Decentralized Parameter Estimation with Wireless Sensor Networks

Signal Processing and Information Theoretic Aspects

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Centre Tecnològic de Telecomunicacions de Catalunya

NEWCOM++ Summer School 2008 - Bressanone (Italy)

Notes

CTTC PhD Fellowship Program 2008

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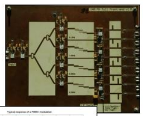
An Image is Worth...

MORE INFORMATION:

http://www.cttc.es/PhDCandidates



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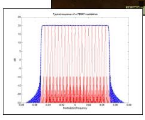

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Motivation

This seminar addresses the following issues:

- How to carry out *decentralized* parameter estimation via WSNs.
- To define new network *performance criteria*: estimation accuracy. How accurate can an estimate be?
- Impact of *WSN-related constraints* on performance and network design of: energy consumption and computational complexity constraints, decentralization, scalability properties, etc.
- How to exploit channel fading and and *opportunistic scheduling* to save energy/improve estimation?
- Impact of *quantized observations* on performance?

Notes

References

The sources of this seminar are, mainly:

- Wireless Sensor Networks. Signal Processing and Communications Perspectives. A. Swami, Q. Zhao, Y-W. Hong, L. Tong Eds., John Wiley & Sons Ltd., 2007.
- Fundamentals of Statistical Signal Processing: Estimation Theory. S. M. Kay, Prentice Hall Signal Processing Series, 1993.
- Elements of Information Theory. T. M. Cover and J. A. Thomas, Wiley Series in Telecommunications, 1993.
- Fundamentals of Wireless Communications, D. Tse, P.Wiswanath, Cambridge University Press, 2005.
- Several articles by Slepian & Wolf, Ribeiro & Giannakis, Gesbert & Alouini, Goldsmith, etc.

Notes

Outline I

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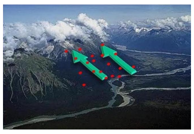
Outline II

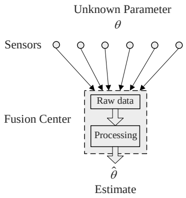
Notes

What is a WSN?

A network with a number of resource-constrained nodes, densely (and often randomly) deployed for one specific purpose, such as sensing or detecting a given phenomenon.



- Typically, measurements are conveyed over wireless channels to a Fusion Center (FC), where data is processed.



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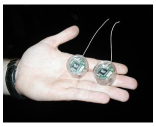
Some practical considerations on WSNs

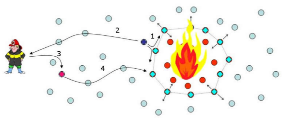


- Sensor nodes with limited sensing, processing and transmission capabilities.
- Sensors are inexpensive devices prone to failures.
- Need for energy-efficient operation
  - A means to maximize network lifetime
  - Sensor nodes are equipped with finite batteries which are difficult/impossible to replace
  - Performance-energy efficiency tradeoffs
- Network size and scalability issues:
  - Do protocols/algorithms/... scale well with the number of nodes?
  - Same software running on all nodes?
- Deployment: massive/random? pre-designed sensor location?

Notes

Typical tasks for WSNs

Detection: To determine whether an event has occurred or not

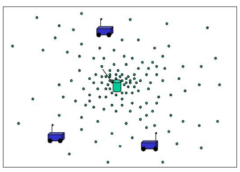


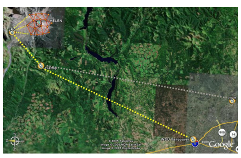
- Applications: earthquake/fire detection, military surveillance,...
- Performance metrics: probability of detection, probability of false alarm, latency, etc.

Notes

Typical tasks for WSNs

Localization/tracking: To determine and/or track the position of goods/people/.. of interest.





- Applications: border control, inventory tracking, logistics and transportation,...
- Performance metrics: positioning accuracy, coverage area, number of simultaneous targets, etc.

Notes

Typical applications of WSNs

Estimation: To accurately represent the observed phenomenon

- Applications: environmental/disaster area monitoring, patient monitoring, pollution measurements,...
- Performance metrics: distortion in the estimates, distortion vs. latency, etc.

Notes

Parameter estimation with WSNs: a good deal?

Pros:

- Increased sensing/monitoring area
- Ability to track spatial variations

Cons:

- Communication with the fusion center/other sensor nodes: fading, path-loss.
- Node reliability: what if 1/+ nodes stop working?

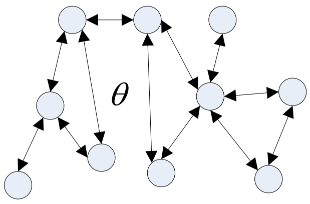
Challenges:

- Need for decentralized estimation techniques - centralized ones to act as a benchmark!
- Availability of local channel state information (CSI) only at the sensor nodes: channel gains, residual energy, etc.
- Energy consumption to acquire channel state information.
- A priori information on the parameter... and how to use it

Notes

Network Topologies

- **Infrastructure-based networks**: A central device (FC) gathers and processes the sensor measurements.
- **Infrastructureless Networks**: Homogeneous network
  - Consensus Networks



Issues: Sensors operate autonomously (no FC coordination), synchronization, belief propagation through P2P communications, stability and convergence time,...

Notes

Background Material: Classical Estimation Theory

Notes

Classical Estimation

Problem Statement:

- Parameter to estimate: θ, deterministic, unknown, scalar (or vector $\boldsymbol{\theta}$)
- Observation vector: $\mathbf{x} = [x_1, \ldots, x_N]^T$ with $\mathbf{x} \in \mathcal{X}$ a $N \times 1$ **random** vector
- Probability density function: $p(\mathbf{x}; \theta)$, known, parameterized by θ.
- Estimator: $\hat{\theta}(\mathbf{x}) = g(\mathbf{x})$

Example: Estimation of DC level in AWGN (σ_n^2)

$$x_k = \theta + n_k \quad k = 1, \ldots, N$$

- Estimator: e.g. : $\hat{\theta}(\mathbf{x}) = \frac{1}{N}\sum_{k=1}^{N} x_k$
- Variance: $\text{Var}\left(\hat{\theta}(\mathbf{x})\right) = \mathbb{E}_x\left[\left(\hat{\theta}(\mathbf{x}) - \mathbb{E}_x\left[\hat{\theta}(\mathbf{x})\right]\right)^2\right] = \frac{\sigma_n^2}{N}$

Notes

Design Criteria For Classical Estimators

Minimum Mean Squared Error (MMSE):

$$\begin{aligned}\text{MSE}\left(\hat{\theta}(\mathbf{x})\right) &= \mathbb{E}_x\left[\left(\hat{\theta}(\mathbf{x}) - \theta\right)^2\right] \\ &= \underbrace{\mathbb{E}_x\left[\left(\hat{\theta}(\mathbf{x}) - \mathbb{E}_x\left[\hat{\theta}(\mathbf{x})\right]\right)^2\right]}_{\text{Var}(\hat{\theta}(\mathbf{x}))} + \underbrace{\left(\mathbb{E}_x\left[\hat{\theta}(\mathbf{x})\right] - \theta\right)^2}_{\text{bias term}}\end{aligned}$$

Intuitive ... but often not feasible !

Minimum Variance Unbiased Estimator (MVU):

- Impose unbiasedness: $\mathbb{E}_x\left[\hat{\theta}(\mathbf{x})\right] = \int_{\mathcal{X}} \hat{\theta}(\mathbf{x})\, p(\mathbf{x}; \theta) = \theta$
- Design estimator with minimum variance:

$$\hat{\theta}(\mathbf{x}) = \arg\min_{\hat{\theta}(\mathbf{x})} \mathbb{E}\left[\left(\hat{\theta}(\mathbf{x}) - \theta\right)^2\right]$$

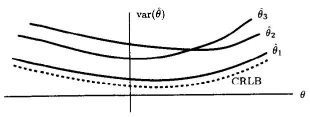
Notes

Cramer-Rao Lower Bound

Definition: Cramer-Rao Lower Bound

An absolute benchmark for any unbiased estimator:

$$CRLB(\theta) = \left(-\mathbb{E}\left[\frac{d^2 \log p(\mathbf{x}; \theta)}{d\theta^2}\right]\right)^{-1} \leq var\left(\hat{\theta}(\mathbf{x})\right)$$



Definition: Efficient Estimator

If it attains the CRLB... for all θ !

Notes

Cramer-Rao Lower Bound

Example: Estimation of DC level in AWGN noise

- Estimator: $\hat{\theta}(\mathbf{x}) = \frac{1}{N}\sum_{k=1}^{N} x_k$
- PDF: $p(\mathbf{x}; \theta) = \frac{1}{(2\pi\sigma_n^2)^{\frac{N}{2}}} \exp\left[-\frac{1}{2\sigma_n^2}\sum_{k=1}^{N}(x_k - \theta)^2\right]$
- $CRLB(\theta) = \sigma_n^2/N$
- $\text{Var}\left(\hat{\theta}(\mathbf{x})\right) = \sigma_n^2/N$

... so this estimator is the best we can do (MVU)!!

If the minimum variance estimator exists, it attains the CRLB!

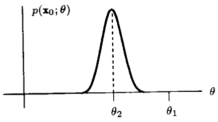
Notes

Some problems associated to MVU estimation

Problems:

- Maximum Likelihood Estimator (MLE): Straightforward & popular

$$\hat{\theta}_{\mathrm{ML}}(\mathbf{x}) = \arg\max_{\theta} p(\mathbf{x};\theta)$$



- Best Linear Unbiased Estimator (BLUE): For linear/linearized data models

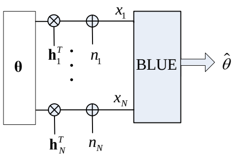
Notes

Best Linear Unbiased Estimator (BLUE)

For linear/linearized vector $(p \times 1)$ observations

$$\mathbf{x} = \mathbf{H}\boldsymbol{\theta} + \mathbf{n} \qquad \mathbf{n} \sim \mathcal{CN}(0, \mathbf{C}_n)$$

$$\mathbf{x} = \begin{bmatrix} \mathbf{h}_1^T \\ \vdots \\ \mathbf{h}_N^T \end{bmatrix} \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_p \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_N \end{bmatrix}$$



BLUE estimator is linear in the observations and given by:

$$\hat{\theta}_{\mathrm{BLUE}} = \left(\mathbf{H}^T\mathbf{C}_n^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{C}_n^{-1}\mathbf{x}$$

- Only requires first & second order moments of the observation
- Same as MVU if linear data & gaussian noise

Notes

Best Linear Unbiased Estimator (BLUE)

Particular case: Scalar parameter (i.e. $p = 1$) in uncorrelated noise

- Scalar: $\theta \to \theta$, $\mathbf{H} \to \mathbf{h}$
- Uncorrelated noise: $\mathbf{C}_n = \begin{bmatrix} \sigma_{n_1}^2 & & 0 \\ & \ddots & \\ 0 & & \sigma_{n_N}^2 \end{bmatrix}$
- BLUE estimator:

$$\hat{\theta}_{\mathrm{BLUE}} = \left(\mathbf{h}^T\mathbf{C}_n^{-1}\mathbf{h}\right)^{-1}\mathbf{h}^T\mathbf{C}_n^{-1}\mathbf{x} = \left(\sum_{k=1}^{N}\frac{|h_k|^2}{\sigma_{n_k}^2}\right)^{-1}\left(\sum_{k=1}^{N}\frac{h_k x}{\sigma_{n_k}^2}\right)$$

- with distortion (variance) given by: $\mathrm{Var}(\hat{\theta}) = \left(\sum_{k=1}^{N}\frac{|h_k|^2}{\sigma_{n_k}^2}\right)^{-1}$
- In case of identical noise variances $(\sigma_{n_k}^2 = \sigma_n^2)$: $\mathrm{Var}(\hat{\theta}) = \frac{\sigma_n^2}{N}$

Notes

Background materials:
Bayesian Estimation

Notes

Bayesian Estimation

Problem Statement:

- Parameter to estimate: θ, random, unknown, scalar (or vector $\boldsymbol{\theta}$)
- Observation vector: $\mathbf{x} = [x_1, \dots, x_N]^T$ with $\mathbf{x} \in \mathcal{X}$ a $N \times 1$ **random** vector
- Probability density function: $p(\mathbf{x}, \theta)$, known, joint distribution of $\mathbf{x}$ and θ.

$$p(\mathbf{x}, \theta) = \underbrace{p(\theta)}_{\text{prior pdf}} \; p(\mathbf{x}|\theta)$$

Pros & cons

- Additional (prior) knowledge to improve estimation
- Risk of mismatched/unknown pdf ... CRITICAL!

Design Criteria:

- MAP(Maximum a Posteriori), MMSE (Minimum MSE)

Notes

Bayesian MMSE Estimator

Bayesian Estimation

- Mean Squared Error (MSE)

$$\begin{aligned} \text{MSE}\left(\hat{\theta}(\mathbf{x})\right) &= \mathbb{E}_{\mathbf{x},\theta}\left[\left(\hat{\theta}(\mathbf{x}) - \theta\right)^2\right] \\ &= \int_{\mathbf{x}}\int_{\theta}\left(\hat{\theta}(\mathbf{x}) - \theta\right)^2 p(\mathbf{x}, \theta) d\mathbf{x} d\theta \end{aligned} \qquad (1)$$

- Bayesian Minimum Mean Squared Error (BMMSE) estimator: The *posterior* mean

$$\hat{\theta}_{\text{BMMSE}}(\mathbf{x}) = \mathbb{E}_{\theta|\mathbf{x}}[\theta]$$

with

$$p(\theta|\mathbf{x}) = \frac{p(\mathbf{x}|\theta)p(\theta)}{\int_\theta p(\mathbf{x}, \theta)d\theta} = \frac{p(\mathbf{x}|\theta)p(\theta)}{p(\mathbf{x})}$$

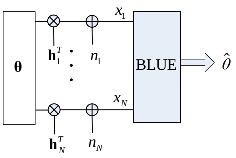
Notes

Bayesian MMSE estimator

Bayesian Estimation

For linear (or linearized) observations

$$\mathbf{x} = \mathbf{H}\boldsymbol{\theta} + \mathbf{n}$$
$$\mathbf{n} \sim \mathcal{CN}(0, \mathbf{C}_\mathbf{n})$$
$$\boldsymbol{\theta} \sim \mathcal{CN}(0, \mathbf{C}_{\boldsymbol{\theta}})$$



The Bayesian (linear) MMSE estimator is:

$$\hat{\boldsymbol{\theta}}_{\text{BMMSE}}(x) = \left(\mathbf{H}^T\mathbf{C}_\mathbf{n}^{-1}\mathbf{H} + \mathbf{C}_{\boldsymbol{\theta}}^{-1}\right)^{-1}\mathbf{H}^T\mathbf{C}_\mathbf{n}^{-1}\mathbf{x}$$

with covariance (distortion) matrix given by,

$$\mathbf{C}_{\theta|\mathbf{x}} = \left(\mathbf{H}^T\mathbf{C}_\mathbf{n}^{-1}\mathbf{H} + \mathbf{C}_{\boldsymbol{\theta}}^{-1}\right)^{-1}$$

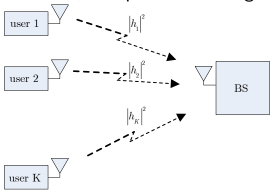
Notes

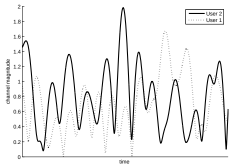
Background Materials:
Multi-user diversity and Opportunistic Scheduling

Notes

Multi-user Diversity and Opportunistic Scheduling

Scenario: Wireless data network, multiple-access (uplink) channel, K users, independent fading for user-BS channels ($|h_1|^2, \ldots, |h_K|^2$)



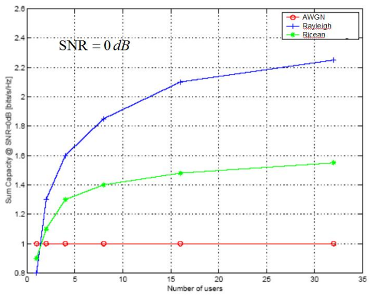


Performance metric: Sum-rate

- Round-robin scheduling: $r_{t,\text{RR}} = \log_2\left(1 + \frac{P|h_{k(t)}|^2}{\sigma_w^2}\right)$
- Opportunistic Scheduling: $r_{t,\text{opp}} = \log_2\left(1 + \frac{P\max_{k(t)}|h_{k(t)}|^2}{\sigma_w^2}\right)$

Notes

Multi-user Diversity and Opportunistic Scheduling



- Multi-user diversity: $\mathbb{E}[r_{t,\text{opp}}] \sim \log\log K$ (Rayleigh Fading)

Notes

Selective Multi-user Diversity

Problem: large amount of feedback (from all users)

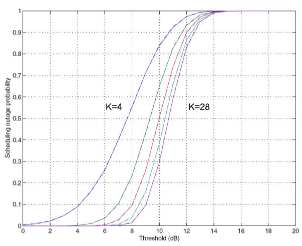
Solution: Selective Multi-user Diversity (SMUD) [Gesbert-Alouini'04]

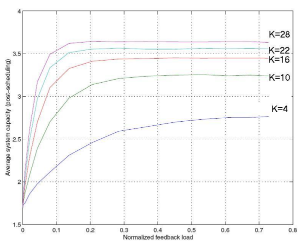
- Algorithm:
  1. Only a subset of users feedback their channel gains: $|h_k|^2 \geq \gamma_k$
  2. BS schedule the user with highest gain out of those above the threshold
- Scheduling Outage: No feedback from sensors - e.g. random selection

$$\Pr(\text{Outage}) = \Pr\left(|h_{\max}|^2 \leq \gamma_{\text{th}}\right)$$

Notes

Selective Multi-user Diversity (cont'd)





- Average SNR = 5dB
- Interplay threshold ⇔ feedback load ⇔ performance
- For $K \geq 25$ users, 10% of users reporting SNRs suffices.

Notes

Multi-user Diversity with Quantized Feedback
Multi-user Diversity and Opportunistic Scheduling

Problem: channel gains feedback in "analog" form
Solution: use quantized channel gains [Nosratinia'07]

- Algorithm:
  1. Users: just notify if above/below threshold $1\{|h_k|^2 \geq \gamma_{\text{th}}\}$
  2. BS: select at random out of those above the threshold
- Performance: Sum-rate
  - If k users report:

$$\bar{r}_k = \sum_{i=1}^{k} \Pr\left[\text{ the } i^{\text{th}} \text{ best user is selected}\right] r_i = \frac{1}{k}\sum_{i=1}^{k} r_i$$

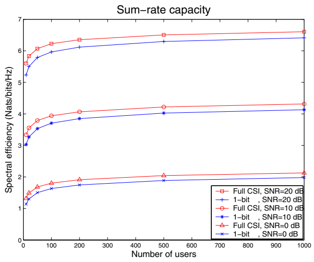
  - For any number of users ($k = 1 \dots K$):

$$\bar{r} = \sum_{k=1}^{n} p_k \bar{r}_k$$

  - Need for optimized choice of the threshold (γ_{th})

Notes

Multi-user Diversity with Quantized Feedback
Multi-user Diversity and Opportunistic Scheduling



- Same growth-rate as with "analog" feedback ($K \to \infty$)
- Small penalty wrt "analog" feedback

Notes

Opportunistic Scheduling and WSNs
Multi-user Diversity and Opportunistic Scheduling

- Differences: The goal now is to maximize estimation accuracy (rather than sum-rate)
- Similarities: To opportunistically schedule sensors results into enhanced performance

Example: Two sources of "randomness"

- Observation noise: $x_k = \theta + n_k$; $n_k \sim \mathcal{CN}(0, \sigma_{n_k}^2)$
- Sensor-FC channels: $|h_k|^2$

Intuition: Distortion minimized for sensors with high $|h_k|^2$ and low observation noise.

Notes

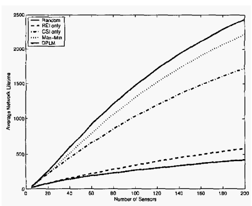
Opportunistic Scheduling & Network Lifetime
Multi-user Diversity and Opportunistic Scheduling

- Extending Network Lifetime (LT) is crucial in WSNs
- Definitions: time elapsed until first sensor dies/part of the WSN is not reachable/estimation quality cannot be achieved/...
- Scheduling with Residual Energy Information (REI)[Chen-Zhao'05]
  - Let $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n$ be the residual energies.
  - New scheduling rule:

REI & CSI $\quad i^* = \arg\max_i \dfrac{\varepsilon_i}{E_{\text{packet}}(|h_i|^2)}$

CSI only $\quad i^* = \arg\max_i |h_i|^2$

REI only $\quad i^* = \arg\max_i \varepsilon_i$



Combined use of REI & CSI maximizes lifetime (1st sensor dies)

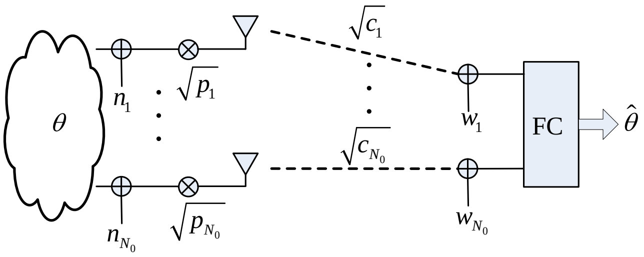
Notes

Notes

Decentralized Estimation with Analog Transmission

Notes

Consider a WSN composed of one Fusion Center (FC) and a large population of N_0 energy-constrained sensors aimed at estimating a scalar, slowly-varying and spatially-homogeneous parameter θ.



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- Each sensor observes $x_i = \theta + n_i$; $i = 1, \ldots, N_0$
- Then, it re-transmits a scaled version to the FC:

$$\mathbf{y} = \mathbf{h}\theta + \mathbf{z} \qquad \begin{array}{l} \mathbf{h} = \left[\sqrt{p_1 c_1}, \ldots, \sqrt{p_{N_0} c_{N_0}}\right]^T \; ; \; c_i = |h_i|^2 \\ \mathbf{z} \sim \mathcal{CN}(0, \mathbf{C_z}) \\ \text{diag}\left[\mathbf{C_z}\right] = \left[p_1 c_1 \sigma_n^2 + \sigma_w^2, \ldots, p_{N_0} c_{N_0} \sigma_n^2 + \sigma_w^2\right] \end{array}$$

- The FC uses a Best Linear Unbiased Estimator (BLUE) to reconstruct θ:

$$\hat{\theta} = \left(\mathbf{h}^T \mathbf{C_z}^{-1} \mathbf{h}\right)^{-1} \mathbf{h}^T \mathbf{C_z}^{-1} \mathbf{y}$$

with distortion given by:

$$D = \text{Var}(\hat{\theta}) = \left(\sum_{i=1}^{N_0} \frac{p_i c_i}{p_i c_i \sigma_n^2 + \sigma_w^2}\right)^{-1}$$

Notes

- Minimization of Distortion:

$$\min_{p_1, \ldots, p_{N_0}} D \qquad \text{s.t.} \qquad \sum_{i=1}^{N_0} p_i \le P_t$$

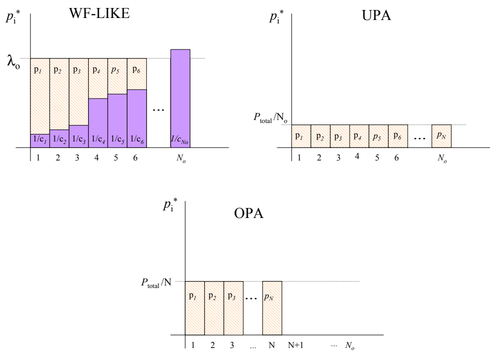
- Solution: Waterfilling-like [Goldsmith et al]

$$p_i^* = \frac{\sigma_w^2}{\sigma_n^2 c_i}\left[\frac{\sqrt{c_i}}{\sqrt{\lambda_0}\sigma_w} - 1\right]^+ \qquad i = 1 \ldots, N_0$$

- Drawback: Extensive signalling sensors $\leftrightarrow$ FC required
- Fix: Opportunistic Power Allocation (OPA) [Matamoros & Antón-Haro]
  - Let only a subset of sensors $N \le N_0$ participate on average in the estimation process and keep FC-sensor signalling low
  - Performance loss??

Notes

Comparison of Power Allocation Schemes
Decentralized Estimation with Analog Transmission

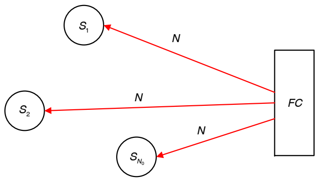


- OPA vs. WF-like and Uniform Power Allocation (UPA)

Notes

Opportunistic Power Allocation (OPA)
Decentralized Estimation with Analog Transmission

1. **Initialization**: Compute the channel gain threshold $\gamma_{\rm th}$.
2. **Identification of the active sensor set**: In each time-slot, each sensor node notifies the FC whether it will participate in the estimation process or not.



3. **Power Allocation and Transmission**: The N active sensor nodes send their observations to the FC.
4. **Go to Step 2.**

Notes

OPA - Threshold Computation
Decentralized Estimation with Analog Transmission

- **Power Allocation Rule**: Uniformly allocated to the subset of **active** sensors, that is,

$$p_i = P_t/N; \qquad i = 1 \dots N$$

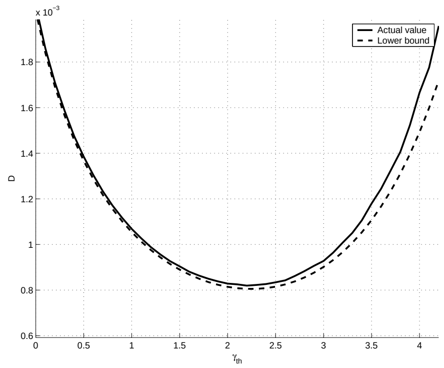
- **Threshold Computation**:
The threshold $\gamma^*_{\rm th}$ which minimizes the bound is given by,

$$\gamma^*_{\rm th} = \left[2\mu_c W_0 \left(\frac{1}{2} \sqrt{\frac{N_0 \sigma_w^2 e}{P_t \sigma_n^2 \mu_c}} \right) - \mu_c \right]^+$$

where $W_0(\cdot)$ is the Lambert function, i.e. the inverse function of $f(W) = We^W$.

Notes

A Sanity Check on the Lower Bound...
Decentralized Estimation with Analog Transmission

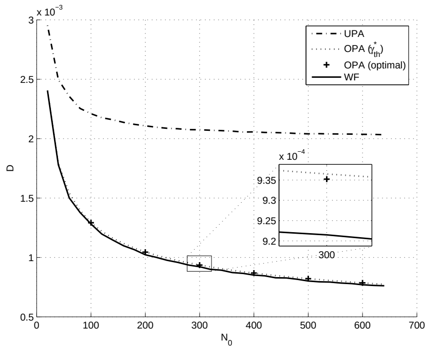


- The lower bound is convex and tight !

Notes

OPA - Performance Results

Decentralized Estimation with Analog Transmission



- OPA outperforms Uniform Power Allocation and very close to WF.
- Almost no penalty for using the approximate threshold γ^*_{th}

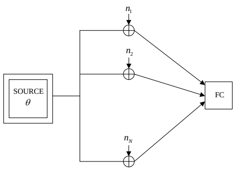
Notes

Decentralized Estimation with Digital Transmission

Notes

Distortion with centralized observations

Decentralized Estimation with Digital Transmission



- θ: deterministic, scalar
- $x_k = \theta + n_k$; $k = 1, \dots, N$
- $n_k \sim \mathcal{N}(0, \sigma_n^2)$

- Sample mean estimator (BLUE & MVU):

$$\hat{\theta}_{\text{BLUE}} = \frac{1}{N} \sum_{k=1}^{N} x_k \quad ; \quad \text{Var}(\theta) = \frac{\sigma_n^2}{N} \tag{2}$$

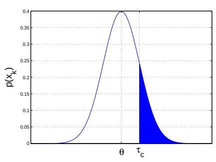
- What can we expect with *decentralized* estimation and *1 bit per sensor* (bandwidth constraints)?

Notes

Quantization with 1 Bit per sensor

Decentralized Estimation with Digital Transmission

- **Sensors**: Send indicator variable $b_k \in \{0, 1\}$... a set of 1-bit quantizers!! [Ribeiro-Giannakis'05]



- b_k: Bernoulli random variable with parameter

$$q_k(\theta) = F(\tau_c - \theta)$$

- F: Complementary CDF of noise
- τ_c: design parameter

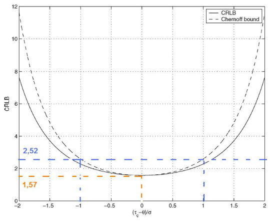
- **FC**: Produces an ML estimate

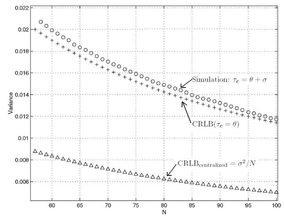
$$\hat{\theta}_{\text{ML}} = \tau_c - F^{-1}\left(\frac{1}{N}\sum_{k=1}^{N} b_k\right)$$

with $\hat{q} = \frac{1}{N}\sum_{k=1}^{N} b_k$.

Notes

- Cramer-Rao Lower Bound: $CRLB(\theta) = \text{Var}(\hat{\theta})$ for large N





- $\text{CRLB}_{\text{min}} \approx 1.57\frac{\sigma^2}{N}$ Variance of the centralized MVU estimator
- To attain CRLB_{min}, we need $\tau_c = \theta$ (... but θ is unknown!)
- For $\tau_c \in [\theta - \sigma_n, \theta + \sigma_n]$, $\text{Var}\left(\hat{\theta}_{\text{ML}}\right) \leq 2.52\frac{\sigma_n^2}{N}$
- Performance loss wrt centralized & $\tau_c = \theta$

Notes

Problem setup and assumptions:

- Parameter: θ, deterministic and scalar.
- Observations: $x_k = \theta + n_k$ with $k = 1 \dots N$
  - n_k: zero-mean, variance $\sigma_{n,k}^2 = \sigma_k^2$ (different observation qualities!)
  - Noise PDF unknown.
  - Finite dynamic range: $x_k \in [-W, W]$
- Digital sensor-FC communications: uncoded M-QAM modulation
- Uniform L_k-bit quantizers in each sensor node (k)
- BLUE estimator at the FC

Goal: Optimal number of quantization levels $(2^{L_k} - 1)$ in each sensor node?

- Target distortion constraint: $D = \mathbb{E}_x\left[\left(\hat{\theta}(\mathbf{x}) - \theta\right)^2\right] \leq D_0$
- Minimum power consumption.

Notes

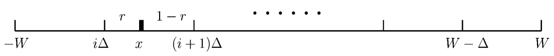
Benchmark: *Centralized* BLUE with *analog* observations

$$\hat{\theta}_{\text{BLUE}} = \left(\sum_{k=1}^{N} \frac{1}{\sigma_k^2}\right)^{-1} \sum_{k=1}^{N} \frac{x_k}{\sigma_k^2}$$

with $\text{Var}(\hat{\theta}) = \left(\sum_{k=1}^{N} \frac{1}{\sigma_k^2}\right)^{-1}$

Quantizer: uniform intervals

- Divide $[-W, W]$ into $2^{L_k} - 1$ intervals, of length $\Delta = 2W/(2^{L_k} - 1)$



- Quantize x_k with an L_k-bit message, $m_k(x_k, L_k)$, while fulfilling $\mathbb{E}\left[m(x_k, L_k)\right] = x_k$

Notes

Case #1: Distortion without errors in sensor-FC channels:

- The message sent is

$$m_k(x_k, L_k) = \theta + n_k + \underbrace{\nu_k}_{\text{Quant. Noise}}$$

with

$$\text{Var}\left(\nu_k\right) \leq \delta_k^2 = \frac{W^2}{\left(2^{L_k} - 1\right)^2}$$

- Quasi-BLUE estimate at the FC:

$$\hat{\theta}_{\text{Q-BLUE}} = \left(\sum_{k=1}^{N} \frac{1}{\sigma_k^2 + \delta_k^2}\right)^{-1} \left(\sum_{k=1}^{N} \frac{m_k}{\sigma_k^2 + \delta_k^2}\right)$$

with distortion lower-bounded by:

$$D = \text{Var}(\hat{\theta}_{\text{Q-BLUE}}) \geq \left(\sum_{k=1}^{N} \frac{1}{\sigma_k^2 + \delta_k^2}\right)^{-1}$$

Notes

Case #2: Distortion with errors in sensor-FC channels:

- Bit Error Rate (BER) of the k-th link: p_b^k
- Received and sent messages differ:

$$m'_k(x_k, L_k) \neq m_k(x_k, L_k) \Longrightarrow \text{increased estimation variance}$$

- Distortion of the quasi-BLUE estimate:

$$D \leq (1 + p_o) \left(\sum_{k=1}^{N} \frac{1}{\sigma_k^2 + \delta_k^2} \right)^{-1}$$

with

$$p_o = \max_{1 \leq k \leq N} \frac{8W}{\sigma_k} \sqrt{\frac{N p_b^k}{3}}$$

Notes

Optimal bit and power allocation:

- For a given distortion target D_o and BER, minimize transmit power
- Optimal number of quantization bits (sensor-wise):

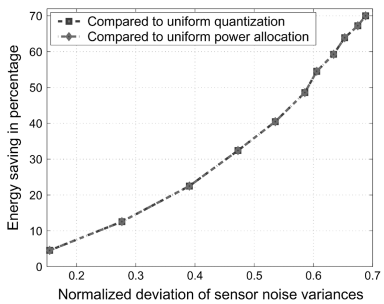
$$L_{\text{opt}}^k = \begin{cases} 0 & \text{if excessive attenuation } a_k \\ \log\left(1 + \frac{W}{\sigma_k}\sqrt{\frac{\eta_0}{a_k}} - 1\right) & \text{otherwise} \end{cases}$$

- Transmit power required:

$$p_k \sim a_k \left(2^{L_k} - 1\right)$$

and $P_{\text{total}} = \sum_{k=1}^{N} p_k$

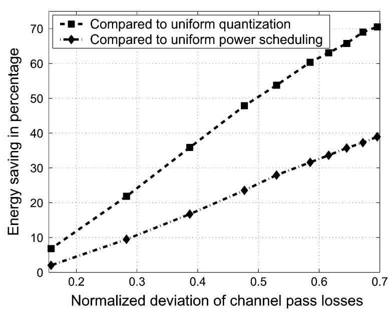
Notes



"Uniform quantization/power allocation": Same number of quantization bits/transmit power in all sensors (to meet D_o)

- The larger the spread in sensor noise variances, the higher the savings.

Notes



"Uniform quantization/power allocation": Same number of quantization bits/transmit power in all sensors (to meet D_o)

- The larger the spread in channel path-losses, the higher the savings.

Notes

Concluding Remarks

- General observations:
  - BLUE and Maximum-Likelihood (ML) estimators are often used in decentralized settings.
  - Benchmarks: centralized estimator and Cramer-Rao Lower Bound.
  - Maximization of sum-rate vs. maximization of estimation accuracy.
- Decentralized Estimation with Analog Transmission (A&F):
  - Opportunistic Power Allocation (OPA) performs close to Optimal Power Allocation (i.e. waterfilling-like solutions) with less complexity and feedback requirements.
  - OPA neatly outperforms Uniform Power Allocation approaches.
- Decentralized Estimation with Digital Transmission:
  - With 1 bit per sensor, ML estimation and rough knowledge on quantization threshold, the distortion is close to the "analog" system.
  - With L_k bits (variable) per sensor, power allocation, and BLUE estimation substantial energy savings w.r.t. the cases with fixed quantization levels/fixed power allocation (for a given distortion).

Notes

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Notes

THANKS FOR YOUR KIND ATTENTION !!
Questions?

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