Modeling and optimizing the operation policies of energy harvesting devices under realistic assumptions

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Brixen SSIE seminar, 20130705

Outline

- Energy harvesting models for wireless sensors
- Optimization problems
- Introduction of realistic modeling
- Conclusions

- Special thanks for this presentation to:
  - Nicolò Michelusi, Kostas Stamatiou, Michele Zorzi
  - Consorzio Ferrara Ricerche
  - SWAP exchange project and WSAN local project

Energy harvesting

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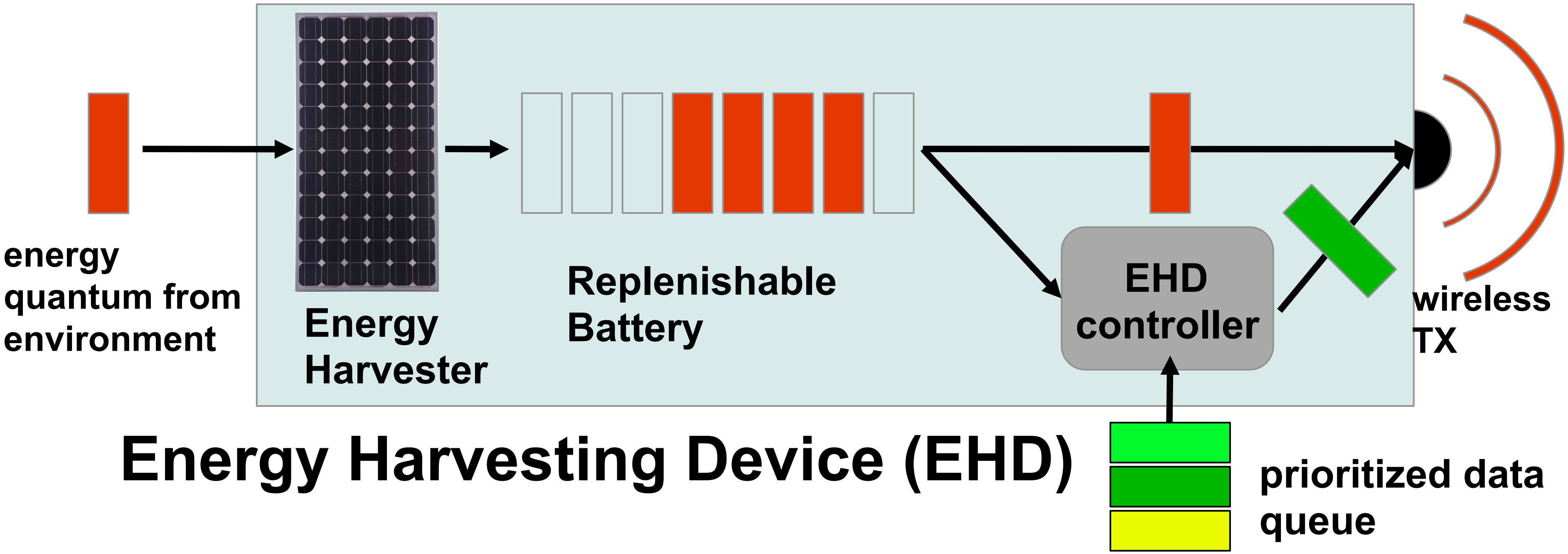
- Sensors have limited energy supply (batteries)
  - → sensor networks have limited lifetime
- Energy harvesting can solve the problem but:
  - the capacity of the battery is limited
  - the energy source is erratic
- The system model is inherently different from that of "traditional" energy-aware protocols.

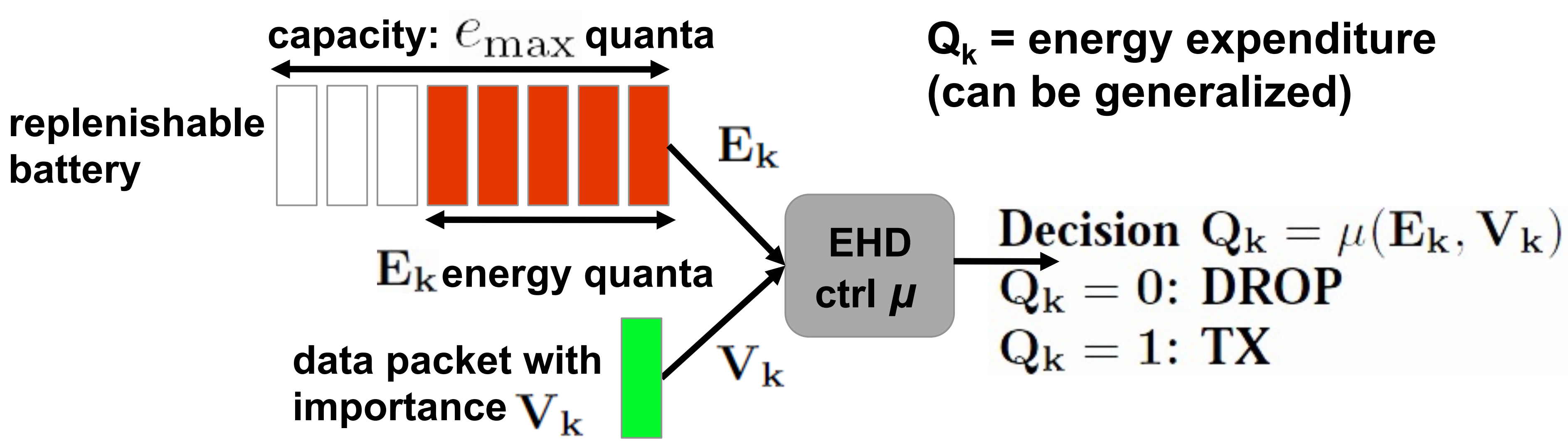
System model

- Slotted time: Bernoulli energy arrivals (average $\bar{b}$) to a battery of maximum capacity e_{max}
- Packet priorities are i.i.d.



EHD controller model

- The EHD controller, at time *k*:
  - knows energy level and data packet importance
  - given this, decides TX (Q_k=1) or DROP (Q_k=0)



- DROP: consumes no energy, gains nothing
- TX: consumes 1 energy quantum, accrues V_k

Reward function g(η)

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- If $\eta(\mathbf{e})$ = Prob{TX when energy is **e**}, use a strictly convex+increasing g(η) as the **reward**
  - i.e., payoff by transmitting with probability $\eta(\mathbf{e})$

$$\textbf{Average reward } \mathrm{g}(\eta(\mathrm{e})) = \mathbb{E}\left[\mathrm{V}\mu(\mathrm{e},\mathrm{V})\right]$$

- E.g.: expectation of the importance of the packets transmitted

Modeled as a Markov chain

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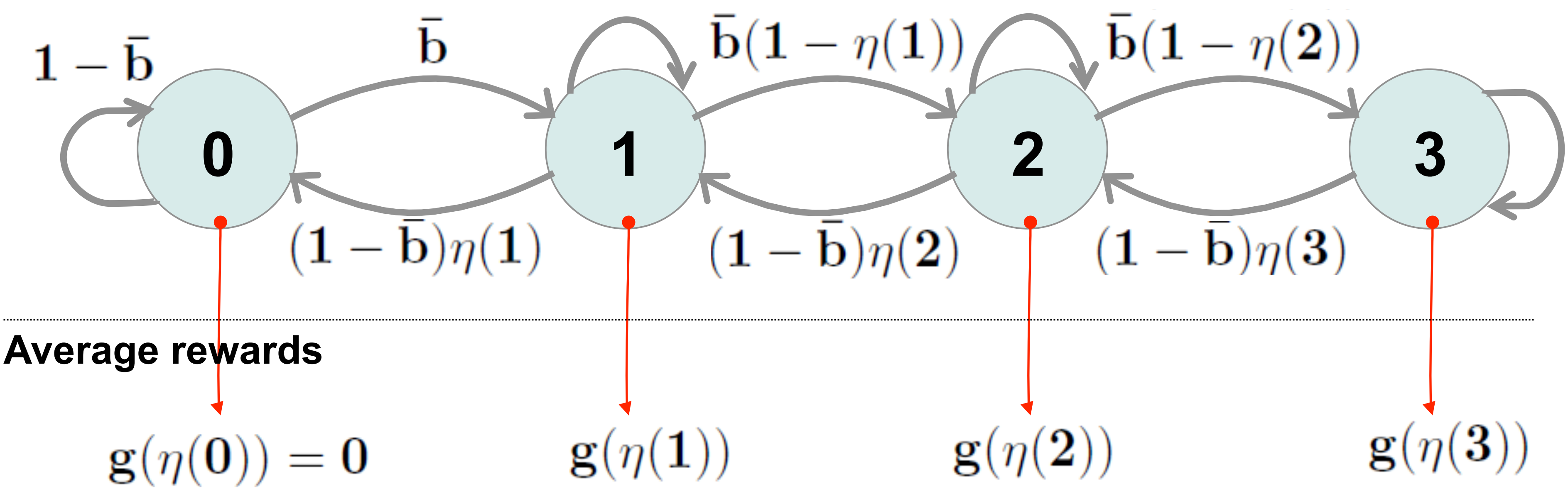
- Energy state **e** = discrete number of quanta
- State transitions depend on the TX prob. η(**e**) and the energy arrival probability



Events to avoid

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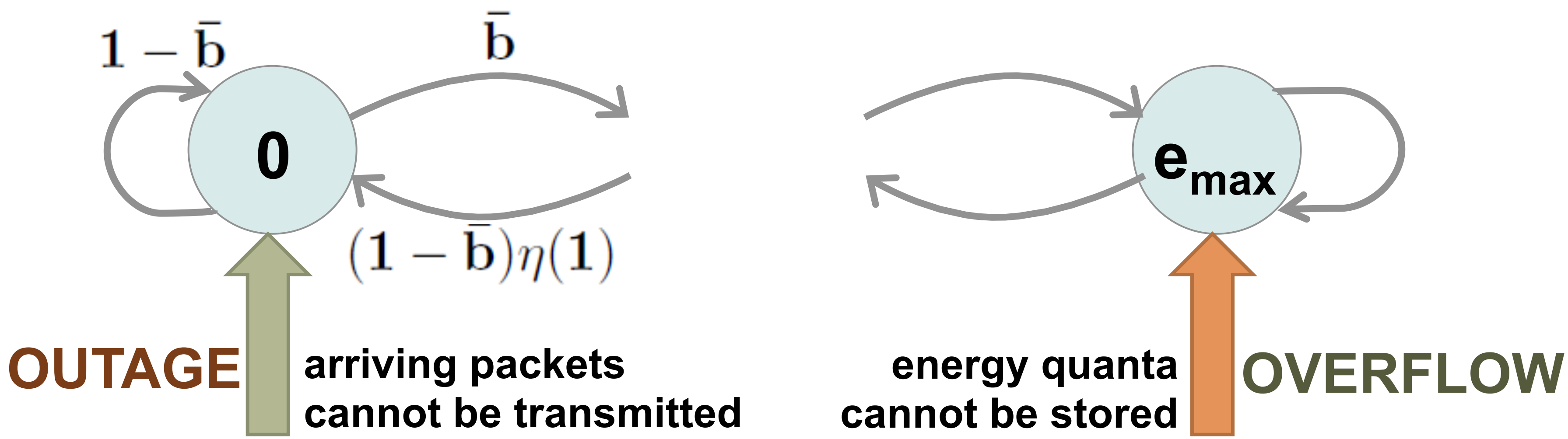
- **Outage**: a packet arriving when the energy is **e** = 0 cannot be transmitted (reward is lost)
- **Overflow**: a quantum arriving when **e** = e_{max}, it cannot be stored (must transmit a packet)



Optimization problem

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- maximize: average long-term importance (i.e. reward) accrued at RX

$$G(\mu; E_0, V_0) = \lim_{K\to\infty} \inf \frac{1}{K} \mathbb{E}\left[\sum_{k=0}^{K-1} \mu(E_k, V_k) V_k \,\middle|\, E_0, V_0 \right]$$

$$\mu^* = \arg\max_{\mu} G(\mu; E_0, V_0)$$

- find structural properties of the optimal policy
- low-complexity policy with good performance

Lemma: there's a threshold

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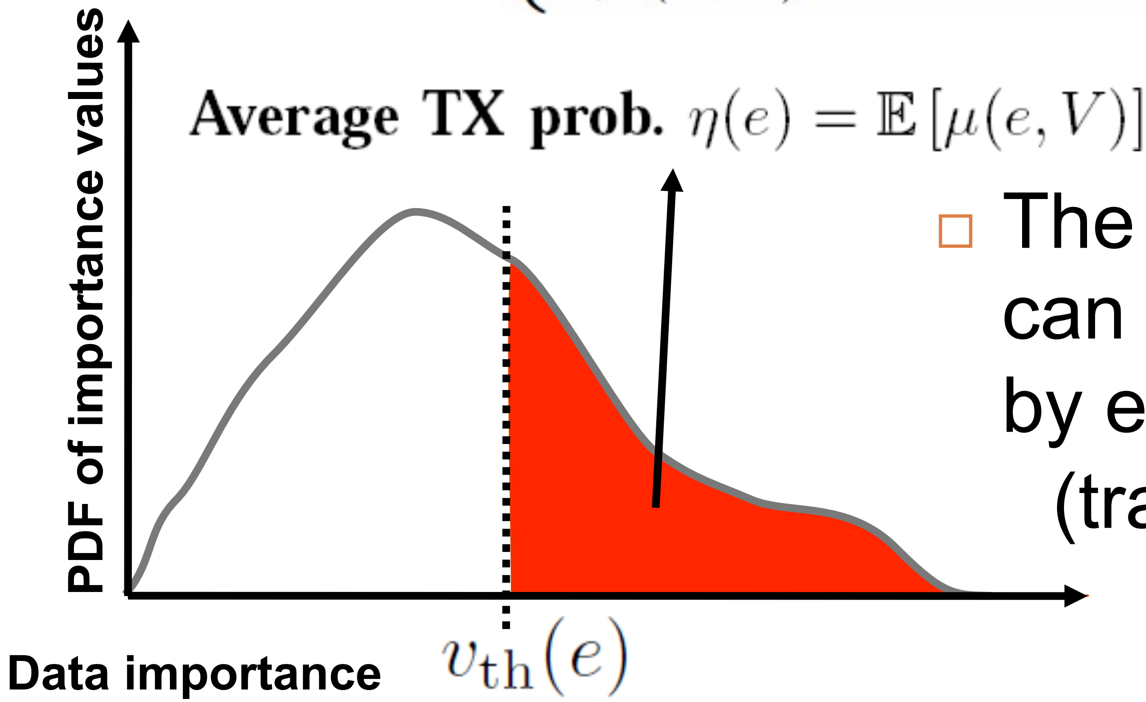
- The optimal policy μ has a threshold behavior

$$\begin{cases} \mu(e,v) = 1 & v \geq v_{\mathrm{th}}(e) \\ \mu(e,v) = 0 & v < v_{\mathrm{th}}(e) \end{cases}$$



- The optimal threshold can be characterized by either v_{th} or $\eta(\mathbf{e})$ (transmission prob.)

Solving the optimization

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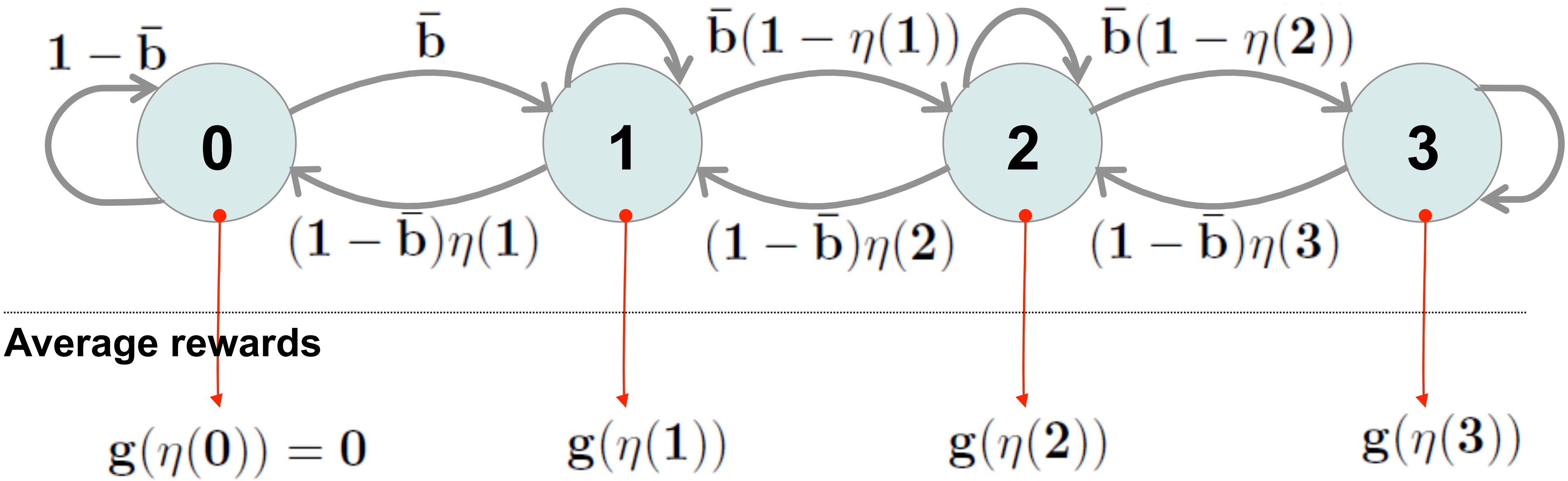
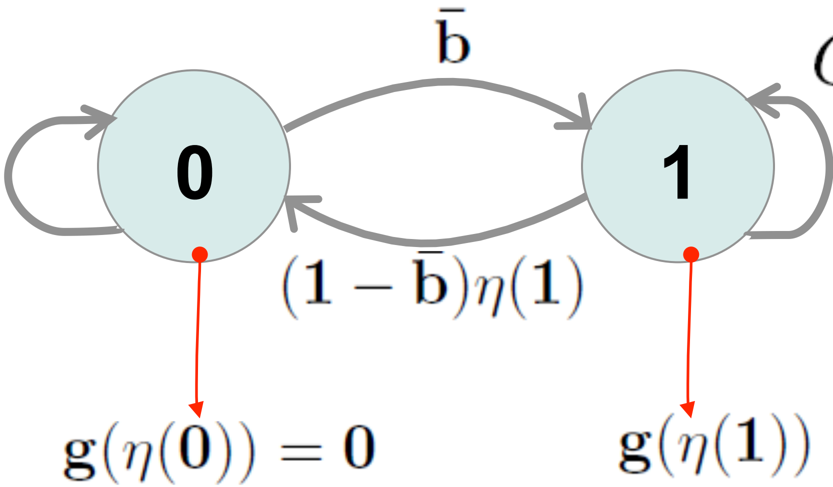


- Average long term reward: $G(\eta) = \sum_{e=1}^{e_{\max}} \pi_\eta(e) g(\eta(e))$
- To optimize: $\eta^* = \arg\max_\eta G(\eta)$ (must be a threshold!)

Example, $e_{max} = 1$



$$G(\eta) = \pi_\eta(1) g(\eta(1)) = \frac{\bar{b}}{\bar{b} + (1-\bar{b})\eta(1)} g(\eta(1))$$

- Just decide $\eta(1)$ with an optimal trade-off:
  - small $\eta(1)$ = low outage chances, low avg reward
  - large $\eta(1)$ = higher avg reward, outage more likely

Theorem

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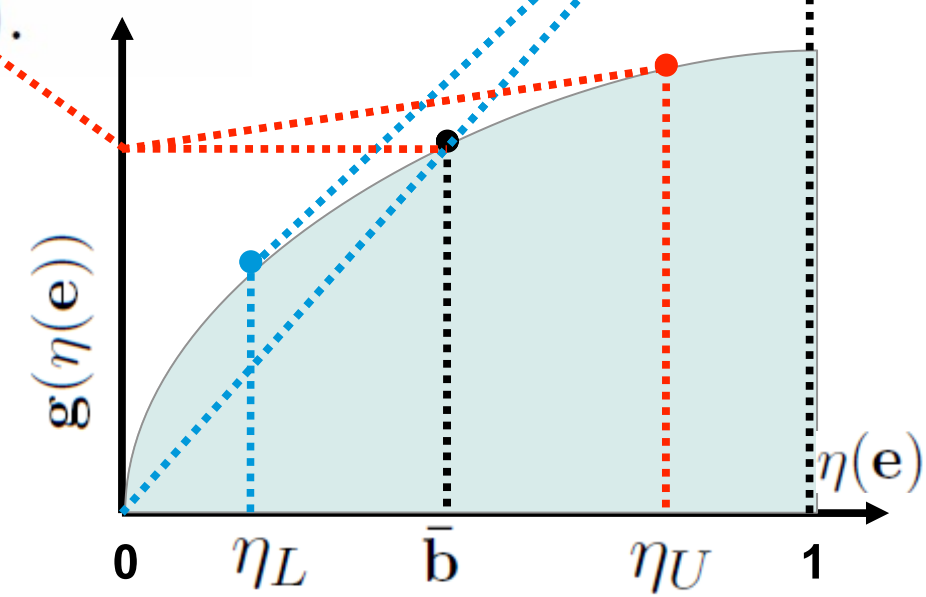
- Optimal policy $\eta^*(e)$ is strictly increasing in *e*
- $\eta^*(e) \in (\eta_L, \eta_U)$ where bounds η_L, η_U solve

$$g(\eta_L) + (1 - \eta_L)g'(\eta_L) = \frac{g(\bar{b})}{\bar{b}},$$

$$g(\eta_U) - \eta_U g'(\eta_U) = g(\bar{b}).$$

geometric interpretation



Exact result from ILP

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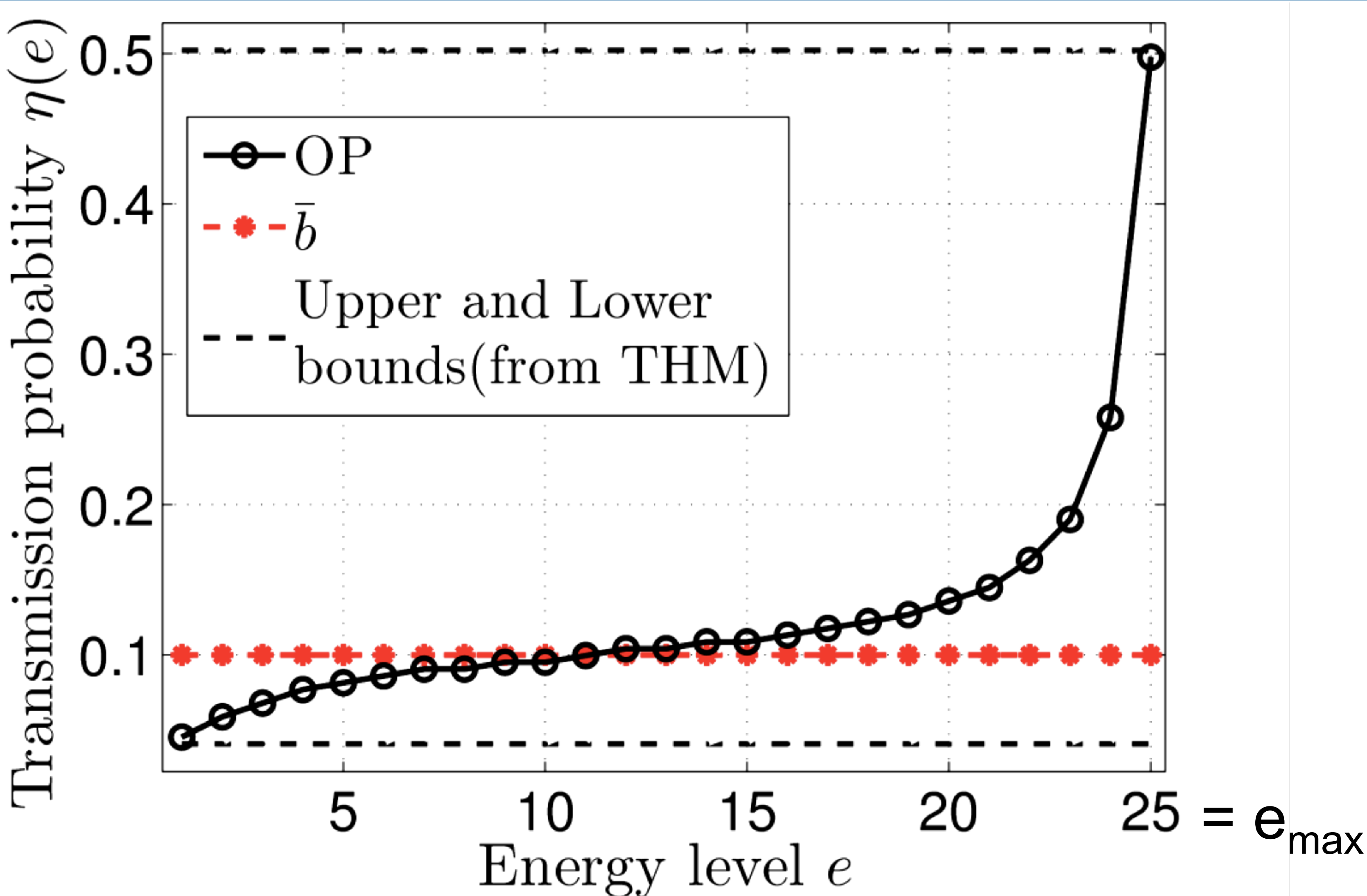

So it is optimal to be...

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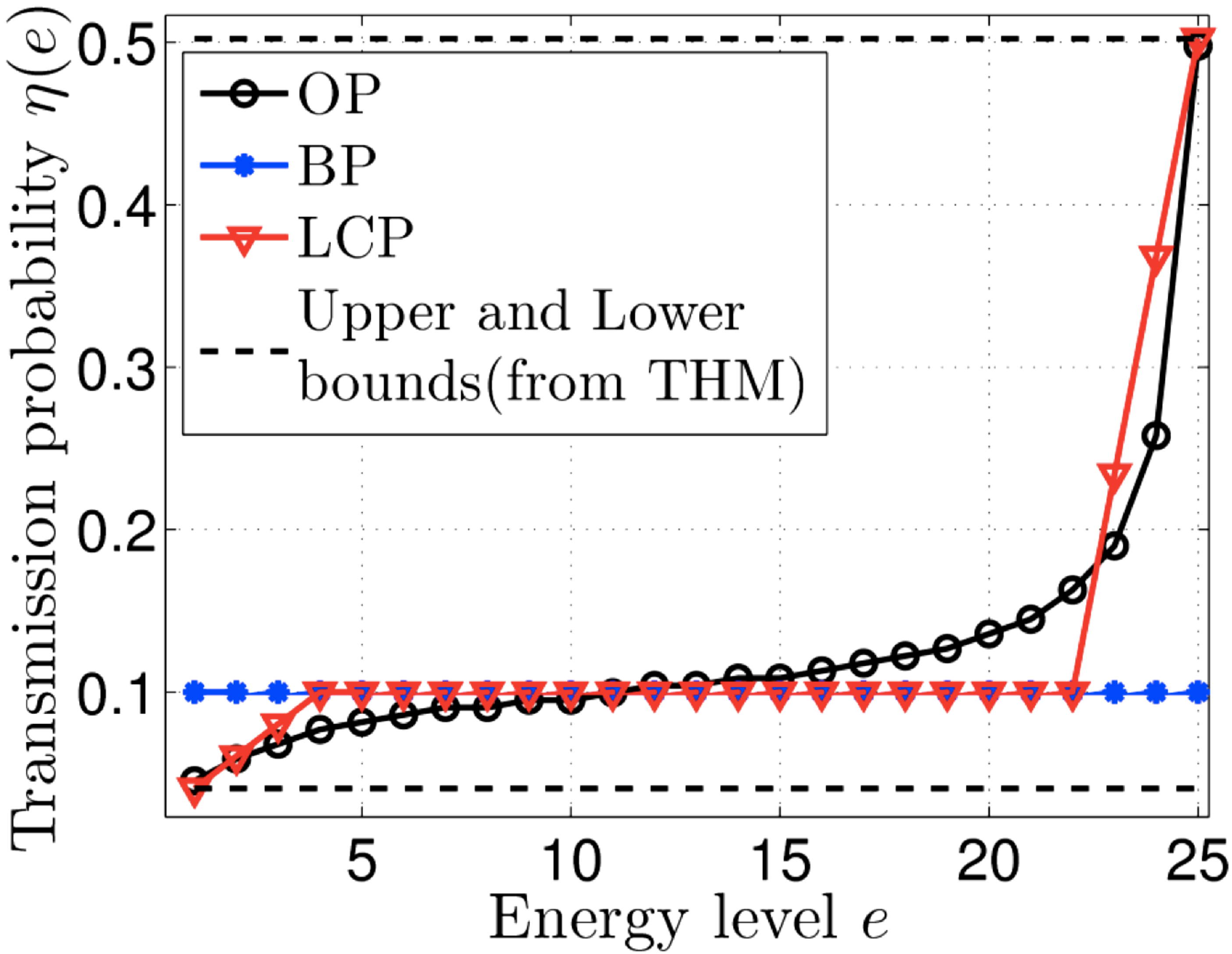
- aggressive when the energy is high
- conservative when the energy is low
- balanced when the energy is intermediate
  - i.e., on average, to use energy with rate $\bar{b}$ (the same as it arrives)
- Being ALWAYS balanced is suboptimal. Idea:
  - at low energy, linear increase from η_L to $\bar{b}$
  - at intermediate energy, constant = $\bar{b}$
  - at high energy, linear increase from $\bar{b}$ to η_U

Low complexity policy



Steady state

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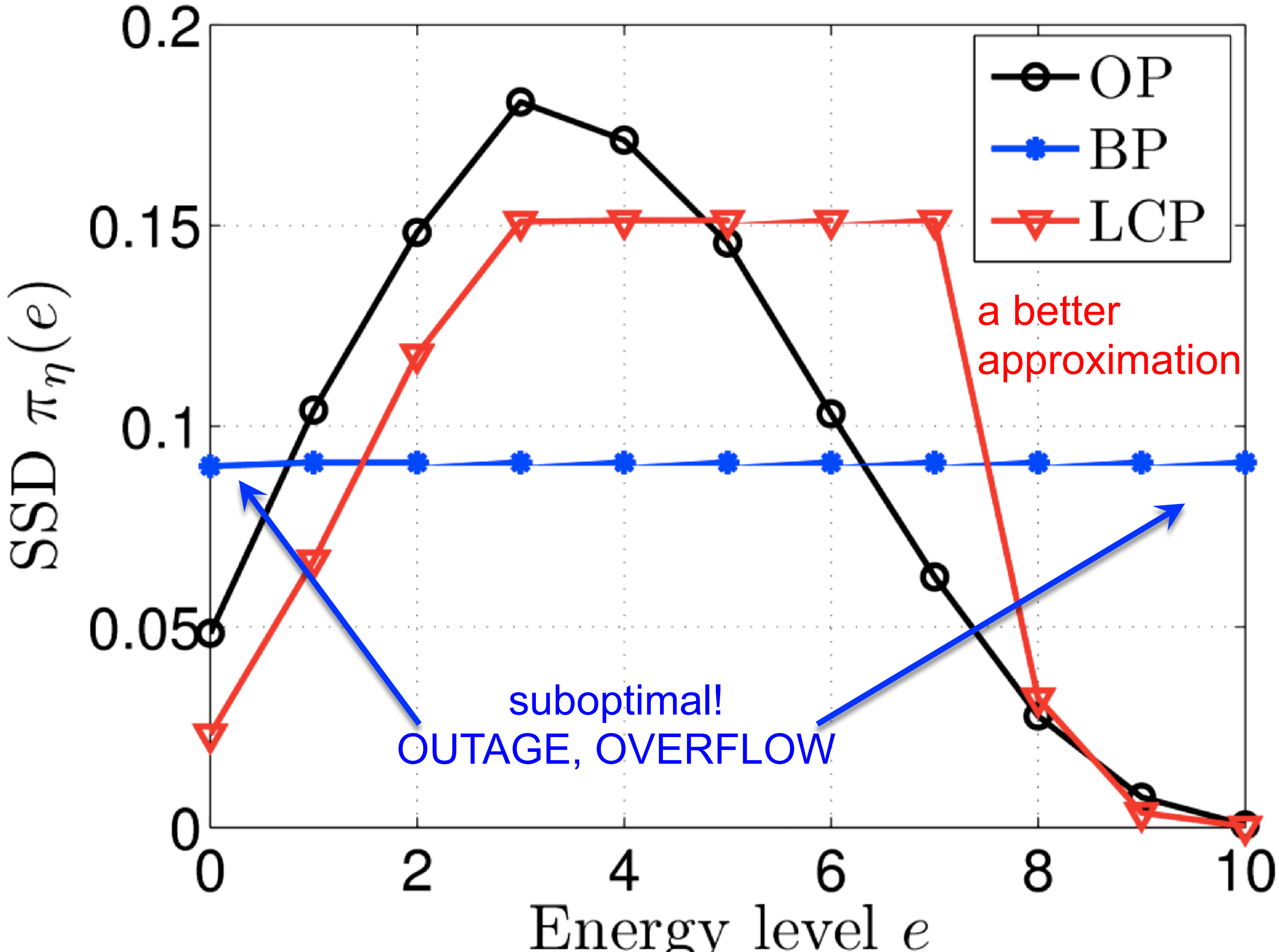

A practical example

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- Objective: to maximize the network throughput
  - Rayleigh fading: channel gain $H_k \sim \mathcal{E}(1)$
  - the importance of a packet is the achievable rate

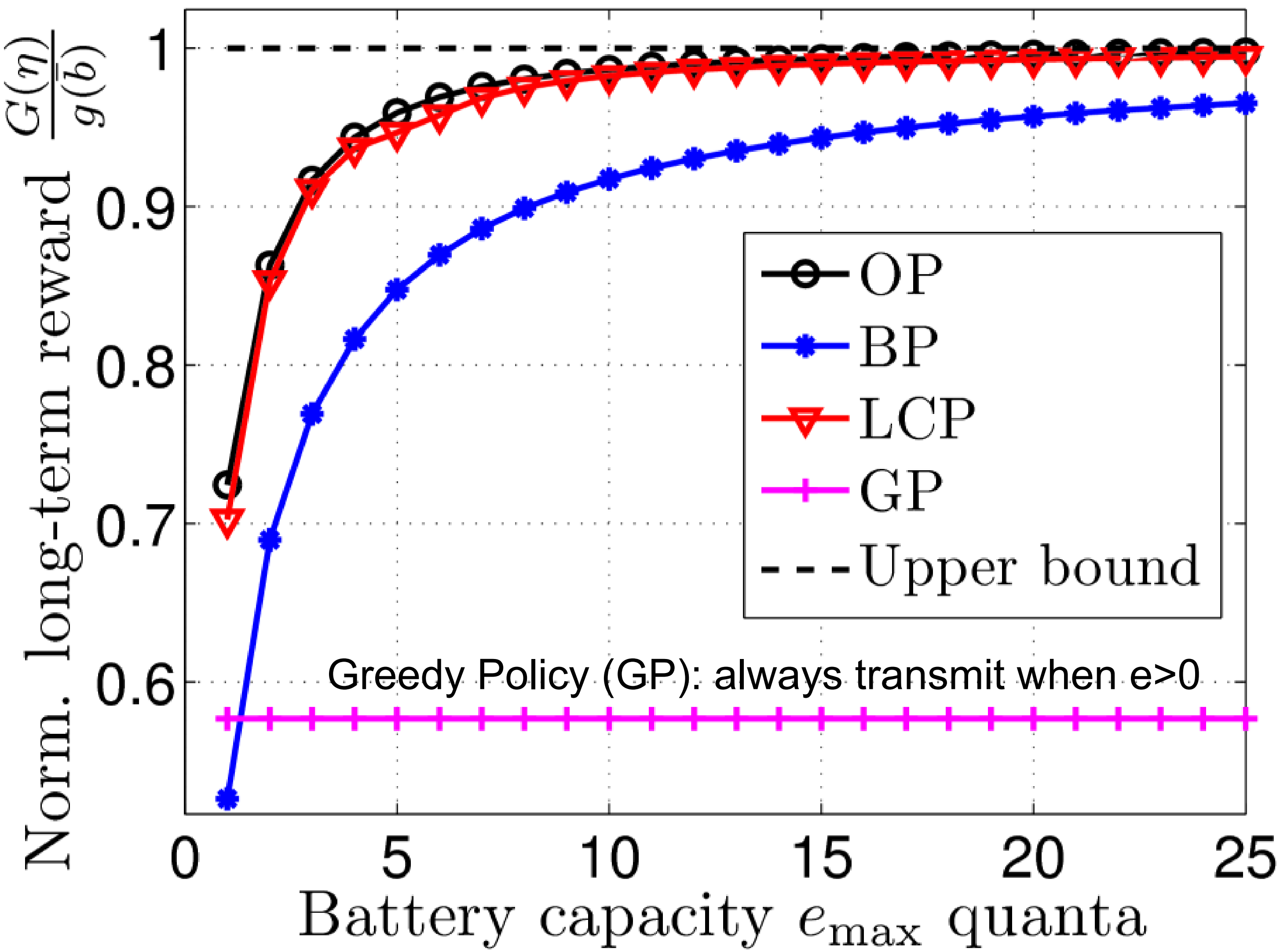
$$V_k = \log_2(1 + \mathrm{SNR} H_k)$$

  - A threshold $h_{\mathrm{th}}(e)$ can be set for the channel

$$\eta(e) = \int_{h_{\mathrm{th}}(e)}^{+\infty} \mathrm{e}^{-h} \mathrm{d}h = \mathrm{e}^{-h_{\mathrm{th}}(e)}$$

$$g(\eta(e)) = \int_{h_{\mathrm{th}}(e)}^{+\infty} \log_2(1 + \mathrm{SNR} h)\mathrm{e}^{-h} \mathrm{d}h$$

Results


Norm. long-term reward $\frac{G(\eta)}{g(\bar{b})}$
1
0.9
0.8
0.7
0.6
OP
BP
LCP
GP
Upper bound
Greedy Policy (GP): always transmit when e>0
0
5
10
15
20
25
Battery capacity $e_{\max}$ quanta

State-of-charge knowledge

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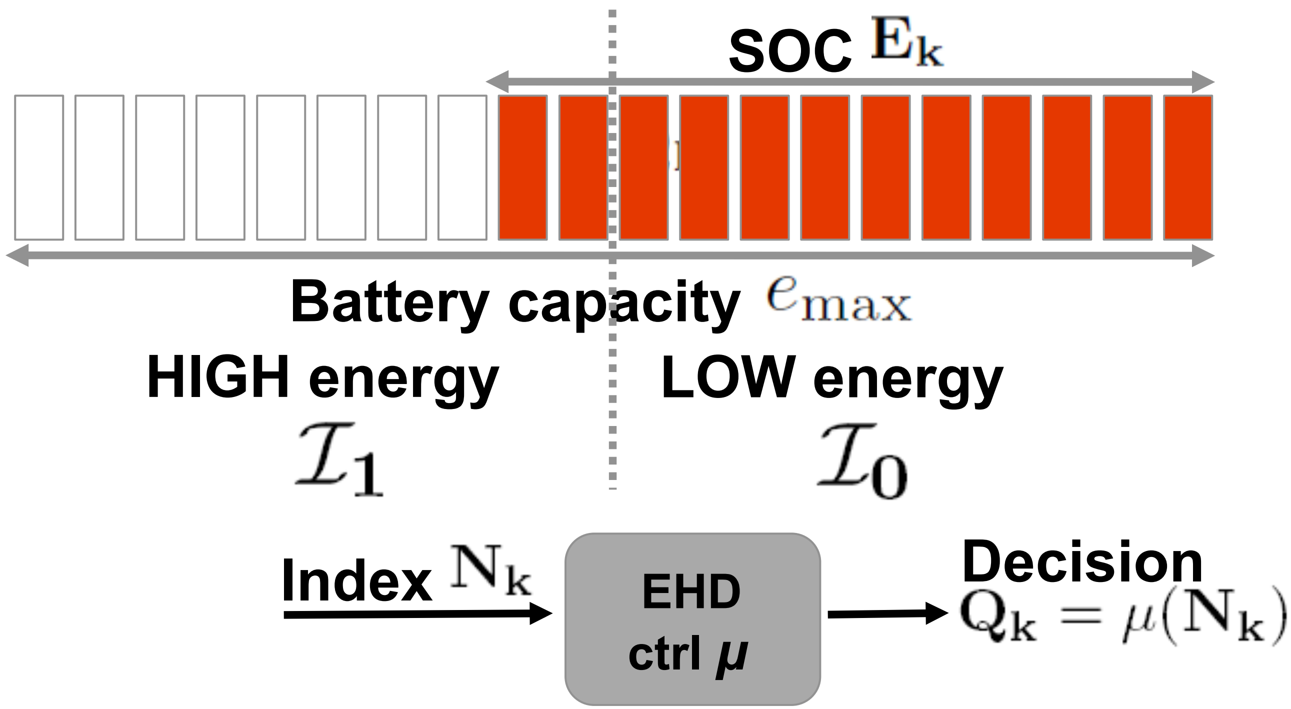
- These policies rely on a perfect knowledge of the state-of-charge (SOC) of the battery.
- Is this realistic? Battery SOC estimations are:
  - imprecise (up to 30% of errors)
  - time-consuming / energy-consuming
- New model: $[0,e_{max}]$ is quantized in intervals
  - it is just known which interval SOC **e** belongs to
  - e.g., two intervals: $\mathcal{I}_0$, $\mathcal{I}_1$ (LOW-HIGH)
  - we also generalize $Q_k > 1$ (that is, we have tasks consuming a variable amount of energy)

EHD with uncertain SOC

- Model of the EHD at time *k*



- The controller may be informed on the past history (all past Q_k's, intervals, outages)

EHD with uncertain SOC

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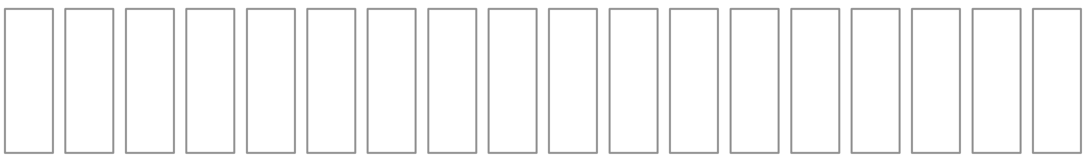
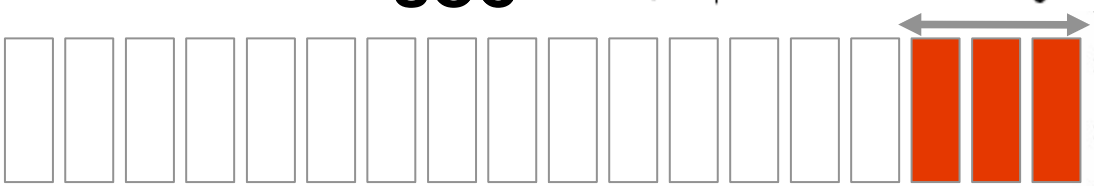
- Again, compute a reward:

SOC $E_{k \to k+1} = 0$



If $E_k < Q_k$
$\Rightarrow$ **energy outage**
$\Rightarrow$ **reward** $g(Q_k, E_k) = 0$

SOC $E_{k \to k+1} = E_k - Q_k$



If $E_k \geq Q_k$
$\Rightarrow$ **OK**
$\Rightarrow$ **reward** $g(Q_k, E_k) = \tilde{g}(Q_k)$
concave, increasing in Q_k

- Same trade-off between:
  - aggressive: avoids OVERFLOW, accrues reward
  - conservative: avoids OUTAGE, and also exploits the concavity of the reward function

Optimization problem

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- Maximize long-term reward

$$\mathbf{G}(\mu, \mathbf{e_0}) = \lim_{\mathbf{K}\to\infty} \inf \frac{\mathbf{1}}{\mathbf{K}} \mathbb{E}\left[\sum_{\mathbf{k}=\mathbf{0}}^{\mathbf{K}-\mathbf{1}} \mathbf{g}(\mathbf{Q_k}, \mathbf{E_k}) \,\middle|\, \mathbf{E_0} = \mathbf{e_0} \right]$$

$$\mu^* = \arg\max_{\mu} \mathbf{G}(\mu, \mathbf{e_0})$$

we only know the initial value of the energy!

- Partially Observable Markov Decision Process
  - Optimal policy depends on the past history
  - Focus on policies using just current SOC interval
  - Solution by exhaustive search

Example and results

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- Scenario: maximize throughput
  - Reward: achievable rate $\tilde{g}(q) = \dfrac{\ln(1+\alpha q)}{\ln(1+\alpha \bar{b})}$
  - Static channel with $\alpha = 1$
  - Geometric arrival rate, $\bar{b} = 20$

- Policies compared:
  - balanced policy (BP): $Q_k = \bar{b}$
  - optimal policy with perfect SOC knowledge
  - imperfect SOC knowledge, 2 intervals
  - imperfect SOC knowledge, 1 interval

Numerical results

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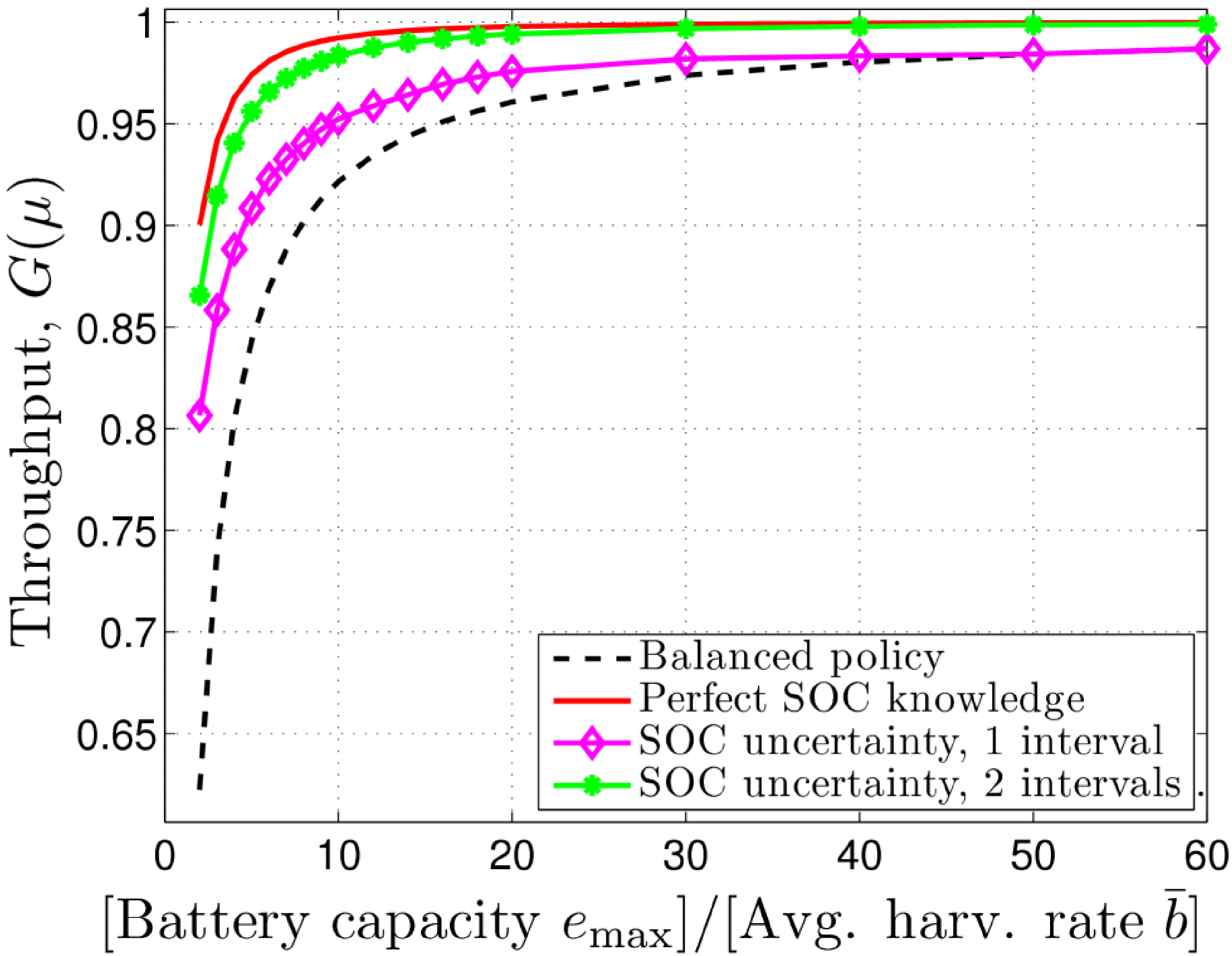

Other arrival distributions

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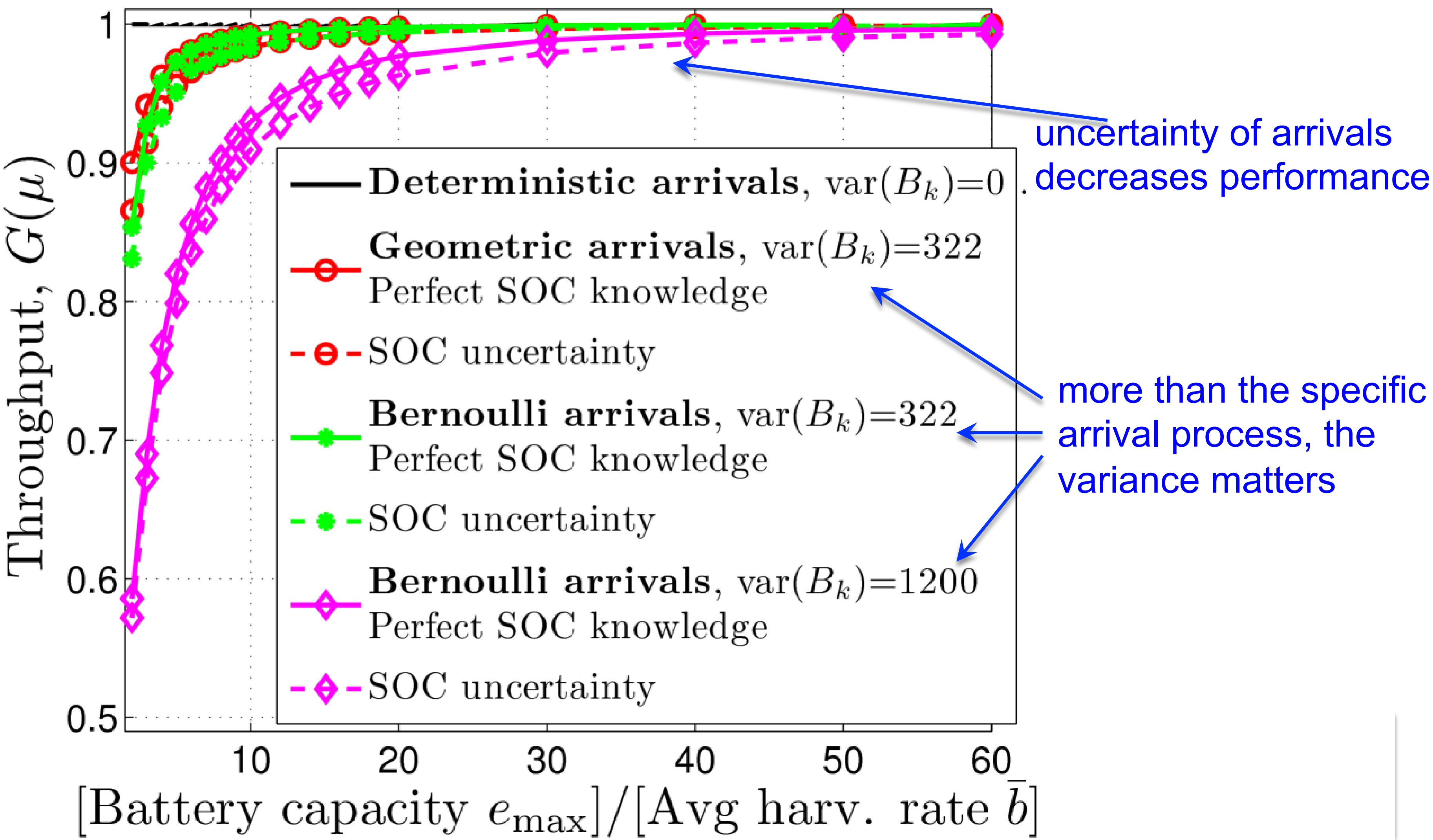

Energy arrival process

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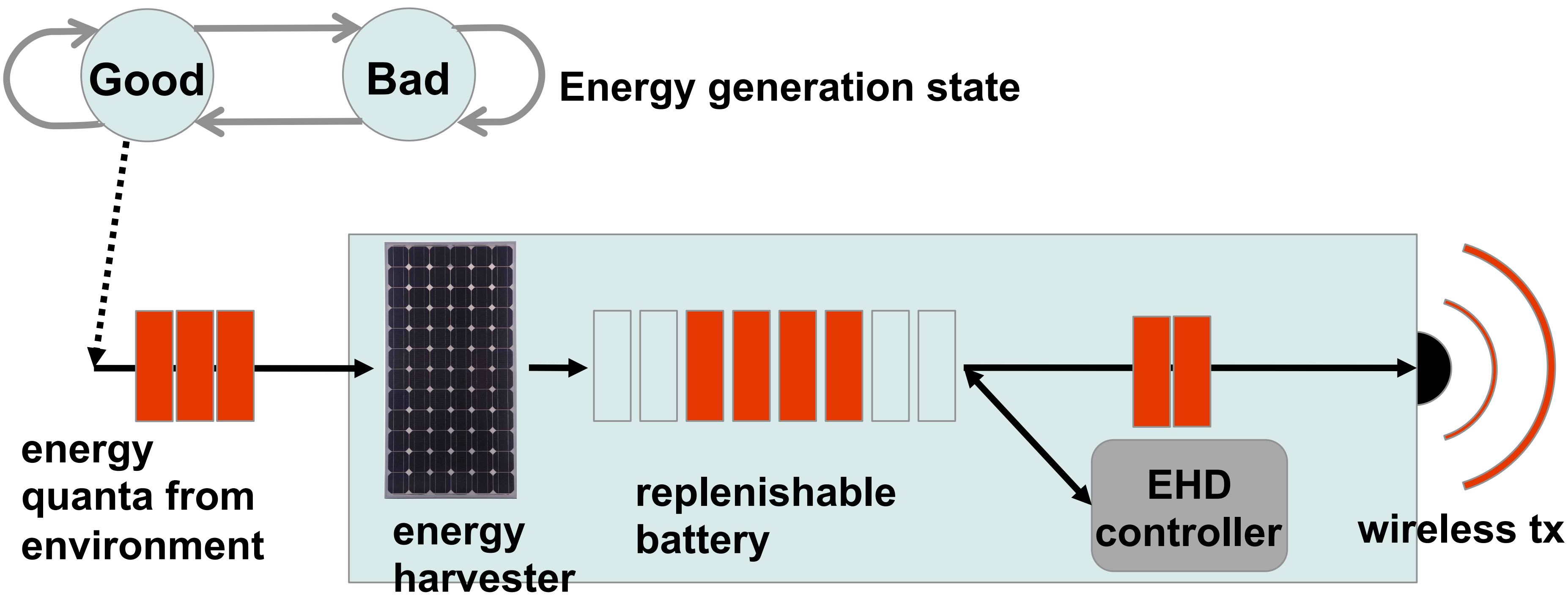
- Energy harvesting process is not really i.i.d.
  - Correlation is present (e.g., it can be Markov-like)



Energy Harvesting Device (EHD)

Combining all the elements

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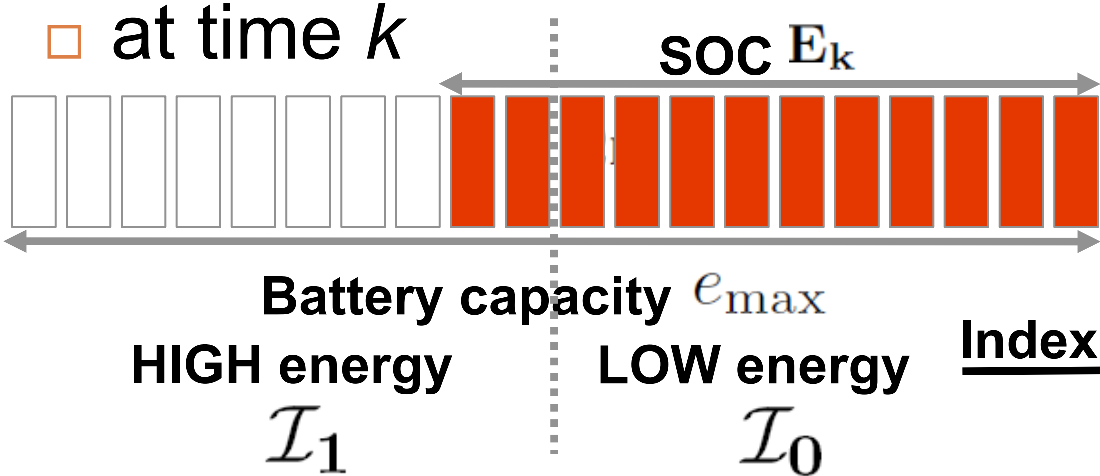
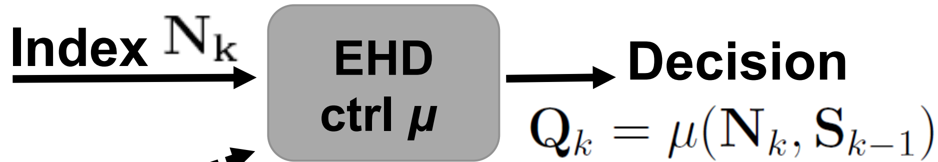
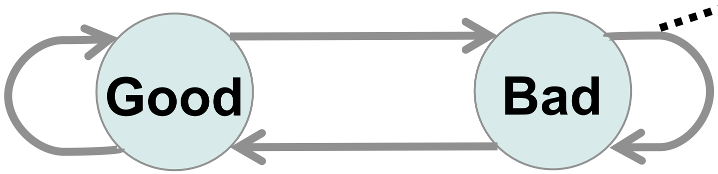
- at time *k*







Energy generation state S_{k-1}
In previous timeslot *k-1*,

$$\Pr(S_{k-1}=s|B_0,\ldots,B_{k-1}) = \frac{p_B(B_{k-1}|s)\sum_{\sigma\in\mathcal{S}} p_S(s|\sigma)\Pr(S_{k-2}=\sigma|B_0,\ldots,B_{k-2})}{\sum_{\tilde{s}\in\mathcal{S}} p_B(B_{k-1}|\tilde{s})\sum_{\sigma\in\mathcal{S}} p_S(\tilde{s}|\sigma)\Pr(S_{k-2}=\sigma|B_0,\ldots,B_{k-2})}$$

Example of scenario

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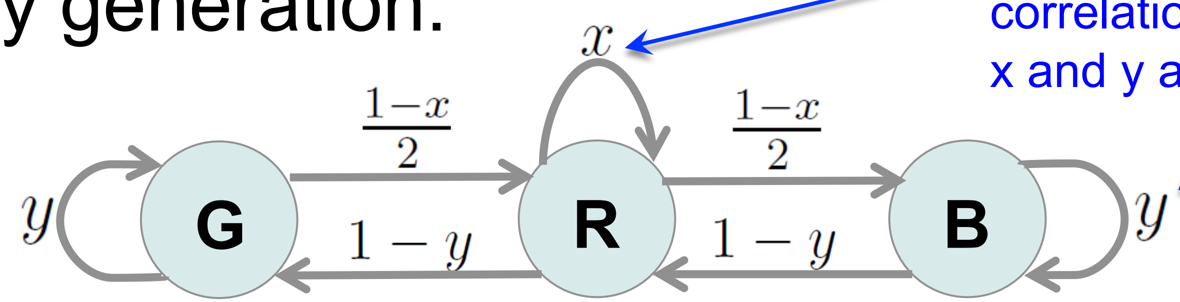
- Throughput maximization
  - Reward = achievable rate $\tilde{g}(q) = \frac{\ln(1+\alpha q)}{\ln(1+\alpha \bar{b})}$ with $\alpha = 1$, $\bar{b} = 20$
- Energy generation:



if not stated otherwise, correlation parameters x and y are set to 0.95

  - **G** (GOOD), deterministic $B_k = 2\bar{b}$
  - **B** (BAD), deterministic, $B_k = 0$
  - **R** (RANDOM), geometric, mean $\mathbb{E}[B_k | S_k = \mathbf{R}] = \bar{b}$

Policies

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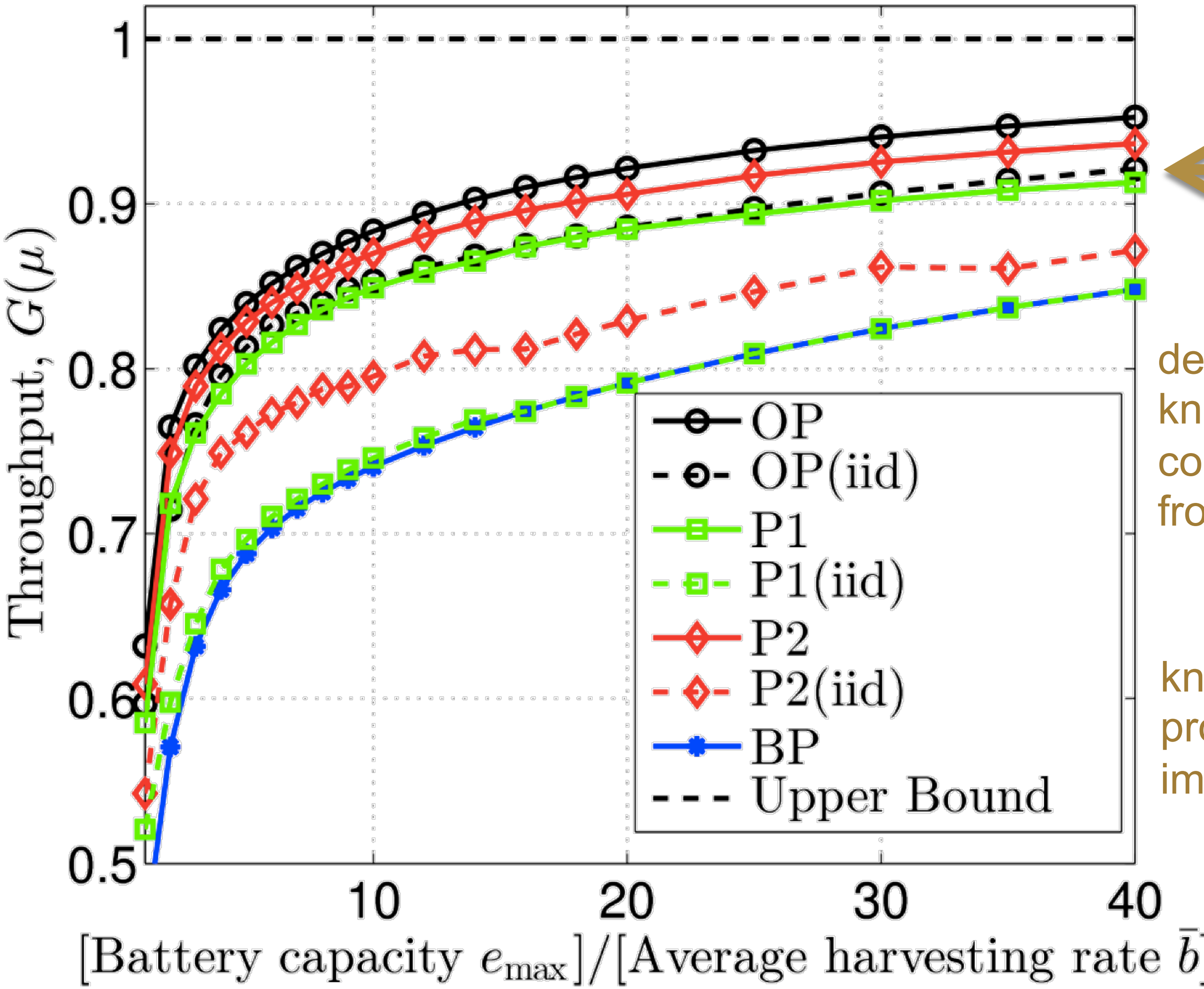
- **OP**: optimal. Knows SOC E_k , generation state S_{k-1}
- **P1**: 1-level uncertainty, knows S_{k-1}, SOC unknown
- **P2**: 2-level uncertainty, the EHD just knows whether the SOC is HIGH/LOW, and knows S_{k-1}
- **BP**: Balanced Policy $\mathbf{Q}_k = \bar{b}, \; \forall k$, asymptotically optimal for “large” batteries
- Policies labeled by **iid**: neglect correlation in energy generation process, assume iid process.

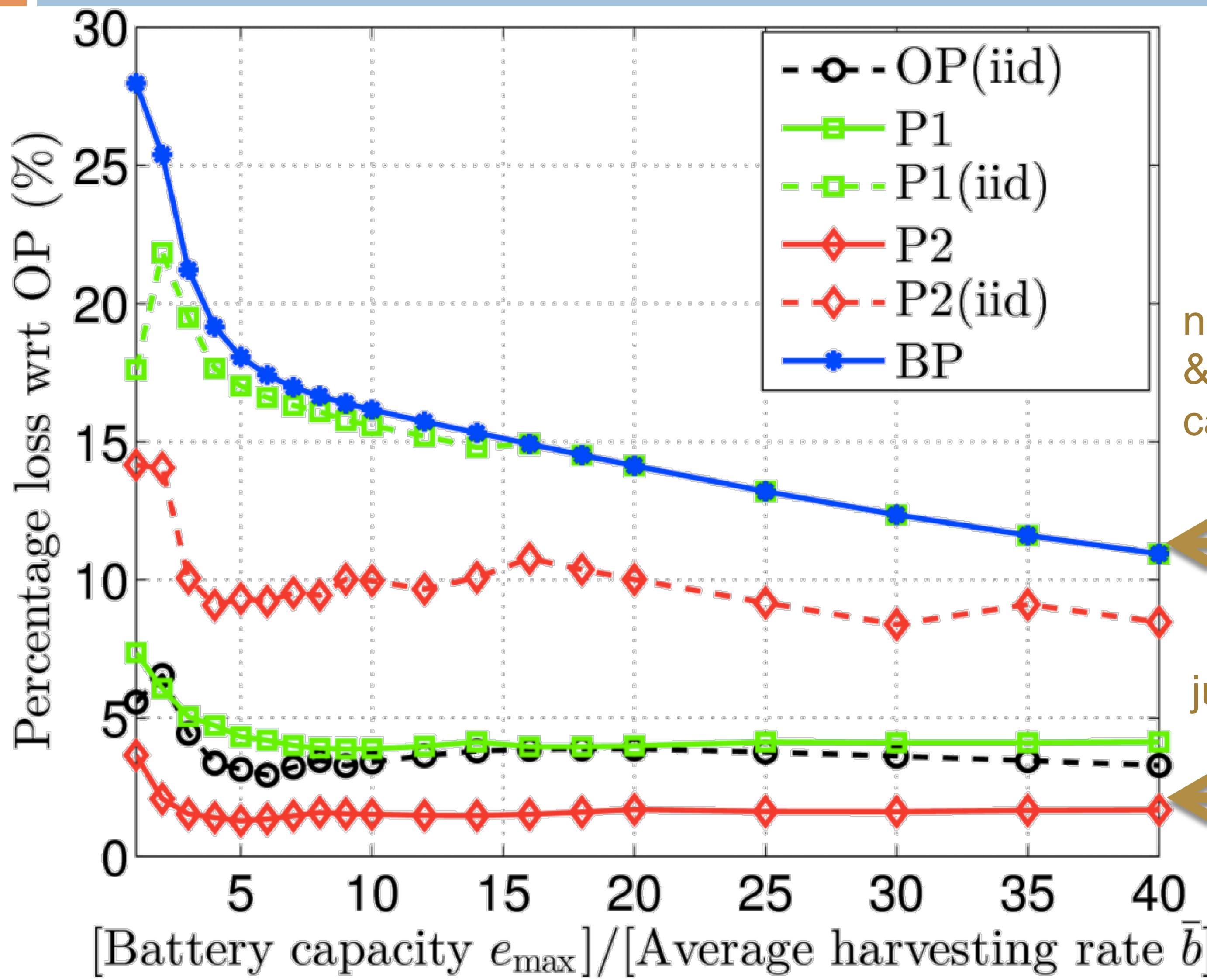
Comparison



degradation from not knowing arrivals is comparable with that from not knowing SOC

knowing the arrival process is as much important as the SOC

Loss from the optimum



not knowing the SOC & the arrival process can be very bad

just some knowledge: not so bad!

Energy arrival correlation

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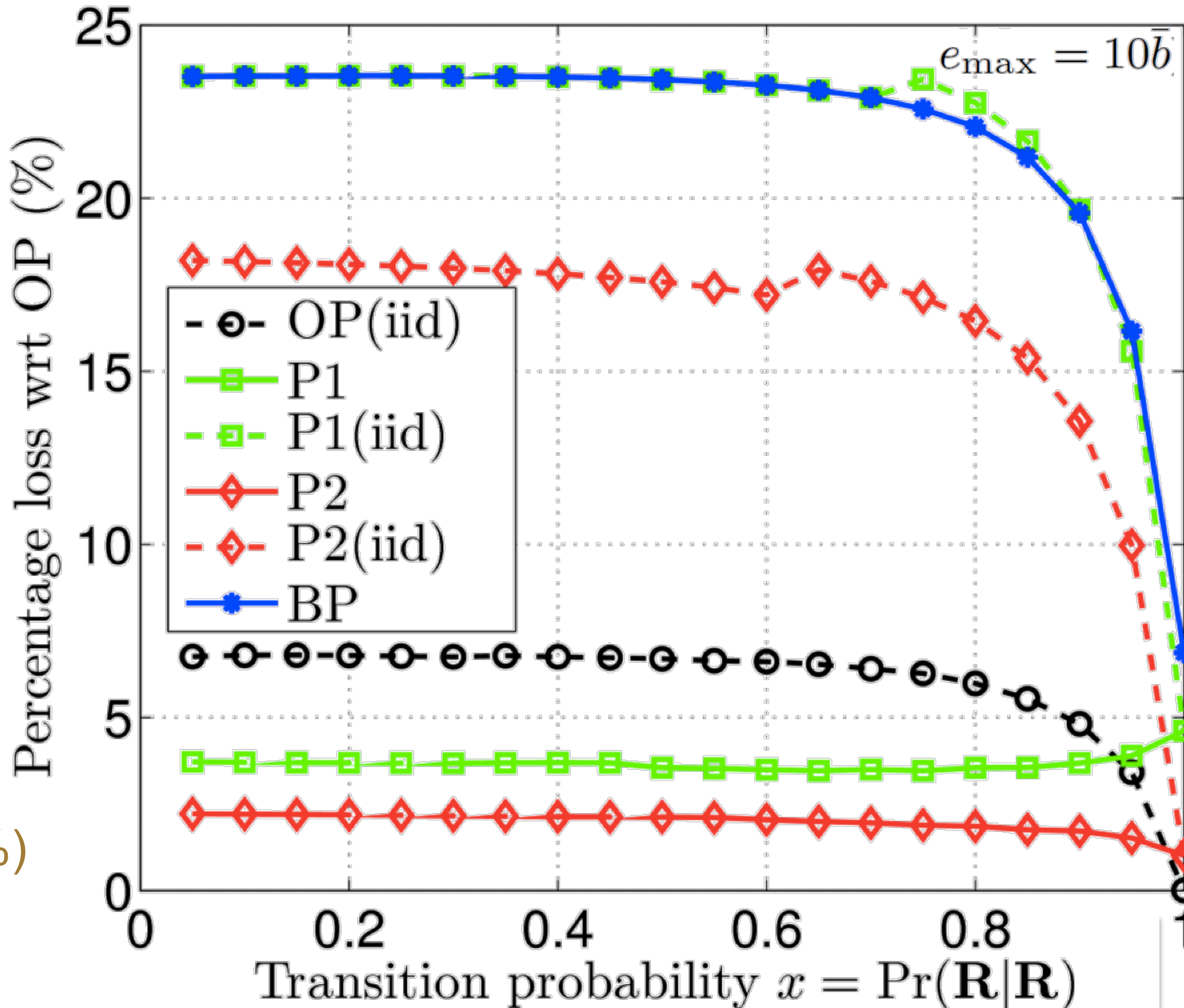

when the arrival process → iid, policies assuming this are not so bad, but if x < 0.8 they lose a lot (up to 20%)

Conclusions

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- Analysis of sensors energy harvesting as QS
  - Modeling aspects / optimization frameworks
- Lessons from increased realism
  - simplified policies (whose main goal is to avoid wasting energy or losing packets) work fine
  - not knowing the battery SOC at all is bad, but knowing even a rough quantization may be fine
  - knowledge of the arrival process is also important