Design and Analysis of a Microfluidic Bus Network with Bypass Channels
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Abstract—Microfluidics is a multidisciplinary field of research that deals with elementary hydraulic circuits with channels of micrometer size. At this scale, fluids exhibit very specific patterns that cannot be observed at the macro-scale. In particular, vortex forces become negligible, so that the behavior of fluids in the circuit becomes easily controllable and predictable. This technology is currently used in medicine and chemistry to perform specific tasks, such as blood analysis, DNA sequencing, and others. The interest on this technology has been increasing over the last few years and, recently, microfluidic circuits capable of performing simple logical operations have been proposed and experimentally tested, paving the way to a new research branch known as microfluidic networking. In this paper we analyze the design of a microfluidic network with a bus topology, where multiple microfluidic machines are connected to a main channel by means of passive switching elements, realized as T-junctions with bypass (shunt). We mathematically model the system and find the rules to be followed for proper design and dimensioning of such a microfluidic network. We then propose a preliminary performance analysis that gives some insights into the complex interrelations among the different elements of the microfluidic network.

Index Terms—microfluidics, droplet, T-junction, purely hydrodynamic switching, bypass channel, bus topology, throughput.

I. INTRODUCTION

Microfluidics networking is a new research branch that aims at bringing some networking functionalities into microfluidic systems, which are elementary hydraulic circuits (typically made of polydimethylsiloxane) with channels of submillimeter diameters (from hundreds of nanometers to hundreds of micrometers). At this scale, the Reynolds number, which accounts for the impact of fluids momentum to viscosity, can become very low and fluids may exhibit specific behaviors that are unobserved at macro scales [1]. These properties are at the basis of a number of applications, ranging from the inkjet printer heads to DNA sequencing chips, and have been recently exploited in the development of Lab-on-Chip (LoC) systems or, more generally, Microfluidic Machines (MMs), which are used to perform precise chemical/physical processes with very limited amount of reactants [2], [3]. For some of these applications, the microfluidic circuit is designed in such a way that a reactant of some kind, named dispersed phase, is inserted in the form of droplets into another immiscible fluid, called continuous phase, that acts as carrier and conveys the reactant to a MM where reactions take place. Recently, physics researchers have advanced the idea of using this technology to build tiny computing units [4], [5], and the possibility of realizing simple boolean functions by using droplet-based microfluidic technology has been experimentally proved.

Along this stream, a very interesting research challenge consists in developing basic microfluidic structures able to interconnect specialized MMs by means of a flexible and modular microfluidic network, thus dramatically enlarging the capabilities of microfluidic systems. This challenge has attracted the interest of researchers from many disciplines, included chemistry, physics, and engineering, and different approaches to enable the control of droplets throughout a microfluidic network have been advanced. A possible solution consists in electro-hydrodynamic actuation [6], [7] that, however, requires specific and complex circuitry with a large number of electrical connections and high power source. Furthermore, the contact of electrodes with fluids may give rise to corrosion problems and unwanted reactions.

Most of these problems are solved by adopting a purely hydrodynamic approach that only relies on the actuators (pumps and reservoirs) at the periphery of the chip (the boundary system), and on the channel geometries and hydrodynamic forces that act on the fluids to control the droplets in the network. The basic principle is that droplets flow along the path with minimum instantaneous fluidic resistance, meanwhile increasing the resistance of the channel they are crossing. Thus, an isolated droplet entering a T or Y junction through the inlet will proceed toward the outlet with minimum instantaneous fluidic resistance, while a closely following droplet may be driven to the other outlet [4], [8]. Therefore, it is possible to steer a “payload” droplet through a series of junctions by modulating its distance with respect to a certain number of “control” droplets [9], [10].

A first attempt to formalize the problem of designing a purely hydrodynamic microfluidic network was made in [11], [12], where the authors proposed some approximate equations to derive the fluidic resistance of a channel as a function of its geometry, the characteristics of the continuous flow and the variation of the fluidic resistance produced by the presence of a dispersed droplet in a channel. Furthermore, the authors applied the developed framework to the design of a microfluidic network with bus topology, where many MMs were attached to the main channel (the bus) by means of simple T-junctions.

In this paper we further advance the analysis of microfluidic networks by considering a new kind of switching unit, recently proposed by Cristobal et al. [13], that consists in connecting
the two outlets of a T-junction by means of a specially designed bypass channel that acts as a microfluidic shunt. As it will be explained later on in this paper, the bypass channels make it possible to increase the performance of the network and simplify the design of the system.

The rest of the paper is organized as follows. In Sec. II we recall the basic principles of microfluidics networking. Sec. III describes the switching unit based on a T-junction with bypass channel and derives the mathematical model that defines the switching properties of the structure. In Sec. IV we discuss the design of a microfluidic network with bus topology, realized by means of T-junctions with bypass, while Sec. V analyzes the throughput of the network when varying some design parameters. Finally, Sec. VI draws conclusions and discusses some possible extensions of the work. Note that, for reader’s convenience, the main notation has been collected in Tab. I, whose rightmost column contains the actual parameter values used in the analysis that will be developed in Sec. V.

II. MICROFLUIDICS BASICS

For self consistency, in this section we recall the mathematical relations, first introduced in [12], that model the basic microfluidic features.

A. Droplet generation

In a droplet-based microfluidic network, the continuous phase is injected into the channels by external pumps, such as syringe pumps and peristaltic pumps, that can regulate the volumetric flow rate \( Q \) [m\(^3\)/s] of the fluid in the channel, or the pressure drop \( \Delta P \) [Pa] across a channel. The dispersed phase is injected into the continuous flow by other, independent pumps, as sketched in Fig. 1. The creation of droplets is governed by the capillary number, a dimensionless parameter that describes the relative magnitude of the viscous shear stress compared with the interfacial tension, given by

\[
C_a = \frac{\mu_c u_c}{\sigma} = \frac{\mu_c Q_c}{\sigma w h},
\]

where \( w \) and \( h \) are width and height of the channel section, \( u_c = \frac{Q_c}{w h} \) is the average velocity of the continuous stream, \( \mu_c \) [Pa s] is its dynamic viscosity, and \( \sigma \) [N/m] is the interfacial tension coefficient between the dispersed and the continuous phases. The shape of the droplets is highly controllable only in the so-called squeezing regime [14], [15], which requires \( C_a < C_a^* \approx 10^{-2} \).

Considering (1), the squeezing regime sets the following upper bound to the flow rate:

\[
u_c < \frac{\sigma}{\mu_c} 10^{-2}.
\]

The length \( \ell_d \) of the droplets created at the T-junction in the squeezing regime can be approximated as [15]

\[
\ell_d = w \left(1 + \xi \frac{Q_d}{Q_c}\right),
\]

where \( \xi \) is a dimensionless parameter of order 1. Thus, droplet length can be tuned by adjusting the volumetric flow rates \( Q_c \) and \( Q_d \) of the continuous and dispersed phases, respectively, provided that (2) holds. Given \( \ell_d \), from the mass conservation principle, it follows that the inter-droplet distance is given by [12]

\[
\delta = \ell_d \frac{Q_c}{Q_d} = w \left(1 + \xi \frac{Q_d}{Q_c}\right) \left(\frac{Q_d}{Q_c}\right)^{-1}.
\]

B. Fluidic resistance

When arriving at a junction, a droplet will follow the branch with instantaneous lowest fluidic resistance. For a microchannel with rectangular section, the resistance can be expressed as

\[
R(\mu, L) = \frac{a \mu L}{w h^3},
\]

where \( L \) is the length of the channel ([meter]), whereas \( a \) is a dimensionless parameter that depends on the channel geometry [16].

When a droplet is injected into a duct, the friction generated with the carrier fluid and the forces produced by the inhomogeneity between the dynamic viscosity of continuous and dispersed phases determine an increase in the fluidic resistance of the channel [17], [18], [19], so the actual fluidic resistance in different parts of a microfluidic network may change over time, depending on the paths followed by the droplets, and on their size.

Mathematically, the variation of resistance produced by a droplet of length \( \ell_d \) injected into a channel of length \( L \) can be approximated as

\[
\rho(\ell_d) = R(\mu_c, L - \ell_d) + R(\mu_d, \ell_d) - R(\mu_c, L) = (\mu_d - \mu_c) \frac{\ell_d a}{w h^3},
\]
where \( \mu_c \) and \( \mu_d \) are the dynamic viscosities of continuous and dispersed phases, respectively.\(^1\)

### C. Microfluidic-electronic duality

In [21], [22], the authors have advanced an interesting duality principle between continuous-flows (layered) microfluidic systems and electric systems, according to which the pressure difference \( \Delta P \), volumetric flow rate \( Q \), and hydraulic resistance \( R \) of a microfluidic channel can be associated to the voltage drop \( \Delta V \), current intensity \( I \), and ohmic resistance \( R_E \) of an electric line, respectively. Furthermore, Ohm’s and Kirchhoff’s current and voltage laws,

\[
\Delta V = R_E I, \quad \sum_{m=1}^{M} I_m = 0, \quad \sum_{k=1}^{K} V_k = 0, \quad (7)
\]

would correspond to the Hagen-Poiseuille law, and the flow and energy conservation laws in microfluidic circuits:

\[
\Delta P = RQ, \quad \sum_{m=1}^{M} Q_m = 0, \quad \sum_{k=1}^{K} \Delta P_k = 0, \quad (8)
\]

where \( M \) is the number of branches emanating from a junction, while \( K \) is the number of contiguous channel segments in any closed loop. Accordingly, the equivalent hydraulic resistance of a series of \( H \) microfluidic resistors is the sum of all \( H \) resistances, while the conductance of the parallel of \( H \) resistors is the sum of their conductances.

The analogy between electric and microfluidic circuits can be further extended to the sources in the corresponding domains (see Fig. 2). In particular, an ideal current generator which provides constant electric current through a line, irrespective of the electrical resistance of the line, is equivalent to an ideal syringe pump that maintains a constant volumetric flow rate through a channel, independently of the hydraulic resistance of the channel. On the other hand, an ideal voltage generator, which maintains a certain voltage across a line, is similar to a peristaltic pump that maintains a constant pressure difference across a channel.

This duality principle, originally proposed for layered microfluidic systems, has been successively extended in [12] to droplet-based microfluidic circuits by introducing time-varying resistors in the electrical dual circuit to account for the increase of resistance induced by a droplet when crossing a channel. This model has been applied to the analysis of a bus network and verified by means of OpenFOAM simulations.

### D. Junction crossing and switching

When crossing a T-junctions, a droplet can either split into the two outgoing branches (breakup regime) or stay compact and flow along the outlet branch with minimum instantaneous fluidic resistance (non-breakup regime). According to the mathematical model proposed in [19], the non-breakup regime is observed when

\[
\ell_d < \ell_d^* \approx \chi_w C_d^{-0.21},
\]

where \( \chi \) is a dimensionless parameter that decreases as the viscosity ratio \( \lambda = \mu_d / \mu_c \) increases [20]. To remain in the non-breakup regime, then, the droplet velocity shall not exceed the limit

\[
u^* = \frac{\sigma}{\mu_c} \left( \frac{\chi w}{\ell_d^*} \right)^{\frac{1}{2\pi \tau}}.
\]

Putting together conditions (2) and (10), the maximum input velocity \( u^* \) at any given time can be expressed as:

\[
u^* = \frac{\sigma}{\mu_c} \min \left\{ 10^{-2} \left( \frac{\chi w}{\max\{\ell_d^h, \ell_d^p\}} \right)^{\frac{1}{2\pi \tau}} \right\}, \quad (11)
\]

where \( \ell_d^h \) and \( \ell_d^p \) are the lengths of the header/payload droplet pair injected into the channel.

In this case, a droplet entering a T-junction will be steered to the outlet with the lowest instantaneous fluidic resistance, say \( R_1 \), or, equivalently, the greatest volumetric flow rate, \( Q_1 \). As a consequence, the resistance of the selected outlet will be increased to \( R_1 + \rho \). If the increased resistance is larger than the resistance \( R_2 \) of the other outgoing branch, the second droplet (namely, the payload) closely following the first one (namely, the header) will switch into the secondary outlet. Conversely, if \( R_1 + \rho < R_2 \), the payload will flow along the same path followed by the header. Therefore, controlling the length of header droplets, it is possible to deterministically control the path of payload droplets through a junction.

### III. T- Junction with bypass channel

The switching principle described in the previous section is actually quite fragile and sensitive to small imperfections in the fabrication of the junction, or noise and fluctuations in the operational parameters (pressure, flow rate, frequency and size of incoming droplets). A solution to increase the robustness of the switching elements to non-idealities and, thus, provide reliable control of the droplet behavior is hence desirable. To this end, Christobal et al. [13] proposed a simple solution to realize robust switching elements. It consists in placing a bypass channel between the two outlet branches of a bifurcation, as sketched in Fig. 3 that also reports the associated electrical

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\(^1\)According to (6), a droplet increases the hydraulic resistance of a channel only if \( \lambda = \frac{\mu_d}{\mu_c} > 1 \). However, some experiments have revealed an increment in resistance also for \( \lambda < 1 \). The reason for this discrepancy is that (6) does not account for a number of secondary forces that act on the narrow region between the droplet boundaries and the channel walls, which are generally negligible for \( \lambda > 1 \). Some more comments on this regard can be found in [12].
being. Fig. 3, we get: the network. This last aspect is particularly welcome since it simplifies the design of a complex network.

In order to present these desired properties, the bypass channel must satisfy the following conditions.

- The resistance \( R_{bp} \) of the bypass channel shall be much lower than the resistance \( R_1 \) and \( R_2 \) of the two outlet branches. In the electrical dual circuit, this means that the voltage at points A and B (see Fig. 3) shall be almost the same.

- The bypass channel shall not let the droplets flow through it. To achieve this goal, the junction of the bypass channel with the outlet branches is obstructed by barriers that are designed to block the passage of droplets, while letting the continuous phase flow freely through them. Hence, droplets are forced to exit the switch only through the channels with resistance \( R_3 \) and \( R_4 \), directly connected to the two outlet branches.

As the total flow rate \( Q \) splits between \( Q_1(t) \) and \( Q_2(t) = Q - Q_1(t) \), the sign of \( Q_1(t) - Q_2(t) \) determines the direction taken by an isolated droplet that enters the switch. Quantitatively, by applying Kirchhoff laws to the electrical scheme of Fig. 3, we get:

\[
\begin{align*}
Q &= Q_1 + Q_2 \\
Q_1 &= Q_3 + Q_{bp} \\
Q_2 &= Q_4 - Q_{bp} \\
R_1Q_1 &= R_2Q_2 - R_{bp}Q_{bp} \\
V_1 + R_3Q_3 - R_{bp}Q_{bp} - R_4Q_4 - V_2 &= 0
\end{align*}
\]  
(12)

being \( Q_i \) the current across resistor \( R_i \). The solution of (12) is hence

\[
\frac{Q_1 - Q_2}{Q} = \frac{(R_2 - R_1)(R_3 + R_4)}{(R_1 + R_2)(R_3 + R_4 + R_{bp}) + R_{bp}(R_3 + R_4)}
\]

\[
R_{bp}\left[\frac{R_2 - R_1}{(R_1 + R_2)(R_3 + R_4 + R_{bp}) + R_{bp}(R_3 + R_4)} + \frac{R_2 - R_1}{(R_1 + R_2)(R_3 + R_4 + R_{bp}) + R_{bp}(R_3 + R_4)}\right]
\]

\[
\frac{Q_1 - Q_2}{Q} = \frac{R_2 - R_1}{R_1 + R_2},
\]

(14)

from which we see that the switching element becomes simultaneously very sensitive to \( R_2 - R_1 \) and insensitive to \( R_3, R_4 \) and \( p_B - p_A \). For the sake of simplicity, in the following we assume that approximation (14) holds, so that we will consider the bypass channel as a “short circuit” in our analysis.

In particular, we observe that, while with standard T-junctions all the different parts of a network are interdependent, and the switching of a single droplet affects the behavior of all the other droplets in the system (see [12]), with bypass junctions the different parts of the network become almost independent. Therefore, the introduction of the bypass channel not only makes the system more robust to perturbations, but it also simplifies the design of the network.

IV. CASE STUDY: BUS-TOPOLOGY DESIGN

To evaluate the potential benefits that can be brought about by bypass channels, we consider the bus network presented in [12] where we replace the standard T-junctions with bypass junctions, as depicted in Fig. 4. We observe that MMs are numbered from 1 to \( N \) starting from the rightmost side of the bus channel and moving upwards toward the droplet source.

Assuming \( R_{bp} \approx 0 \), the equivalent downstream resistance seen after each junction \( n \), can be recursively expressed as

\[
R_{eq,n} = \frac{R_{3,n}R_{2,n}}{R_{1,n} + R_{2,n}} + \frac{(R_{3,n} + R_{eq,n-1})R_{4,n}}{R_{3,n} + R_{eq,n-1} + R_{4,n}}
\]

(15)

with \( R_{eq,0} = 0 \). Note that, resorting to the electric duality principle, given the input velocity of the droplets in the system (i.e., the current \( Q \)), we can determine the speed of the fluids \( Q_{i,n} \) in every channel of the network. Referring to Fig. 4, in
branch to the target MM. The right choice, therefore, depends
directly connected to the target MMs, we see that, according
\[ \beta \]
where 
for some 
\( \zeta > 0 \)
and having 
\[ Q_{3,n+1} = Q = \omega / h u. \]
We now need to determine the system parameters in order to achieve the desired behavior for the droplets. First of all, in plain conditions (i.e., without droplets), the stream along the main bus shall be the strongest, so that isolated droplets will always flow through the bus all the way down till its end. Since, according to (14), the switching behavior at the \( n \)th bypass junction is determined by the parallel between \( R_{1,n} \) and \( R_{2,n} \) only, we simply require
\[ R_{1,n} < R_{2,n}. \] (17)
Second, we need to establish the right resistance increment \( \rho_n \) produced by the header droplet in order to switch a closely following payload droplet toward the \( n \)th outlet branch to the \( n \)th MM. To this end, two different conditions must hold.

1) The payload droplet must flow along the bus till it reaches the \( n \)th bifurcation. Mathematically, this requires
\[ \rho_n < \min \{R_{2,j} - R_{1,j}; j = n + 1, \ldots, N\} \]
where we conventionally set \( R_{2,N+1} - R_{1,N+1} = \infty \).

2) The payload droplet must take the \( n \)th outbound branch when it arrives at the corresponding bifurcation, which requires \( \rho_n > R_{2,n} - R_{1,n} \).
Combining the two conditions we get
\[ R_{2,n} - R_{1,n} < \rho_n < R_{2,n+1} - R_{1,n+1}, \] (18)
from which we see that the length of the header droplet must increase with \( n \). However, there still remain several degrees of freedom in the network design. For the sake of simplicity, we hence set the length \( L_{1,n} \) and \( L_{4,n} \) of all horizontal branches of bypass switches to the same value \( L' \), so that we have \( R_{1,n} = R_{3,n} = R' = \frac{\alpha_n L'}{h u} \) for \( n = 1, \ldots, N \). Under this assumption, (18) yields \( R_{2,n+1} > R_{2,n} \), so that the length \( L_{2,n} \) and \( L_{3,n} \) shall increase with \( n \), as depicted in Fig. 4.
To satisfy both (17) and (18) we thus set
\[ R_{2,n} = R_{1,n}(1 + n \zeta) = R'(1 + n \zeta), \] (19)
for some \( \zeta > 0 \), so that \( \rho_n \) can be expressed as
\[ \rho_n = \beta (R_{2,n+1} - R_{1,n+1}) + (1 - \beta)(R_{2,n} - R_{1,n}) \]
\[ = R' \zeta (n + \beta), \] (20)
where \( \beta \in (0, 1) \). Concerning the length \( L_{4,n} \) of the branches directly connected to the target MMs, we see that, according to (16), large values of \( R_{4,n} \) will favor the flow of droplets along the main bus, while penalizing the speed along the outlet branch to the target MM. The right choice, therefore, depends
\[ \frac{Q_{1,n}}{Q_{2,n}} = \frac{R_{2,n}}{R_{1,n} + R_{2,n}} \],
\[ Q_{2,n} = Q_{3,n+1} \frac{R_{2,n}}{R_{1,n} + R_{2,n}} \]
\[ Q_{3,n} = Q_{3,n+1} \frac{R_{4,n}}{R_{3,n} + R_{eq,n-1} + R_{4,n}} \]
\[ Q_{4,n} = Q_{3,n+1} \frac{R_{4,n}}{R_{3,n} + R_{eq,n-1} + R_{4,n}} \]
for \( n = 1, \ldots, N \) and having \( Q_{3,N+1} = Q = \omega / h u. \)

V. PERFORMANCE ANALYSIS

Once dimensioned the bus network with bypass channels, we can evaluate its performance. To this end, we consider a network with \( N = 10 \) MMs and \( \zeta = 1 \) and analyze the performance, in terms of throughput to a specific MM, when varying the length \( \ell_d \) of the payload droplet. Since the objective of microfluidic networks is to carry reactants to the MMs (to perform some sort of reactions), we define the network throughput in terms of mean volume of payload droplets delivered to the intended MM in unit time, i.e.,
\[ S = \lim_{t \to \infty} \frac{V(t)}{t}, \] (22)
where \( V(t) \) is the total volume of dispersed fluid delivered to the intended receivers in time \( t \).
We consider a simple access policy, named exclusive access, according to which only a single header-payload droplet pair can be crossing the network at any given time.

Fig. 5 shows the network throughput for different target MMs (namely, number 3, 4, and 7). As expected, the throughput is lower for MMs closer to the source (larger \( n \)) because, according to (19) and (21), the lengths \( L_{2,n} \) and \( L_{4,n} \) of the outlet branch rapidly increases with \( n \) and, consequently, droplets need to travel longer distances to reach the target MM, once deviated along the proper outlet branch. Furthermore, the values of \( \ell_d' (n) \) also increases with \( n \), thus limiting the maximum speed of the continuous flow \( u^* \), according to (11).
Given the target MM, we see that the throughput increases with the payload length \( \ell_d (n) \) until it reaches the length \( \ell_d' (n) \) on the distribution of the traffic to the different MMs. In this study, we arbitrarily set
\[ R_{4,n} = 2n R' \] (21)
that, according to the result of some simulations, cuts a good balance between the two opposite effects, thus yielding good performance. The systematic analysis of the optimal setting of this parameter is left for future work.
of the corresponding header droplet that steers the payload towards the $n$th MM. For larger values of $\ell_d(n)$, according to (10) the input velocity (and, hence, the speed of the droplets) must be decreased to avoid break-ups at junctions, so that the throughput is also decreased. This trend is confirmed by the graph of Fig. 6, which shows the time $b(n)$ taken to deliver a payload droplet to the $n$-th MM. We observe, in fact, that the delivery time to a certain target is almost constant with $\ell_d$ till the critical point, corresponding to $\ell_d^c(n)$, after which it quickly increases due to the reduction of the input velocity.

VI. Conclusions

In this paper we studied the properties of a microfluidic switching element realized by means of a T-junction with a bypass channel. We introduced this building block in a pure hydrodynamic microfluidic network with bus topology, for which we derived the throughput, in terms of average volume of dispersed flow delivered to the intended MM in the unit time, and drew a number of considerations. Comparing the throughput of the bus network with bypass channels with that of a topologically identical network without bypass channels, as that in [12], we observe that the bypass solution largely outperforms the other one, even when adopting the exclusive access policy, which is extremely conservative in the utilization of the network. In addition, bypass channels make it possible to treat every junction separately from the rest of the network. In this way, droplets in different parts of the network do not influence each other, except for those inside the same switching element. This property makes it possible to define more advanced access policies, thus leaving space for further significant increase in system performance.

VII. Acknowledgements

This work has been partially funded by the University of Padova through the PRAT2012 project “Microfluidic Networking (MiNET): introducing networking technologies in microfluidic systems.”

References