance as systems with full reception capability. Finally, Fig. 4 compares the throughput $S_n(1)$ of systems with single reception capability $k = 1$ (the metric considered in most of the previous literature) for different values of the capture threshold b. Solid curves refer to the RF case, whereas dashed curves are used for the PLRF scenario. The exact results (marks) are

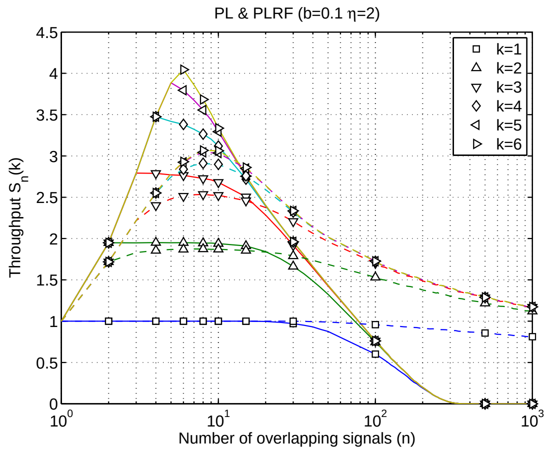

Figure 3. Throughput for different capture capabilities k when varying n, in PL (solid) and PLRF (dashed) scenario ($b = 0.1$, $\eta = 2$, $R = 10$).

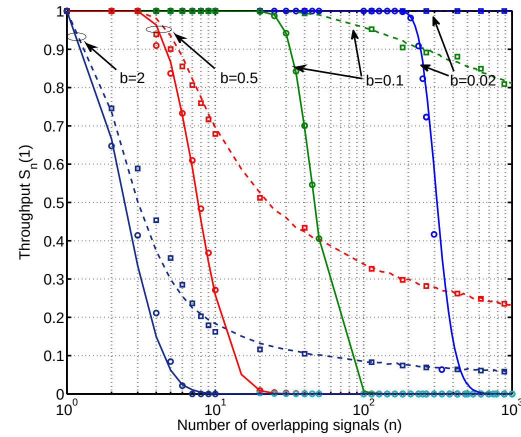

Figure 4. Throughput $S_n(1)$ vs n for single reception systems in PL and PLRF cases, for different values of the capture threshold b. ($\eta = 2$, $R = 10$).

compared with the approximate values (lines) obtained using $\tilde{C}_n(0)$ in place of $C_n(0)$ in (21). As can be noted, the accuracy of the approximation is very good in all the considered cases.

V. Conclusions

The analysis of the capture phenomenon is particularly complex because of the interdependency among the SINR values experienced by the different transmitters. In this paper, we proposed a novel approach for the computation of the probability that r out of n interfering signals can be correctly received. Different from previous approaches presented in the literature, our method deals with these SINR interdependencies in a simple and scalable manner, thus potentially enabling a deeper understanding of the capture phenomenon. We also provided an approximate expression that is much easier to compute and proves to be excellent in estimating the probability that $r \geq 1$ and $r = n$. As an example, we applied the proposed method to study the system throughput when varying the multi-packet reception capabilities of the receiver. The study revealed that increasing the multi-packet reception capability beyond a given level yields marginal benefits. Future directions of research include the extension of this efficient method of analysis to wireless packet networks with more sophisticated physical layers, including power control and iterative interference cancellation [10]. Further manipulation of the analytical expressions may also enable the study of limiting behaviors, following an approach similar to [5].

Appendix

The most demanding operations in the numerical computation of (19) consists in the two Fourier Transforms (15) and (16). A direct application of the Fast Fourier Transform (FFT) algorithm is not practical for large values of n, since it returns N samples equally spaced over the signal bandwidth and, when raising the FTs to a power significantly greater than 1, most of such samples reduce to zero. It is thus more convenient to use the Bluestein's FFT algorithm (BFFT) [11], which provides an efficient way to "squeeze" the N samples of the FT into a fraction λ of the original bandwidth. The BFFT algorithm leverages on the following expression of the FT:

$$\Psi\left(\frac{\lambda k}{NT}\right) = T b_k^* \sum_{h=0}^{N-1} a_h\, b_{k-h}\,, \quad k = 0, 1, \ldots, N \tag{27}$$

$$a_h = f(hT) \exp\left(\frac{-i\pi\lambda h^2}{N}\right) \; ; \quad b_h = \exp\left(\frac{i\pi\lambda h^2}{N}\right) \tag{28}$$

Eq. (27) corresponds to the convolution of the two sequences a_h and b_h of length N, multiplied by N phase factors b_k^*, complex conjugate of b_k. By zero-padding the two sequences to a length $M \geq 2N - 1$, the convolution can be performed efficiently by using, e.g., Cooley-Tukey's algorithm, with a complexity of the order of $M \log(M)$.

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