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Uplink resource allocation in cellular systems: An energy-efficiency perspective

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***Abstract*—In this work we address the problem of optimal resource allocation in the uplink of a wireless cellular network with Rayleigh fading channels, where the aim is to minimize the average total energy spent for packet delivery. The devices are assumed to transmit over orthogonal resources where the total energy used for each resource is modeled as the sum of the transmit energy and an overhead circuit energy. We first derive the optimal allocation for the single-user case when varying our assumptions on both the Channel State Information available at the transmitter and the Automatic Repeat reQuest capabilities. Then, we generalize the analysis to the multi-user case and compare the results obtained for the different scenarios.**

***Index Terms*—Energy efficiency; resource allocation; uplink cellular network; CSI; ARQ.**

I. Introduction

In this paper we study the problem of energy-efficient resource allocation in the uplink of a wireless cellular network with Rayleigh fading channels. The aim is to investigate the impact of different transmission policies on the overall energy spent by a node to deliver a given number of bits to the Base Station (BS), with a given outage probability. Therefore, we depart from the more classic throughput maximization objective and, instead, focus on the optimization of energy efficiency. The study is based on a simple but common model for the energy consumption, according to which the total energy spent by the devices consists of the sum of the transmit power, which impacts on the transmit bitrate and the packet error probability, and a constant power that is assumed to be absorbed by the circuits when the node is active, irrespective of the transmit power.

A similar model is considered in [1] where the authors propose an information-theoretic characterization of the multiple access scenario, defining the theoretical capacity region but without presenting any practical resource allocation algorithm. In [2], instead, the authors considered the impact of optimal sleep schedules in practical networks. Reference [3] reports a study of energy-efficient scheduling for Machine-to-Machine (M2M) communication without considering the overhead energy cost. An algorithm known as MoveRight is derived in [4] to solve general scheduling problems with energy minimization as the goal. However, the algorithm assumes that the energy for each device is monotonically decreasing with the number of allocated resources, an assumption that does not hold in the more realistic setting considered in this work, as will be discussed later. In [5], the authors consider energy efficiency issues for Time Division Multiple Access over fading channels with finite-rate feedback, where only quantized Channel State Information (CSI) is available at the transmitter through a finite number of bits of feedback from the receiver. Such model, however, does not take into account the overhead due to the circuit energy. Based on finite-rate feedback, [6] also investigates the problem of minimizing the transmit power in systems based on orthogonal frequency-division multiplexing. Finally, [7] introduces Quality of Service constraints in the resource allocation problem for multicarrier systems and optimizes the downlink and uplink transmit power consumption over a finite set of available modes of operation.

In this work we define a multi-step procedure for the optimal resource allocation in an uplink network affected by Rayleigh fading where the energy metric to be minimized takes into account both the transmit energy and a circuit energy overhead for connection set-up and maintenance operations. We describe such a method and evaluate the solution in the single user case for different scenarios, depending on both the information we have at the transmitter and the BS capabilities. Then we extend the study to a scenario with multiple users.

The remainder of the paper is organized as follows. In Sec. II we describe our system model and define the optimization problem for resource allocation. In Sec. III we solve the problem by using numerical methods, and then derive suboptimal resource allocation strategies under different assumptions, for the single-user case. Sec. IV, instead, presents the generalization of the problem for the multi-user scenario. Finally, in Sec. V we draw our conclusions.

II. System model

We consider a set $\mathbf{K} = \{1, \dots, K\}$ of devices, where device k has L_k bits to send to a common BS in a single-hop fashion. We assume a scheduled system where the BS allocates a set of N_{tot} time-frequency resources, each of equal duration Δt [s] and bandwidth Δf [Hz], to the users in $\mathbf{K}$. The generic kth user is assigned N_k resources, such that $N_{tot} = \sum_{k=1}^{K} N_k$.

A. Channel model

Let $g_{k,n}$ denote the channel gain of user k on the nth resource. We assume that $g_{k,n}$ can be modeled as an exponential random variable with mean $\overline{g}_k$, so that

$$\Pr[g_{k,n} \le x] = 1 - \exp(-x/\overline{g}_k), \quad x \ge 0.$$

Furthermore, we assume that the channel gains are mutually independent, across both users and resources.

B. Energy model

For each transmission, let $E^{(t)}_{k,r_{k,n}}$ denote the amount of energy that device $k \in \mathbf{K}$ spends transmitting in resource $r_{k,n}, n \in \{1, \ldots, N_k\}$. To make the model more realistic, besides $E^{(t)}_{k,r_{k,n}}$, we assume that each transmission with positive energy incurs also a fixed energy cost $E^{(c)}_k$, which accounts for the rate-independent energy consumptions (e.g., the energy dissipated by the circuit to receive the BS beacon, to wake up from sleeping states and so on). As a result, the overall energy consumed by device k to transmit on resource $r_{k,n}$ is

$$E_{k,r_{k,n}} = [E^{(t)}_{k,r_{k,n}} + E^{(c)}_k]\chi\left\{E^{(t)}_{k,r_{k,n}}\right\}; \tag{1}$$

where $\chi\{\cdot\}$ is defined as

$$\chi\{x\} = \begin{cases} 1 & \text{if } x > 0\,, \\ 0 & \text{if } x = 0\,. \end{cases}$$

The total energy cost for user k to transmit the message over its N_k allocated resources[1] is hence

$$E^{(tot)}_k = \sum_{n=1}^{N_k} \left([E^{(t)}_{k,r_{k,n}} + E^{(c)}_k]\chi\left\{E^{(t)}_{k,r_{k,n}}\right\}\right). \tag{2}$$

In the following, we assume that, as long as there are bits to transmit, each user k transmits with constant energy $E^{(t)}_k$ on the allotted resources, irrespective of the channel gain. Therefore,

$$E^{(t)}_{k,r_{k,n}} = \begin{cases} E^{(t)}_k, & r_{k,n} \text{ allocated to } k \wedge \sum_{i=1}^{n-1} b_{k,r_{k,i}} < L_k; \\ 0\,, & \text{otherwise.} \end{cases} \tag{3}$$

where $b_{k,r_{k,i}}$ are the bits sent by user k in resource $r_{k,i}$.

C. Transmission model

We assume that a device that transmits with energy $E^{(t)}_{k,r_{k,n}}$ on a given resource $r_{k,n}$ can reliably deliver a maximum number of bits given by

$$\rho_{k,r_{k,n}} = \log_2\left(1 + \Gamma_{k,r_{k,n}}\right)\Delta f \Delta t\,, \tag{4}$$

where

$$\Gamma_{k,r_{k,n}} = \frac{\alpha_{k,r_{k,n}}\overline{g}_k}{\eta_0 \Delta f \Delta t} E^{(t)}_{k,r_{k,n}} = \alpha_{k,r_{k,n}}\overline{m}_k E^{(t)}_{k,r_{k,n}} \tag{5}$$

is the Signal to Noise Ratio (SNR) at the receiver.[2] In (5), the term η_0 is the power spectral density of the AWGN at the receiver, so that $\eta_0\Delta f\Delta t$ is the total noise energy in a time-frequency resource, while the term $\overline{m}_k = \frac{\bar{g}_k}{\eta_0\Delta f \Delta t}$, which is proportional to the average channel gain for user k, has been introduced for notational convenience. Finally, the coefficient $\alpha_{k,r_{k,n}}$ represents the normalized effect of Rayleigh fading on the received power, and is hence modeled as an independent exponential random variable with unit mean.

Note that, if the number of bits transmitted by user k in resource $r_{k,n}$ exceeds $\rho_{k,r_{k,n}}$, we assume that the received packets will contain unrecoverable errors. In the following, this event will be referred to as *outage*.

D. Optimization problem

With reference to the above framework, the objective of the study is to find the optimal number N^*_k of resources to be allocated to each user $k \in \mathbf{K}$ to successfully deliver L_k bits with an overall outage probability lower than ϵ, while minimizing the expectation of the total aggregate energy. Mathematically, we can express this problem as follows:

$$\min_{N_k, E^{(t)}_k} \mathrm{E}\left[E^{(tot)}\right]$$
$$\text{subject to: } \Pr[k\text{th user outage}] \le \epsilon\,,\ \forall k \in \mathbf{K}\,,$$

where

$$\begin{aligned} \mathrm{E}\left[E^{(tot)}\right] &= \mathrm{E}\left[\sum_{k=1}^{K} E^{(tot)}_k\right] \\ &= \mathrm{E}\left[\sum_{k=1}^{K}\sum_{n=1}^{N_k}\left([E^{(t)}_{k,r_{k,n}} + E^{(c)}_k]\chi\left\{E^{(t)}_{k,r_{k,n}}\right\}\right)\right] \\ &= \sum_{k=1}^{K} \overline{n}_k\left(E^{(t)}_k + E^{(c)}_k\right), \end{aligned} \tag{6}$$

and $\overline{n}_k = \mathrm{E}[n_k] = \mathrm{E}\left[\sum_{n=1}^{N_k}\chi\left\{E^{(t)}_{k,r_{k,n}}\right\}\right]$ represents the average number of resources that are actually used for transmission.

The solution of the optimization problem, then, develops along the following three steps:

- First: for a given number N_k of assigned resources, we compute the minimum transmit energy per resource $E^*_k = \min\left\{E^{(t)}_k\right\}$ that yields an overall outage probability less than ϵ. Note that the circuit energy is irrelevant here, since it does not affect the success of the transmissions.
- Second: for the above E^*_k, we calculate the average number of resources, $\overline{n}_k$, that are actually used for transmission. Indeed, the energy E^*_k guarantees that, using all the N_k assigned resources, the probability that the user is not able to deliver its L_k is less than ϵ. Depending on the channel realizations, the L_k bits can actually be delivered using $n_k \le N_k$ resources. However, $\overline{n}_k$ does depend on

[1] Note that, as will be more clear in the next section, the BS assigns N_k resources to a user, which is the optimal number of resources that allows that user to send its message with a probability higher than a certain threshold while minimizing the energy cost. However, in some cases, part of these resources may not be used because the user succeeds in sending its payload with fewer resource slots.

[2] This SNR shall be scaled by a proper margin factor to account for the gap between the spectral efficiency of practical modulation schemes and the Shannon bound. Nonetheless, for the sake of simplicity and without loss of generality of the model, here we neglect this constant term.

both N_k and ϵ through E_k^*, while the circuit energy can once again be omitted.

- Finally, for such E_k^* and the corresponding $\overline{n}_k$ we compute the average total energy cost, as given by (6), when varying N_k. The number of resources N_k^* for which the mean total energy is minimized is the solution to our optimization problem.

In the following we address such a problem in different scenarios. We start considering the case of a single user, and we investigate the overall energy consumption when varying our assumptions on the channel state information (CSI) available at the transmitter. Then, we generalize the study to the multi-user case. Note that, for all these scenarios, the receiver has to know the number of users K and their average channel gain $\overline{g}_k$, $k = 1, \ldots, K$ in order to assign the optimal number of resources.

III. Single user analysis

We first consider full CSI at the transmitter, i.e., we assume the transmitter knows the channel gain $g_{k,r_{k,n}}$ in each allocated resource and is able to adapt the modulation rate accordingly. Then, we relax this assumption and consider the dual case where the transmit energy and rate are kept constant over each resource, so that the channel fluctuations may cause packet losses.

A. Full CSI case

Under the full CSI assumption, the optimization problem can be expressed as follows

$$E_k^* = \min\left\{E_k^{(t)} : \Pr\left[\sum_{n=1}^{N_k} \rho_{k,r_{k,n}} \geq L_k\right] \geq 1-\epsilon\right\}, \quad (7)$$

where ϵ is the outage probability. Recalling (4), the probability on the right-hand side of (7) can be rewritten as

$$\Pr\left[\sum_{n=1}^{N_k} \log_2(1 + E_k^{(t)}\overline{m}_k\alpha_{k,r_{k,n}}) \geq \tilde{L}_k\right], \quad (8)$$

where $\tilde{L}_k = \frac{L_k}{\Delta f \Delta t}$.

Then, given E_k^*, we need to evaluate the corresponding $\overline{n}_k$. Let S_i indicate the event that the L_k bits are successfully transmitted within $i \leq N_k$ resources, while the complementary event will be denoted as W_i. For the full CSI case, we hence have

$$S_i = \left\{\sum_{n=1}^{i} \log_2(1 + E_k^*\overline{m}_k\alpha_{k,r_{k,n}}) \geq \tilde{L}_k\right\}. \quad (9)$$

Then, the probability mass distribution (pmd) of the number n_k of resources that are actually used by the node can be expressed as

$$g(i) = \Pr[n_k = i] = \begin{cases} \Pr[S_1] & i = 1\,; \\ \Pr[S_i, W_{i-1}] & i \in \{2, \ldots, N_k - 1\}\,; \\ \Pr[W_{N_k-1}] & i = N_k\,. \end{cases} \quad (10)$$

and the average number of used resources is given by

$$\overline{n}_k = \sum_{i=1}^{N_k} i \cdot g(i) \quad (11)$$

which is a non-decreasing function of N_k, as proved in the Appendix.

Unfortunately, (8) and (11) cannot be expressed in closed form. A possible way to overcome this problem is to resort to Monte-Carlo analysis to find them. Another way is to find closed form expressions that well approximate (8) when operating in the high- or low-SNR scenarios, as explained next.

1) Approximation for low SNR: If we assume a low SNR regime, i.e., $\Gamma_{k,r_{k,n}} \ll 1$, the following approximation holds:

$$\log_2(1 + \Gamma_{k,r_{k,n}}) \simeq \Gamma_{k,r_{k,n}} \log_2 e. \quad (12)$$

Replacing (12) in (8), we get

$$\Pr\left[\sum_{n=1}^{N_k} \alpha_{k,r_{k,n}} \geq \frac{\tilde{L}_k}{E_k^{(t)}\overline{m}_k \log_2 e}\right]. \quad (13)$$

The sum of N_k iid exponential random variables with parameter 1, as appears in (13), has Erlang distribution of parameters $(N_k; 1)$, whose cumulative distribution function (cdf) is

$$\mathrm{F}_{Erl,N_k}(x) = 1 - \sum_{i=0}^{N_k-1} \frac{x^i e^{-x}}{i!}. \quad (14)$$

Using (14) and (13) into (7) we finally get

$$E_k^* = \min\left\{E_k^{(t)} : \mathrm{F}_{Erl,N_k}\left(\frac{\tilde{L}_k}{E_k^{(t)}\overline{m}_k \log_2 e}\right) \leq \epsilon\right\}. \quad (15)$$

Considering that the cdf is monotonic, (15) can be easily solved with simple numerical methods.

We now need to find $\overline{n}_k$. In the low SNR regime, we have

$$\Pr[S_i] = 1 - \Pr[W_i] = \Pr\left[\sum_{n=1}^{i} \alpha_{k,r_{k,n}} \geq \frac{\tilde{L}_k}{E_k^*\overline{m}_k \log_2 e}\right]$$
$$= 1 - \mathrm{F}_{Erl,i}\left(\frac{\tilde{L}_k}{E_k^*\overline{m}_k \log_2 e}\right) = 1 - \mathrm{F}_{Erl,i}(\ell_k).$$

where, for ease of notation, we set $\ell_k = \frac{\tilde{L}_k}{E_k^*\overline{m}_k \log_2 e}$.

The pmd of n_k, given by (10), can then be computed in closed form (though using the implicit functions $\mathrm{F}_{Erl,i}(x)$). In particular, for $i = 2, \ldots, N_k - 1$, we have

$$g(i) = \Pr[S_i, W_{i-1}] = \int_0^{\ell_k} \mathrm{f}_{Erl,i-1}(x)\, e^{-(\ell_k - x)} \mathrm{d}x\,.$$

The value of $\overline{n}_k$ can finally be obtained as in (11), which involves a finite sum of known terms.

2) Approximation for high SNR: Assuming a high SNR regime, i.e., $\Gamma_{k,r_{k,n}} \gg 1$, the following approximation holds

$$\log_2(1 + \Gamma_{k,r_{k,n}}) \simeq \log_2(\Gamma_{k,r_{k,n}}). \quad (16)$$

The probability in (8) can then be approximated as

$$\Pr\left[\sum_{n=1}^{N_k} \log_2(E_k^{(t)}\overline{m}_k\alpha_{k,r_{k,n}}) \geq \tilde{L}_k\right] \quad (17)$$

that, replaced in (7), yields

$$E_k^* = \min\left\{E_k^{(t)} : \Pr\left[\prod_{n=1}^{N_k} \alpha_{k,r_{k,n}} \le \frac{2^{\tilde{L}_k}}{{E_k^{(t)}}^{N_k} \overline{m}_k^{N_k}}\right] \le \epsilon\right\}. \tag{18}$$

In this case, we have the product of $N = N_k$ iid exponential random variables with parameter one, whose non trivial cdf $\mathrm{F}_{PE,N}(x)$ was derived in [8].

Accordingly, (18) becomes

$$E_k^* = \min\left\{E_k^{(t)} : \mathrm{F}_{PE,N_k}\left(\frac{2^{\tilde{L}_k}}{{E_k^{(t)}}^{N_k} \overline{m}_k^{N_k}}\right) \le \epsilon\right\} \tag{19}$$

whose solution is obtained by inverting the cdf $\mathrm{F}_{PE,N_k}(x)$.

To compute $\overline{n}_k$, we express (9) as

$$\begin{aligned}\Pr[S_i] = 1 - \Pr[W_i] &= \Pr\left[\prod_{n=1}^{i} \alpha_{k,r_{k,n}} \le \frac{2^{\tilde{L}_k}}{{E_k^*}^i \overline{m}_k^i}\right] \\ &= 1 - \mathrm{F}_{PE,i}\left(\frac{2^{\tilde{L}_k}}{{E_k^*}^i \overline{m}_k^i}\right).\end{aligned}$$

The computation of the pmd and the average of n_k follows the same rationale used for the low SNR case.

B. Average Channel State Information case

We now assume that the transmitter only knows the average channel gain towards the BS, but not the current fading term for each transmission resource. Furthermore, at first, we do not consider any feedback or ARQ mechanism, so that the failure of a single transmission attempt will determine the loss of the entire packet. As usual, the first step consists in finding the minimum energy level to guarantee that the overall outage probability is less than ϵ when all the N_k available resources are actually used for transmission.

Because of the lack of information about the current channel gain and the outcome of previous attempts, the best strategy for the device is to equally split the L_k bits in a certain number M of packets, and transmit each packet of L_k/M bits in one resource, with constant energy $E_k^{(t)}$. Note that, according to our model, if $L_k/M > \rho_{k,r_{k,n}}$, the transmission is assumed to fail. The overall outage probability is hence given by

$$\begin{aligned}P_{out} &= 1 - \left(\Pr\left[\log_2(1 + E_k^{(t)} \overline{m}_k \alpha_{k,r_{k,n}}) \ge \frac{\tilde{L}_k}{M}\right]\right)^M \\ &= 1 - \exp\left(-M \frac{2^{\tilde{L}_k/M} - 1}{E_k^{(t)} \overline{m}_k}\right),\end{aligned} \tag{20}$$

where the last term follows from the exponential distribution of $\alpha_{k,r_{k,n}}$. A simple functional analysis of (20) reveals that the outage probability is monotonically decreasing in M. Therefore, from the outage probability perspective, the best strategy is to use, for any given $E_k^{(t)}$, all the available resources, i.e., to set $M = N_k$.

The bound on the outage probability, then, yields the following minimum required energy per resource:

$$E_k^* = \frac{N_k\left(2^{\tilde{L}_k/N_k} - 1\right)}{\overline{m}_k \log_e \frac{1}{1-\epsilon}}. \tag{21}$$

Note that, with this assumption, the average number of transmissions is $\overline{n}_k = N_k$ because we are always using all the available resources to transmit.

C. Average Channel State Information case with ARQ

In this section we introduce ARQ capabilities at the receiver. Once again, we assume that the end device can arbitrarily split the payload in M packets of length L_k/M, where $1 \le M \le N_k$, and retransmit the packets that are not acknowledged by the BS in the remaining resource blocks, if any. In this case, then, the system is capable of recovering up to $N_k - M$ packet transmission failures.

The number of potential successes in N_k resources can then be modelled as a Binomial process $X \sim \mathrm{Bi}(N_k; 1 - PER)$ with parameters N_k and $1 - PER$, where PER is the failure probability of a single packet transmission of length L_k/M. The condition on the outage probability can hence be expressed as

$$\Pr[X \le M - 1] = 1 - I_{PER}(N_k - M + 1, M) \le \epsilon \tag{22}$$

where $I_x(a,b) = \frac{\int_0^x t^{a-1}(1-t)^{b-1}\mathrm{d}t}{\int_0^1 t^{a-1}(1-t)^{b-1}\mathrm{d}t}$ is the regularized incomplete Beta function. Given N_k, we can invert the previous equation to obtain the maximum value PER_ϵ of the packet error probability that satisfies (22).

Now, recalling that $\{\alpha_{k,r_{k,n}}\}$ are iid exponentially distributed, the error rate for any resource is

$$PER = 1 - \exp\left(-\frac{2^{\tilde{L}_k/M} - 1}{E_k^{(t)} \overline{m}_k}\right). \tag{23}$$

Setting (23) equal to PER_ϵ and solving in $E_k^{(t)}$ we finally get

$$E_k^* = \frac{2^{\tilde{L}_k/M} - 1}{\overline{m}_k \log_e \frac{1}{1-PER_\epsilon}}, \tag{24}$$

which gives the minimal required transmission energy E_k^* per resource, as a function of M.

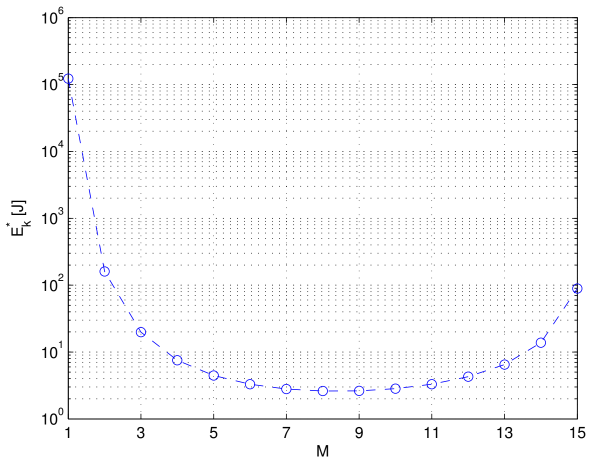

Fig. 1. Minimal energy per resource as a function of M, with $N_k = 15$ and $\epsilon = 0.05$, in the ARQ scenario.

As an example, we report in Fig. 1 the optimal energy per resource when varying M, for $N_k = 15$ and $\epsilon = 0.05$. Note that the rightmost value, i.e., $M = N_k$, corresponds to the energy obtained for the case without ARQ, as given by (21). Therefore, by introducing the ARQ mechanism, it is possible to achieve the same outage probability with less energy consumption per resource. However, the ARQ mechanism requires a feedback channel and some further processing. To account for the extra energy consumption due to the ARQ mechanism at the device side, we add a fixed cost E_{ARQ} per resource[3] in the energy metric defined in (2). Accordingly, the total energy cost per device for the ARQ case becomes

$$E_{k,ARQ}^{(tot)} = \sum_{n=1}^{N_k} \left([E_{k,r_{k,n}}^{(t)} + E_k^{(c)} + E_{ARQ}] \chi \left\{ E_{k,r_{k,n}}^{(t)} \right\} \right) \tag{25}$$

whose expectation is the function to be minimized.

In this case, the distribution of the number n_k of resources that are actually used by the device in case of success can be modeled as a negative binomial function $\mathrm{NB}(M; 1 - PER_\epsilon)$, which models the probability distribution of the number of independent Bernoulli trials, each with success probability $1 - PER_\epsilon$, required to obtain exactly M successes. Instead, in case of outage, we obviously have $n_k = N_k$. We hence have

$$g(i) = \Pr[n_k = i] = \binom{i-1}{i-M} (1 - PER_\epsilon)^M PER_\epsilon^{i-M},$$

for $i = M, \ldots, N_k - 1$, while $g(N_k) = 1 - \sum_{i=1}^{N_k-1} g(i)$.

The average value of n_k is then computed as in (11) and, in this case, it also depends on M.

D. Results for the single user case

For the reader's convenience, we list in Tab. I the main parameters that, unless otherwise specified, are used in the examples discussed throughout the remainder of the paper.

TABLE I
MAIN PARAMETERS CONSIDERED IN THE STUDY.

Target spectral efficiency	$\tilde{L}_k = 20$ bps/Hz
Average channel gain at the receiver	$\overline{m}_k = 5$
Circuit energy	$E_k^{(c)} = 0.5$ J
ARQ energy	$E_{ARQ} = 2$ J

Note that E_{ARQ} and E_c in Tab. I have the same order of magnitude as the average transmit energy per slot in the optimal allocation point, as will be seen later on in this section.

In Fig. 2 we compare the exact solution of the minimization problem for the FullCSI scenario, with the approximate solutions obtained in the low and high SNR regions. As expected, the low SNR approximation (LSNR) is quite far from the exact solution (FullCSI) for small values of N_k, for which the actual SNR needed by the system to satisfy (8) is quite high with the selected parameters. For increasing N_k, instead, the low SNR assumption becomes more reasonable and the approximation

[3]For the sake of simplicity, we assume that the ARQ cost is fixed. However, the framework can also accommodate more complex metrics.

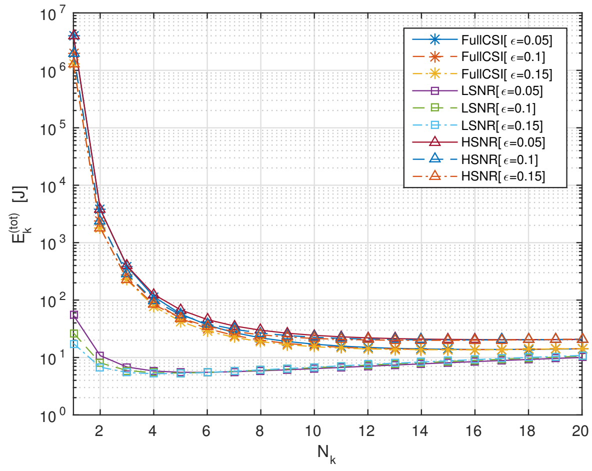

Fig. 2. Total energy in the full CSI scenario ($*$) and comparison with the low SNR ($\square$) and high SNR ($\triangle$) solutions.

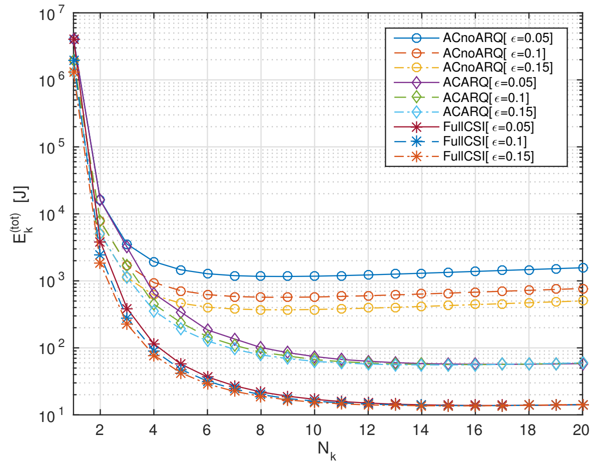

Fig. 3. Total energy in the various scenarios.

turns out to be more accurate. Clearly, the opposite occurs to the high SNR approximation (HSNR).

In Fig. 3, instead, we compare the results obtained for the three scenarios, namely full CSI (FullCSI), average channel state information with no ARQ (ACnoARQ), and average channel state information with ARQ (ACARQ).

For small values of N_k, both ACnoARQ and ACARQ perform similarly to FullCSI, and for $N_k = 1$ the performance is the same for the three cases because with $M = N_k = 1$ equations (20) and (8) are equivalent. However, the performance of ACnoARQ and FullCSI are significantly different for larger values of N_k. This gap can be viewed as the price to be paid for having only partial and rough knowledge of the channel status. In the FullCSI case, in fact, the device can adapt its rate to the channel gain in order to make the most of the channel capacity in each resource. AC schemes, instead, have a fixed rate and thus may incur in packet losses if channel conditions are poor.

Comparing the ACARQ and ACnoARQ results we can iden-

TABLE II
OPTIMAL RESOURCE ALLOCATION FOR THE VARIOUS SCENARIOS

Scenario	Tolerance	Optimal resource allocation
FullCSI	$\epsilon = 0.05$	$N_k^* = 17$
	$\epsilon = 0.1$	$N_k^* = 16$
	$\epsilon = 0.15$	$N_k^* = 15$
ACnoARQ	$\epsilon = 0.05$	$N_k^* = 9$
	$\epsilon = 0.1$	$N_k^* = 9$
	$\epsilon = 0.15$	$N_k^* = 9$
ACARQ	$\epsilon = 0.05$	$N_k^* = 17$
	$\epsilon = 0.1$	$N_k^* = 16$
	$\epsilon = 0.15$	$N_k^* = 15$

tify two different regions. For small N_k, the two performance curves are almost superimposed, though the ACnoARQ case yields slightly higher total energy (not visible on the scale of the figure) due to the contribution of E_{ARQ}. In this situation, in fact, there are few resources available, so that the set of possible values for M is very limited (see Sec. III-B) and the retransmission capability cannot be exploited efficiently. In this case, the best choice is just to equally divide the message over all the available resources, as for the ACnoARQ scenario, to take full advantage of the reduction on the required bitrate per slot. For larger values of N_k, instead, the contribution of the ARQ mechanism becomes significant (see Fig. 1). In this case, the best strategy is to divide the message in a number $M < N_k$ of packets, and exploit the remaining resources to recover some failed transmissions. Clearly, the optimal value of M depends on E_{ARQ} through (25). However, we observed that the ARQ approach yields considerable benefit even when E_{ARQ} is of the same order of magnitude of the transmit energy per slot.

Moving to the main objective of this work, from Fig. 3 we can identify the number of resources N_k^* which yields the minimum energy consumption for the different scenarios. Notice that, from (7), (15), (19), (21), and (24) (through (22)), the required constant transmit energy per slot to achieve the target outage probability turns out to be a non-increasing function of the number N_k of available resources. However, the average number of actually used resources ($\overline{n}_k$) increases with N_k, thus determining an increase of the circuit energy component in the overall energy consumption. The optimal number N_k^* of resources to be allocated to minimize the overall energy consumption for a given outage probability ϵ, hence, is reached when the reduction of the transmit energy per slot brought about by a further increase of the number of allocated resources is compensated by the extra energy costs due to the increase of the (average) number of transmissions. Mathematically, for any given ϵ,

$$N_k^* = \max\left\{N_k : \mathrm{E}\left[E_k^{(tot)}(N_k)\right] < \mathrm{E}\left[E_k^{(tot)}(N_k - 1)\right]\right\}.$$

Tab. II collects the results for the considered scenarios.

Finally, we note that, for small values of N_k, relaxing the constraint on the outage probability (i.e., increasing ϵ) leads to a reduction of the total energy cost in every scenario, due to a reduction of the minimum energy per slot E_k^* required to reliably transmit the packets.[4] However, for FullCSI and ACARQ, when $N_k > N_k^*$ we observe that the total energy cost slightly increases when ϵ increases. This counterintuitive result can be explained observing that, for $N_k > N_k^*$, the mean number of transmissions $\overline{n}_k$ actually increases with ϵ as a consequence of the lowering of the transmit energy per slot. In this situation, it is preferable to increase the energy per slot above the minimum value E_k^* that guarantees the target outage probability, because in this way we reduce $\overline{n}_k$, which is the dominant factor when $N_k > N_k^*$. This reasoning does not hold for ACnoARQ where the average number of transmission attempts is always equal to N_k, independently of ϵ.

IV. MULTI-USER SCENARIO

In this section we consider a multi-user scenario, which is investigated by means of simulations only, due to its higher complexity. Specifically, we performed a simulation for $K = 100$ users, considering a constant payload of $L_k = 200$ bits for each device. Each uplink resource has a duration of $\Delta t = 1$ ms and $\Delta f = 10$ kHz, so that $\tilde{L}_k = L_k/(\Delta t \Delta f) = 20$ bps/Hz. The factor $\overline{m}_k$ for the different users is picked at random and uniformly in the interval $[1, 100]$ to simulate a scenario where users have different average channel gains. In addition, the resource blocks are affected by independent Rayleigh fading.

Given this framework, we exploit the results obtained in Sec. III to determine the optimal scheduling policy for minimizing the overall energy consumption, with outage probability $\epsilon = 0.05$ for each user.[5]

To this end, we applied the following simple iterative procedure. First of all, if the number of available resources equals or exceeds the sum of the optimal number of resources required by each device, i.e., $N_{tot} \geq \sum_k N_k^*$, then each device k will be given just N_k^* resources, since any additional resource will determine an increase of the overall energy cost.

Conversely, when the resources are not sufficient to satisfy the target requirements of all users ($N_{tot} < \sum_k N_k^*$), the allocation is performed as follows. We start by assigning one slot to each device, since all users are expected to transmit within the total allotted resources. (This clearly requires $N_{tot} \geq K$, otherwise the problem would admit no feasible solution.) Then, as long as $\sum_k N_k \leq N_{tot}$, the algorithm adds one resource per step to the device k' that leads to the greatest decrease in the sum energy. Mathematically, we have

$$k' = \arg\max_k \left\{E_k^{(tot)}(N_k) - E_k^{(tot)}(N_{k+1})\right\} \qquad (26)$$

where $E_k^{(tot)}(N_k)$ is given by (6) for the HSNR and ACnoARQ cases and by (25) for ACARQ, respectively. This step is repeated iteratively, until all N_{tot} resources are assigned.

[4]This behavior gives a qualitative proof of the validity of our procedure. Indeed, if the curves for different ϵ were not in this order as $N_k \leq N_k^*$, it would mean that starting with the minimization of the transmit energy per slot that satisfies the outage condition would not have led to the optimum solution.

[5]Note that, with this setting, the HSNR approximation turns out to be reliable and has been used to obtain the results reported in the following in place of the exact solution (FullCSI), which requires cumbersome and long simulations.

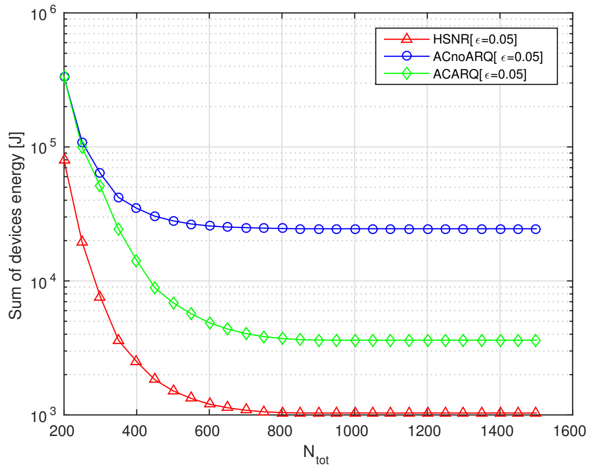

Fig. 4. Sum devices energy vs total resources with optimal resource allocation.

Fig. 4 shows the results, in terms of overall allocated energy, when varying N_{tot}. Note that the value N_{tot}^* beyond which the curves achieve a constant value corresponds to the sum of the optimal resource allocations of the different users, i.e., $N_{tot}^* = \sum_{k=1}^{K} N_k^*$. As expected, we observe that the ACnoARQ scenario yields the highest energy consumption. Conversely, full knowledge of the CSI can dramatically improve the uplink performance of a cellular system also for what concerns the energy efficiency aspects. However, perfect CSI is difficult to attain in practice, in particular with low-end devices (such as, for instance, wireless sensor nodes or machine type devices), and usually requires some form of collaboration between terminal and BS, with an additional expenditure of energy which is not accounted for in this model. A compromise is offered by the ACARQ case, where the limited information provided by the ARQ mechanism (which can also be realized in aggregate form to further reduce the complexity and energy consumption of the devices), can bring a significant gain with respect to the ACnoARQ case.

V. Conclusions

With the aim of minimizing a specific energy metric, we have studied the problem of scheduling a finite set of uplink resources for wide-area cellular networks where many devices communicate a fixed payload to a central BS and channels are affected by Rayleigh fading. Using a simple model to account for both the device transmit energy and the circuit/ARQ energy, we described a procedure to solve our energy optimization problem. Then, we numerically solved it in the full- and average-CSI (with and without ARQ) knowledge scenarios for the single-user case and compared the corresponding performance. Finally, the analysis was generalized to the multi-user case for which we also performed some simulations and illustrated the related results.

As future work, we are planning to give a more thorough mathematical formalization to our analysis. Moreover, we are considering extensions to non-orthogonal cases where devices could potentially share resources.

Appendix

Using the total probability theorem, we can express $\overline{n}_k$ as

$$\overline{n}_k = \epsilon N_k + (1-\epsilon)\,\mathrm{E}\left[n_k | S_{N_k}\right] \tag{27}$$

Now, the expectation of n_k conditioned on S_{N_k}, i.e., given that the L_k bits are successfully transmitted within N_k resources, can be expressed as

$$\begin{aligned}
\mathrm{E}\left[n_k|S_{N_k}\right] &= \sum_{h=1}^{N_k} \Pr\left[n_k \geq h | S_{N_k}\right] = \sum_{h=1}^{N_k} \Pr[W_{h-1}|S_{N_k}] \\
&= \sum_{h=1}^{N_k} (1 - \Pr[S_{h-1}|S_{N_k}]) = N_k - \sum_{h=1}^{N_k} \Pr\left[S_{h-1}|S_{N_k}\right] \\
&= N_k - \sum_{h=1}^{N_k} \frac{\Pr\left[S_{h-1}, S_{N_k}\right]}{\Pr[S_{N_k}]} = N_k - \sum_{h=1}^{N_k} \frac{\Pr\left[S_{h-1}\right]}{1-\epsilon}\,.
\end{aligned} \tag{28}$$

Using (28) in (27) we obtain

$$\overline{n}_k = N_k - \sum_{h=1}^{N_k} \Pr[S_{h-1}; N_k]$$

where we have made explicit that the probability distribution of S_h depends on N_k through E_k^*, as follows from (9). For $\overline{n}_k$ to be increasing with N_k, hence, we must have

$$N_k - \sum_{h=1}^{N_k} \Pr\left[S_{h-1}; N_k\right] \geq N_k - 1 - \sum_{h=1}^{N_k-1} \Pr\left[S_{h-1}; N_k - 1\right],$$

which yields

$$\sum_{h=1}^{N_k-1} \Pr\left[S_{h-1}; N_k-1\right] + 1 \geq \sum_{h=1}^{N_k-1} \Pr\left[S_{h-1}; N_k\right] + \Pr\left[S_{N_k-1}; N_k\right].$$

Now, according to (7), E_k^* is non-increasing with N_k. Then, the probability of success within h resources (S_h), which is positively correlated with the energy used in each transmission, will be non increasing with N_k, i.e., $\Pr\left[S_{h-1}; N_k - 1\right] \geq \Pr\left[S_{h-1}; N_k\right]$ for all $h \geq 1$, which, together with the trivial inequality $1 \geq \Pr\left[S_{N_k-1}; N_k\right]$, concludes the proof.

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