WiLab at the
University of Bologna

First NEWCOM++ Summer School

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Topology Control and Connectivity

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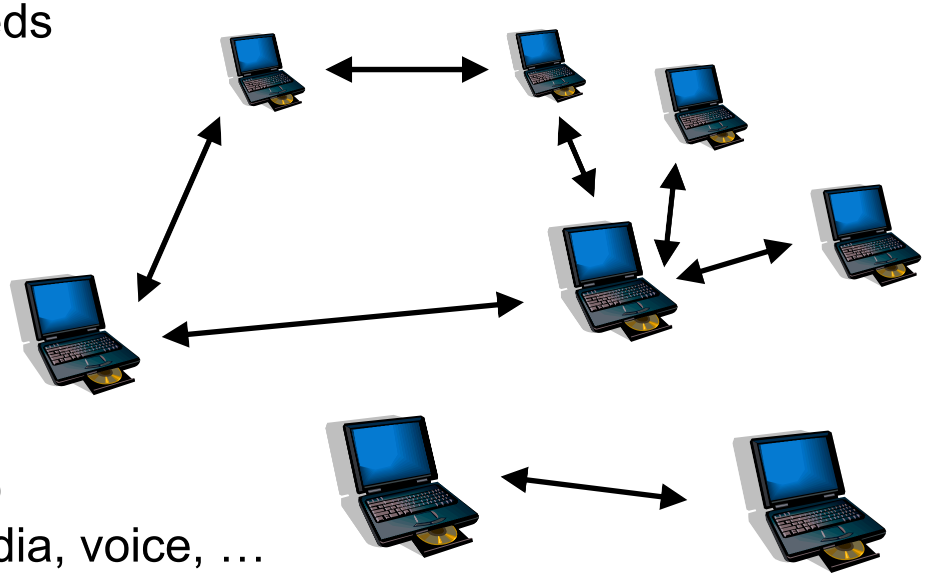
Wireless Ad Hoc Networks

WAHNs (Wireless Ad Hoc Networks) are formed dynamically
by an autonomous system of nodes connected via wireless links without using
an existing network infrastructure or centralised administration.

Nodes are connected through "ad hoc" topologies,
set up and cleared according to user needs
and temporary conditions.



Main Features

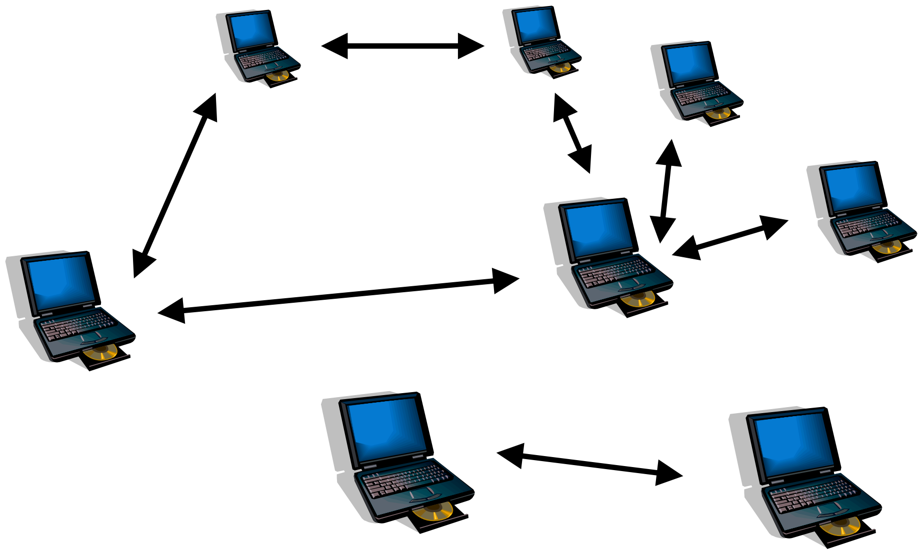
Fixed infrastructure is not needed
Unplanned and highly dynamical

Nodes are "smart" terminals (laptops, …)
Real-time or non real-time data, multimedia, voice, …
Every node can be either source or destination of information
Every node can be a router toward other nodes
Energy is not the most relevant matter
Capacity is the most relevant matter

Wireless Ad Hoc Networks

Application Examples:

- Tactical Networks (military application) – nodes are mobile over battle field
- Emergency Services – nodes are mobile over large areas
- Home and Enterprise Networks – nodes are nomadic, palmtops or laptops
- …



WSNs vs Wireless Ad Hoc Networks (WAHNs)

WAHNs:

Fixed infrastructure is not needed
Unplanned

Nodes are “smart” terminals (laptops, …) **no**
Real-time or non real-time data, multimedia, voice, … **no**
Every node can be either source or destination of information **no**
Every node can be a router toward other nodes **no**
Energy is not the most relevant matter **no**
Capacity is the most relevant matter **no**

WSNs are NOT a special case of Ad Hoc Networks:

Communication Strategies and Protocols should be very different (do not re-use IEEE 802.11 as it is!).

Topology Control versus Connectivity Control

Topology Control aims at controlling the set of links that connect couples of nodes, in order to simplify routing of messages / allow routing of messages between all pairs of nodes.

The ***physical* topology** of a network can be controlled through physical layer, in most cases power control techniques are used.

The ***logical* topology** of a network is controlled by entities working at layer 2 and 3, and is based on a **reduced set of links** wrt the physical topology.

Logical topologies can be *flat* or ***hierarchical*** according to the roles assigned to nodes.

Connectivity Theory aims at describing the potential topologies of a networks, assuming nodes are randomly distributed over space.

It deals mainly with physical topologies, but can be extended
to some aspects of logical topologies taking non-electromagnetic aspects into account
like for instance capacities, interference, etc.

Outline

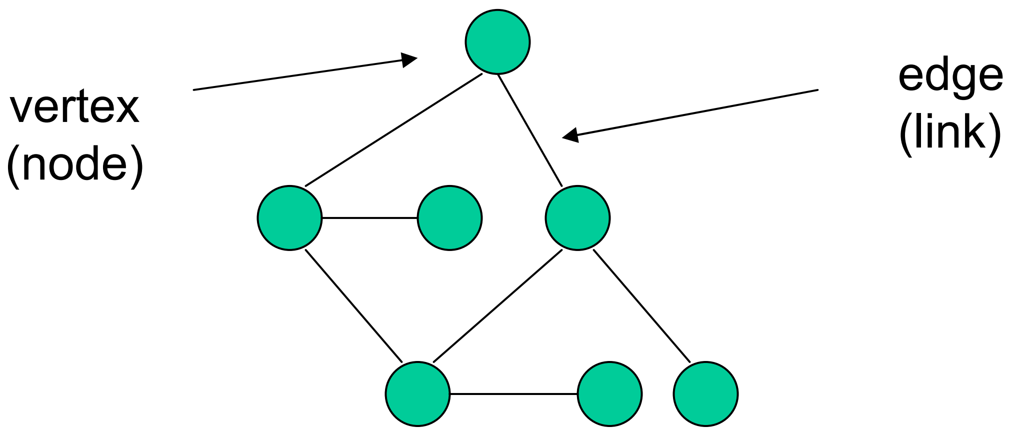
1. Network Topologies in WSNs
2. Connectivity Theory: Preliminaries
3. Small Worlds
4. Critical Transmission Range
5. Connectivity Over an Unlimited Region
6. Connectivity for WSNs
7. Connectivity Over Limited Regions for WSNs

Background: Elements of Graph Theory

Graph

A set of items connected by *edges*. Each item is called a *vertex* or node.
Formally, a graph is a *set* of vertices and a *binary relation* between them, adjacency.
Formal Definition: A graph G can be defined as a pair (V,E),
where V is a set of vertices, and E is a set of edges between the vertices
E = {(u,v) | u, v in V}.

If the graph is **undirected**, the adjacency relation defined by the edges is *symmetric*, or E = {{u,v} | u, v in V} (sets of vertices rather than ordered pairs).

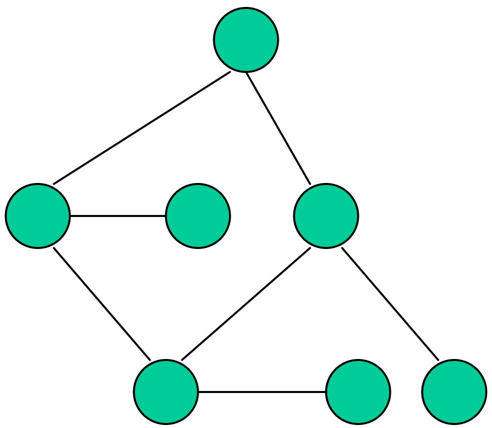


Background: Elements of Graph Theory

Geometric Graph (GG)
Vertices have a geometric location in $\mathbf{R}^d$. In the following we assume d=2.

Random Graph
Edges between pairs of nodes exist according to random statistics.

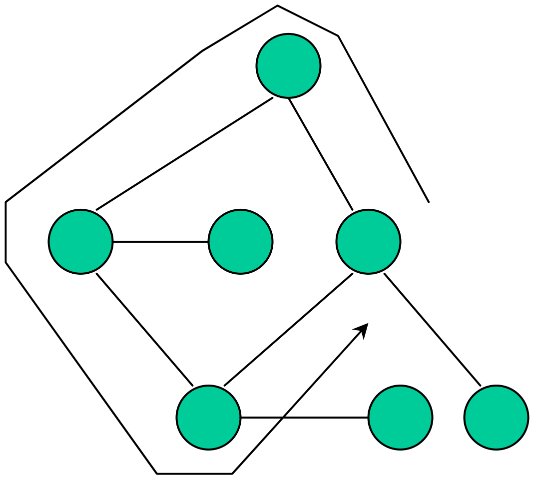
Geometric Random Graph (GRG)
Random Graph where edges exist according to proximity relation between nodes and nodes are in unknown positions.
In GRGs, the set of nodes is normally finite, and their number deterministically known.



Background: Elements of Graph Theory

Complete Graph
An *undirected graph* with an *edge* between every pair of *vertices*

Acyclic Graph
A *graph* with no *path* that starts and ends at the same *vertex*.



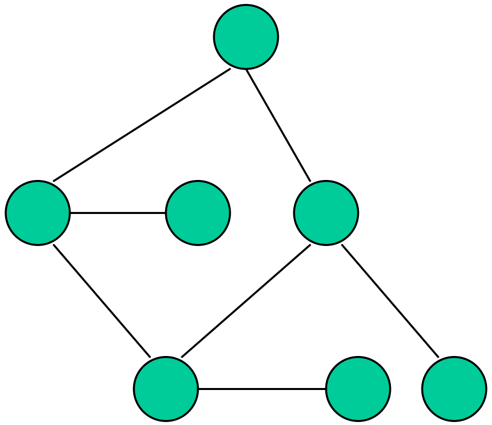
Not complete
Not acyclic

Background: Elements of Graph Theory

Connected Graph
An *undirected graph* that has a *path* between every pair of *vertices*.

Edge Connectivity
The smallest number of *edges* whose deletion will cause a *connected graph* to not be connected.

Node Connectivity
The smallest number of *vertices* whose deletion causes a *connected graph* to not be connected.



Connected
EC=1
NC=1

Background: Elements of Graph Theory
The Communication Graph

A Network is a pair (N,L)
N is a set of wireless nodes, of size n. Assume they are located in an unit square.
L is the function mapping every node u to a position L(u).

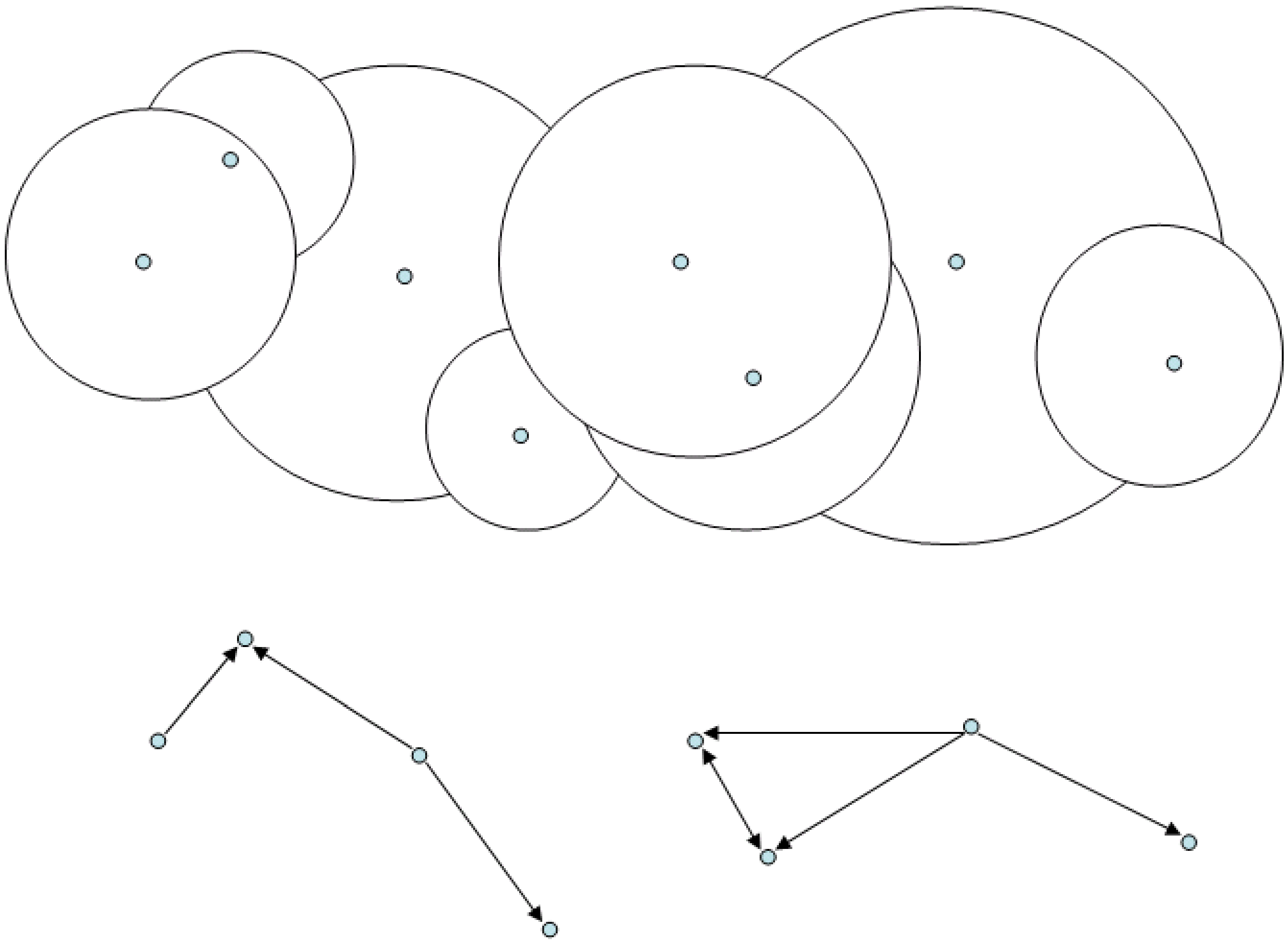
A Range Assignment is a function assigning to every node u a transmit range RA(u).

The Communication Graph is the directed graph (G,E) where the directed edge (u,v) exists if the Euclidean distance between u and v is less or equal than RA(u).
In this case v is neighbour to u. If u is also neighbour to v for all pairs (u,v), the Communication Graph is undirected and all links are symmetrical.

A RA for a Network is **connecting** if the correspondent CG is connected.

A RA where all nodes have the same transmit range is said **homogeneous**.
If the value of the transmit range R is relevant, than the RA is said **R-homogeneous**.

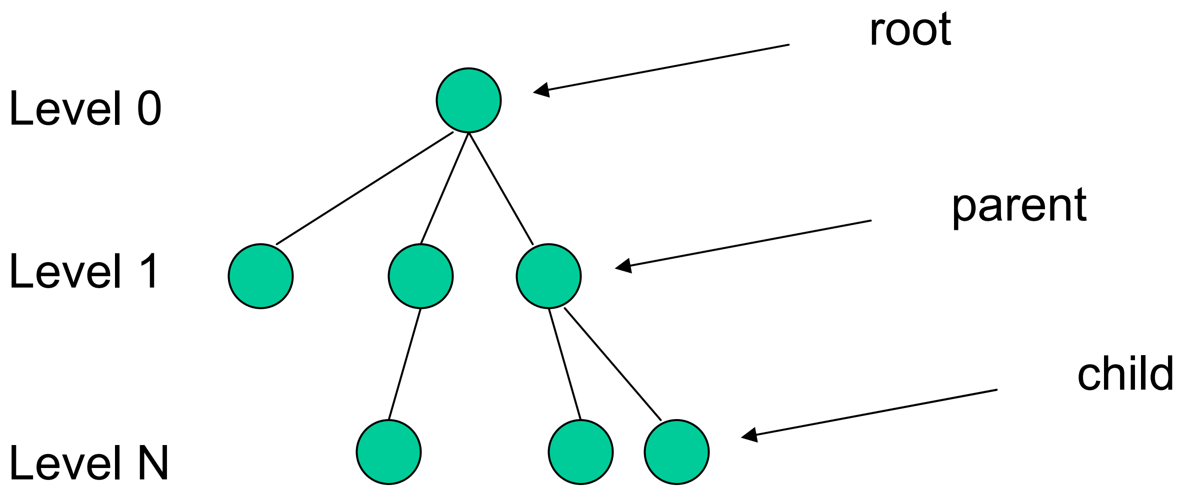
Background: Elements of Graph Theory
The Communication Graph: Example



Background: Elements of Graph Theory

Tree

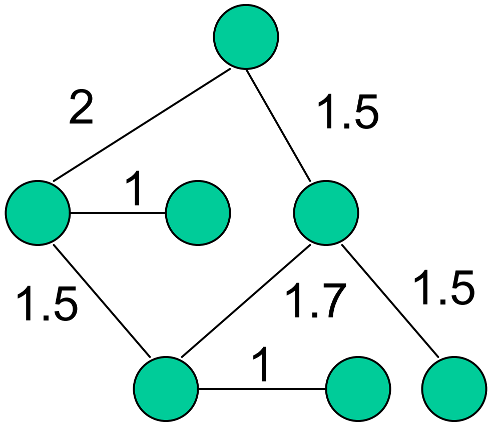
A *connected*, *undirected*, *acyclic graph*.
It is a data structure accessed beginning at the *root* node, where each *node* is either a *leaf* or an *internal node*.
An internal node has one or more *child* nodes and is called the *parent* of its child nodes. All children of the same node are *siblings*.
Contrary to a physical tree, the root is usually depicted at the top of the structure, and the leaves are depicted at the bottom.



Background: Elements of Graph Theory

Weighted Graph
A *graph* having a weight, or number, associated with each *edge*

Euclidean Tree
A *tree* in a weighted GG where weights are assigned to edges based on Euclidean distances.

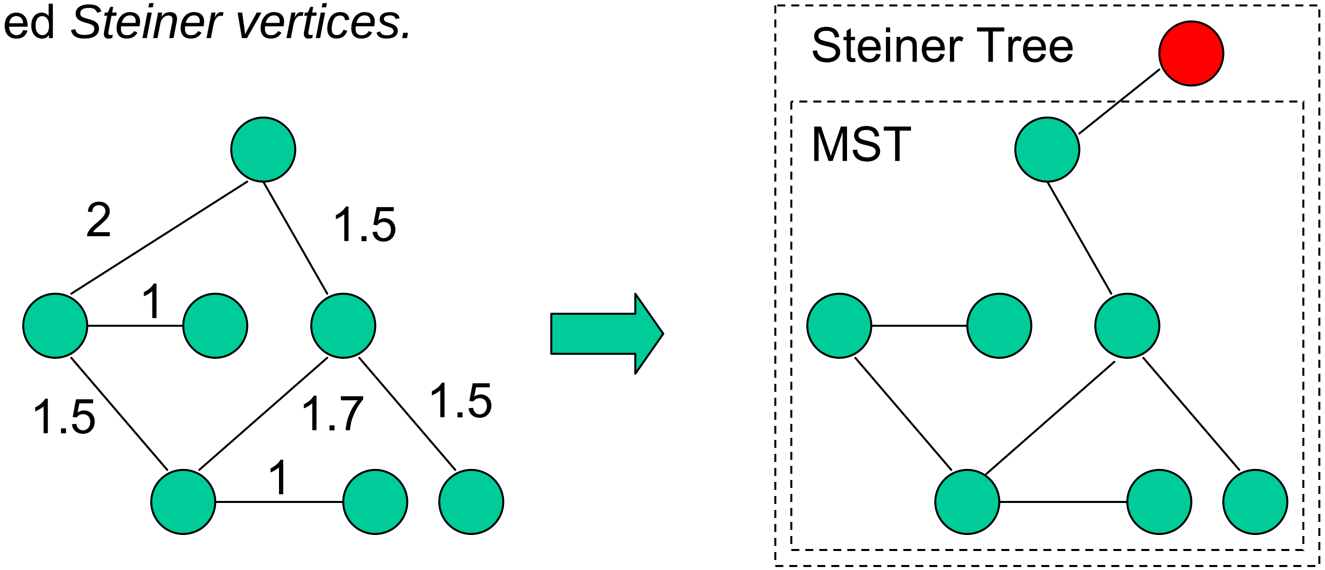


Background: Elements of Graph Theory

Spanning Tree
A connected, acyclic subgraph containing all the vertices of a graph

Minimum Spanning Tree (MST)
A minimum-weight tree in a weighted graph which contains all of the graph's vertices.

Steiner Tree
A minimum-weight *tree* connecting a designated set of *vertices*, called *terminals*, in an *undirected*, *weighted graph*. The tree may include non-terminals, which are called *Steiner vertices.*



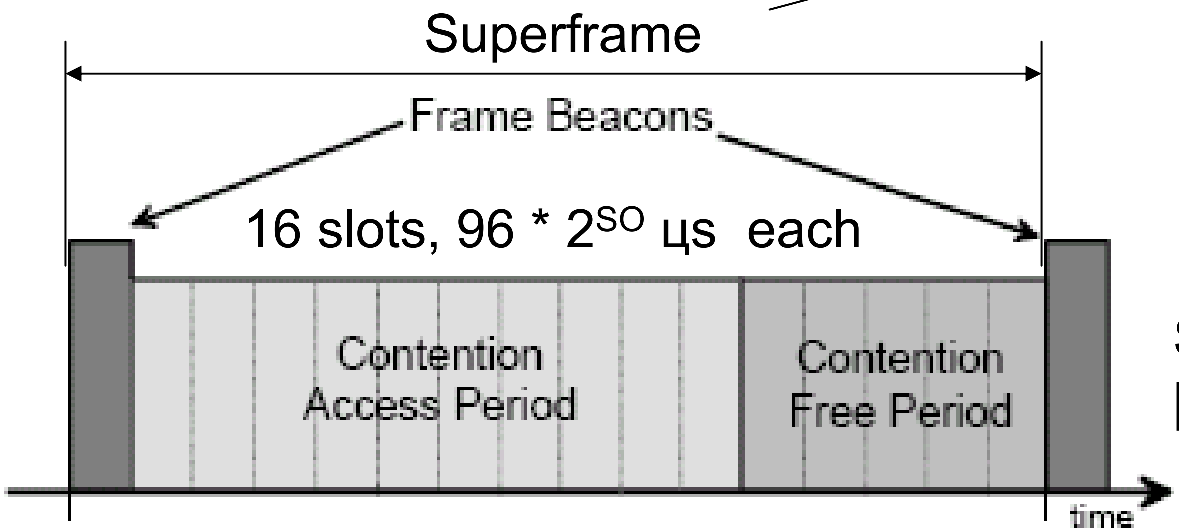
Section 1
Network Topologies in WSNs

802.15.4
Trees

IEEE 802.15.4 - MAC

Approx. duration: **15 ms – 250 s**

Beacon-Enabled Mode



SO = 0, 1, …, 14
[Superframe Order]

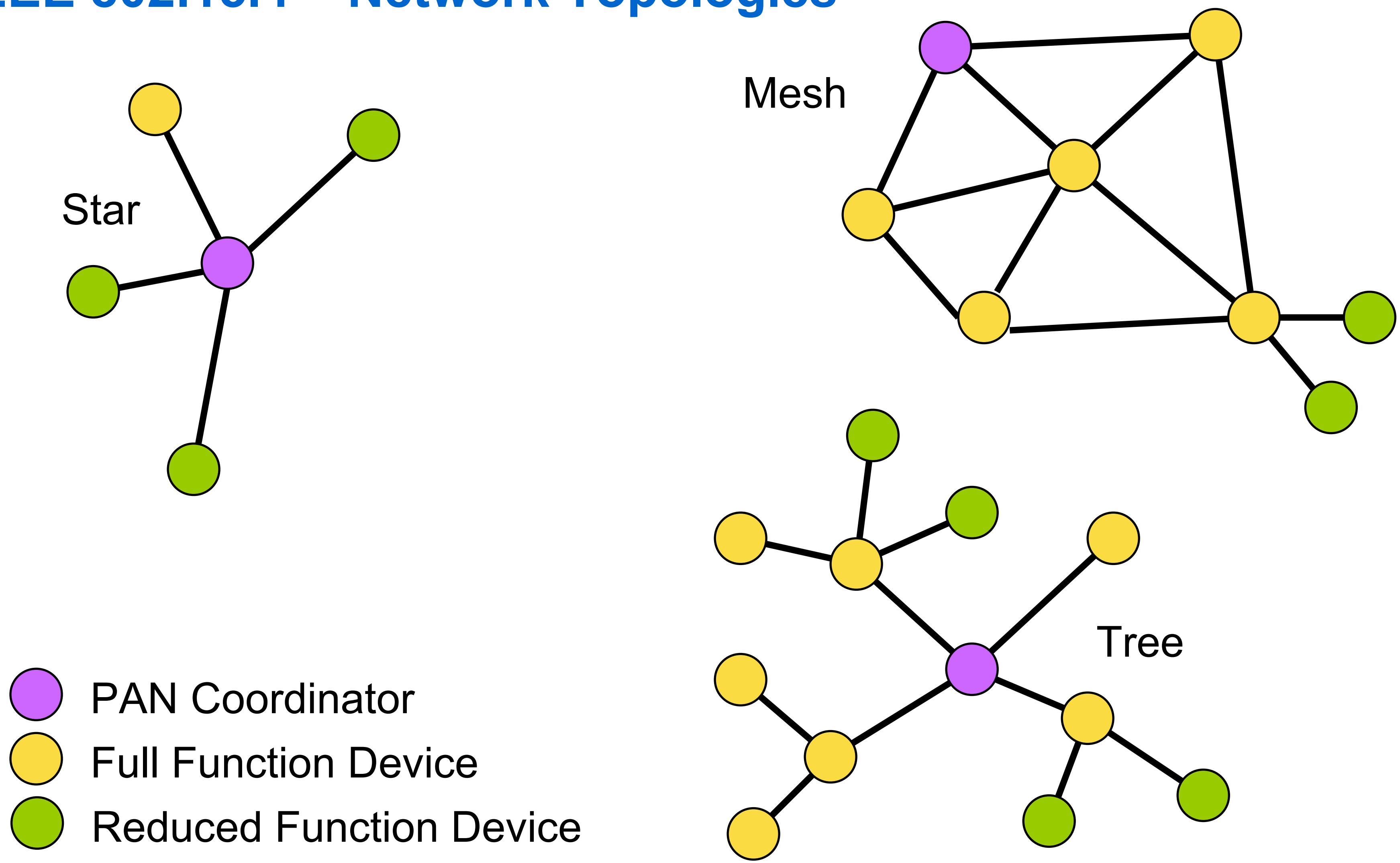
CSMA/CA

Maximum 7 GTSs
(Granted Time Slots)

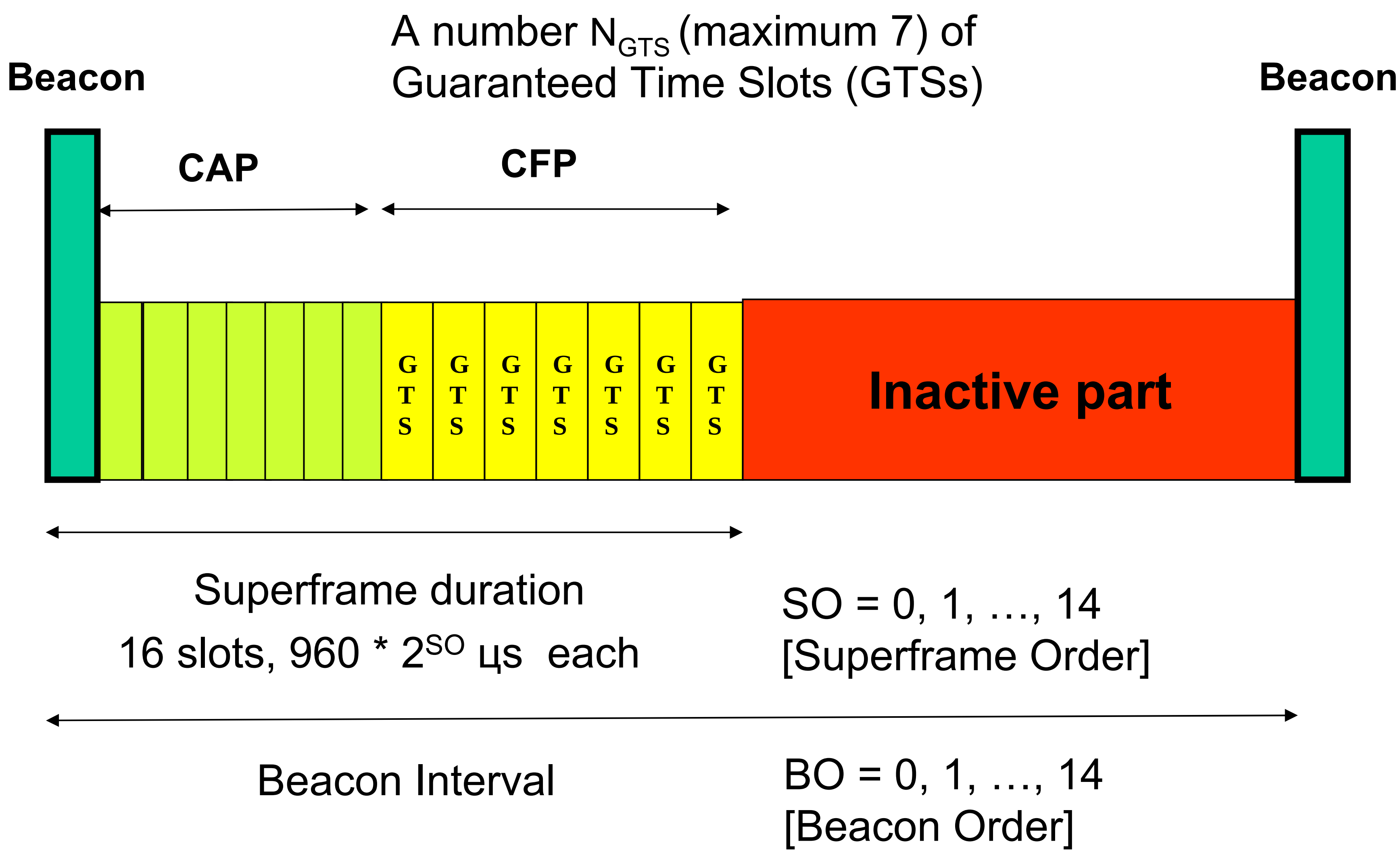
Non Beacon-Enabled Mode

Only CAP, with CSMA/CA

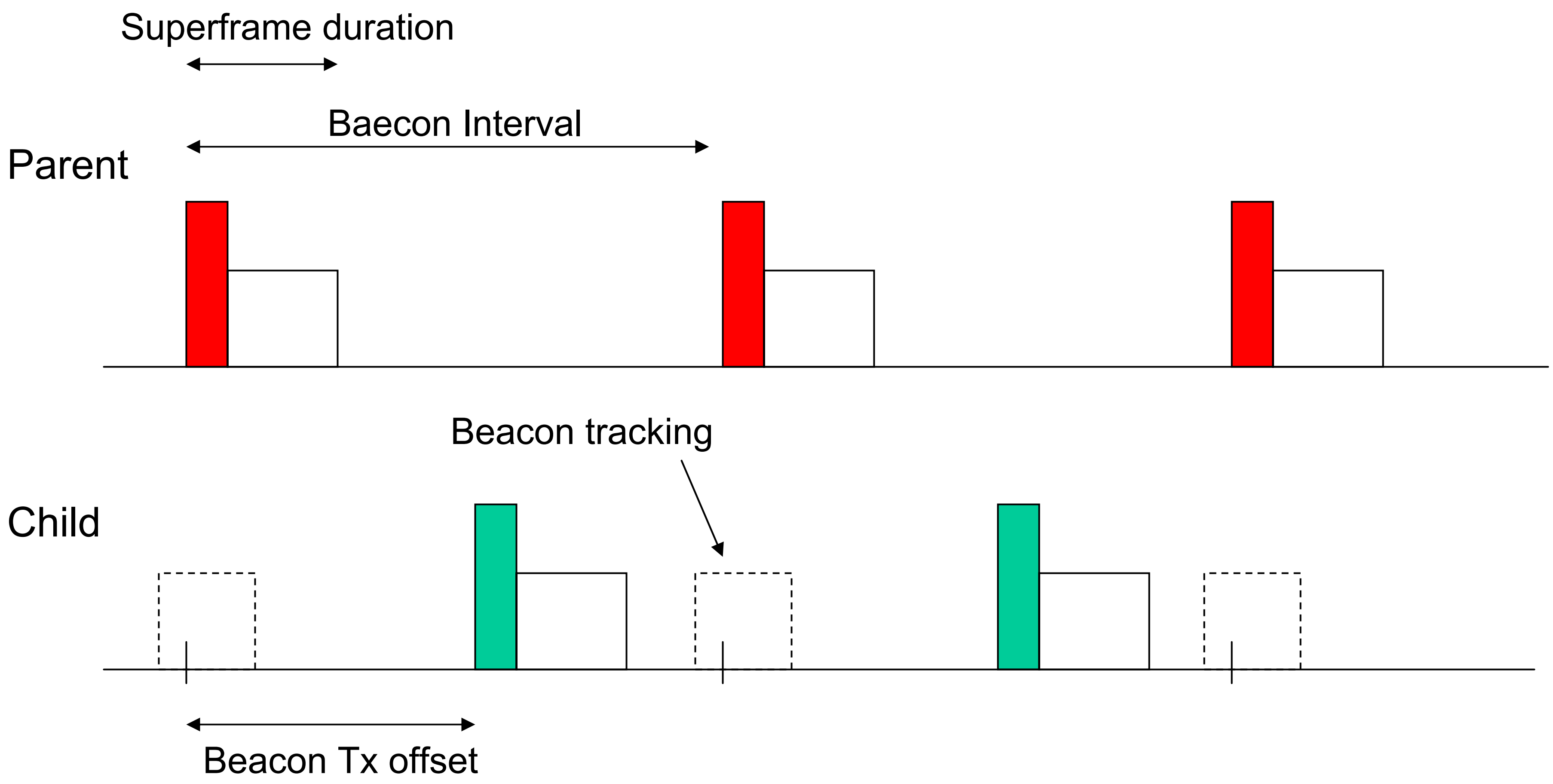
IEEE 802.15.4 – Network Topologies



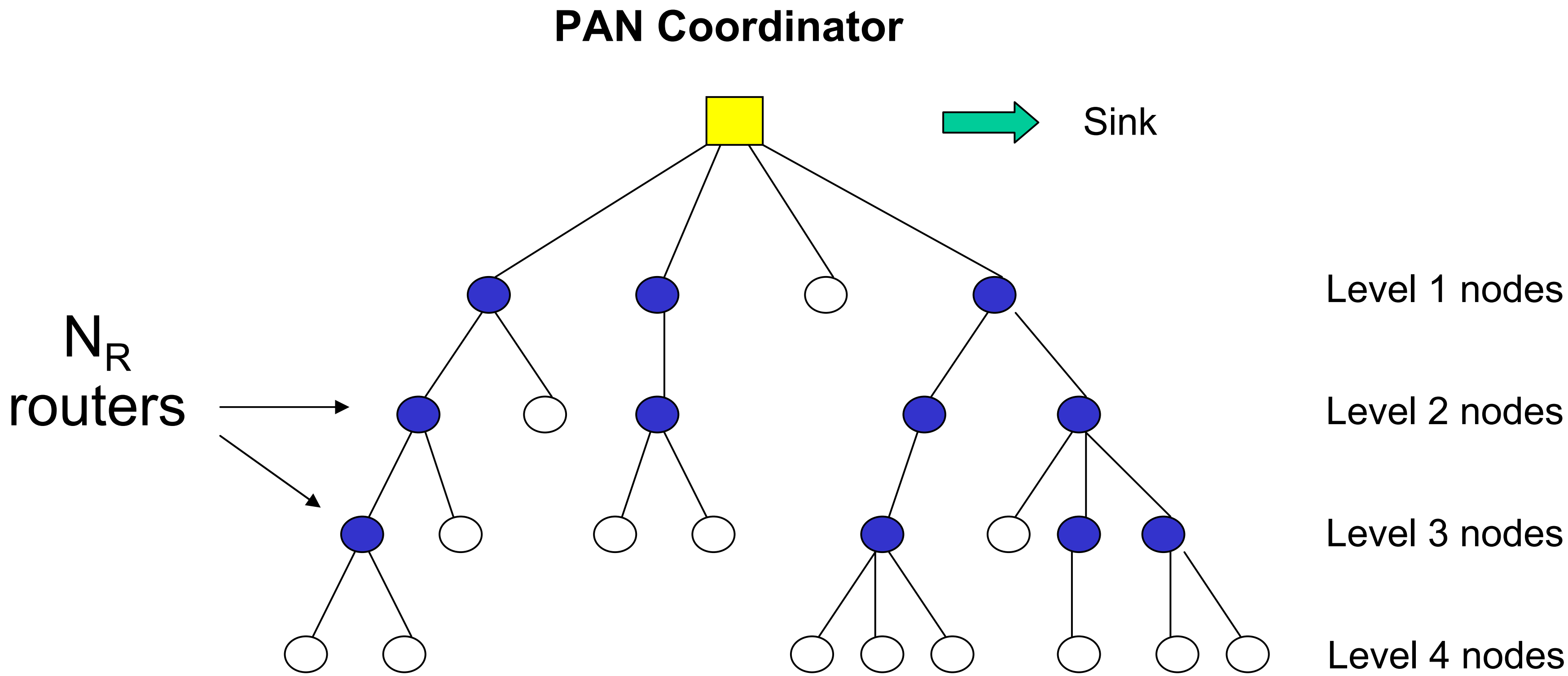
IEEE 802.15.4 - MAC (BE Mode)



IEEE 802.15.4 - Trees



IEEE 802.15.4 - Trees



$$2^{BO} \geq (N_R + 1) \cdot 2^{SO}.$$

Trees

Pros

Routing is simple
Local Addressing is a simple task
Few parameters allow control of the topology
Data aggregation strategies are simplified

Cons

Large average delays

Section 2
Connectivity Theory: Preliminaries

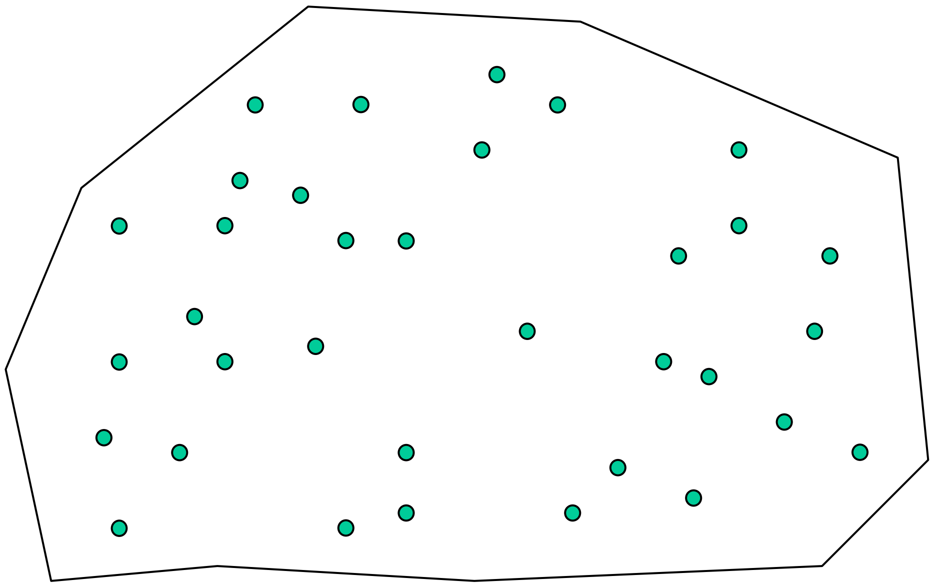
What does it aim to
Link Connectivity
Full Connectivity
Critical Transmission Range

What does it aim to

In networks formed by large numbers of nodes distributed according to some **statistics** over a limited or unlimited region of R^d, Connectivity Theory aims at describing the potential set of links that can connect nodes to each other, subject to some constraints from the physical viewpoint (power budget, or radio resource limitations).

It studies network properties

d = 2 is considered here.

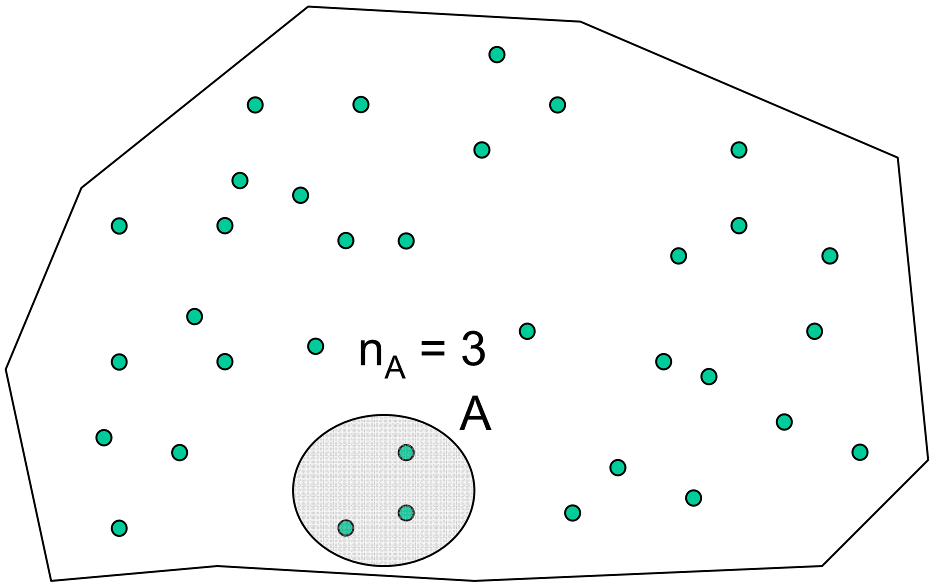


Poisson Point Process (PPP)

Given a (sub-)region A, the number of nodes in A, n_A, is Poisson, with mean $N_A = \rho A$ where ρ is node density.
This is equivalent to **random and uniform independent positions of nodes**.

Prob $(n_A = n) = e^{-\rho A} (\rho A)^n / n! = e^{-N_A} (N_A)^n / n!$
The probability that the region is empty is Prob $(n_A = 0) = e^{-\rho A} = e^{-N_A}$

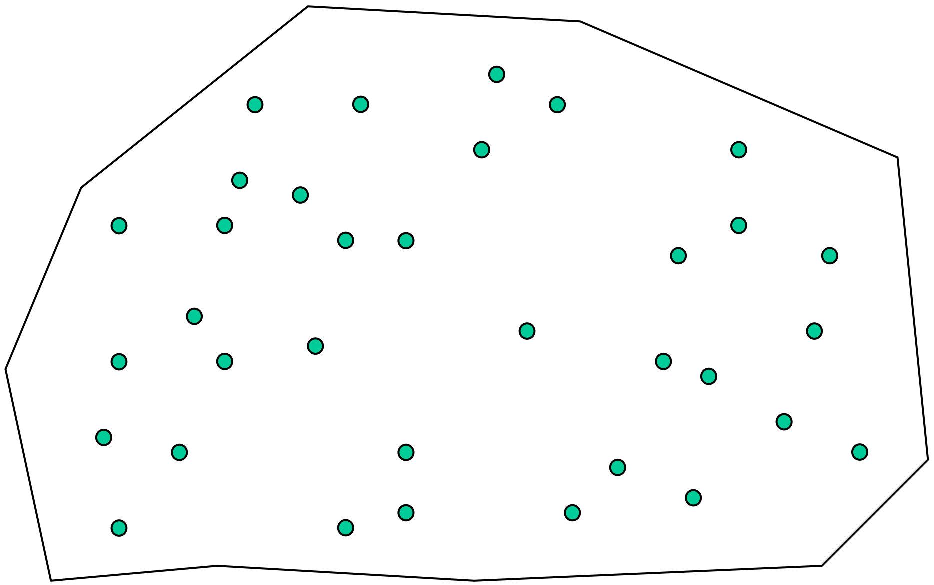
PPPs may be considered either on bounded or unbounded regions.



What does it aim to
Examples

What is the probability that the network is **fully connected**
(i.e. every node can reach any other node through any number of hops)
when nodes are distributed in the region according to a PPP with intensity ρ.

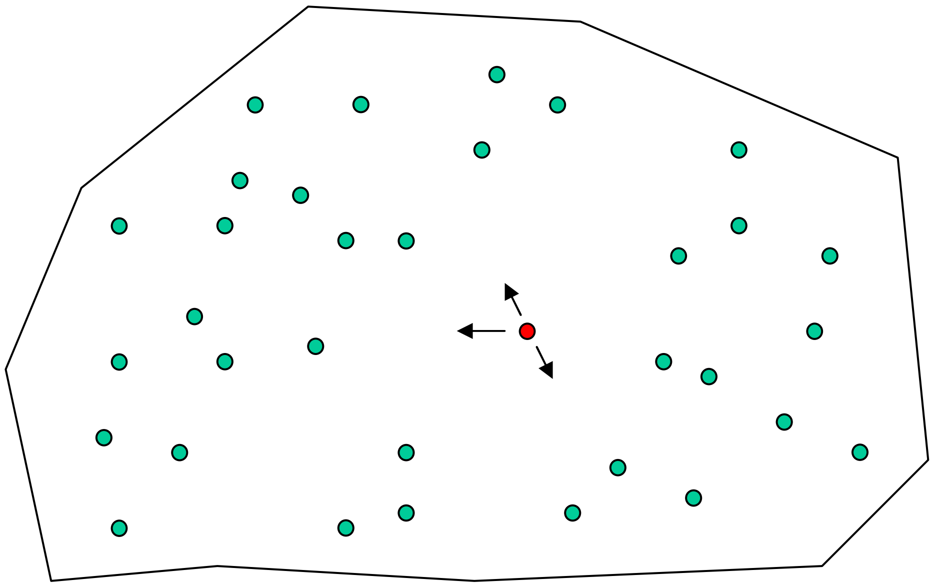
Relevant in ad hoc nets.



What does it aim to
Examples

What is the probability that **a node is isolated**
when nodes are distributed in the region according to a PPP with intensity ρ.

Relevant in WSNs.

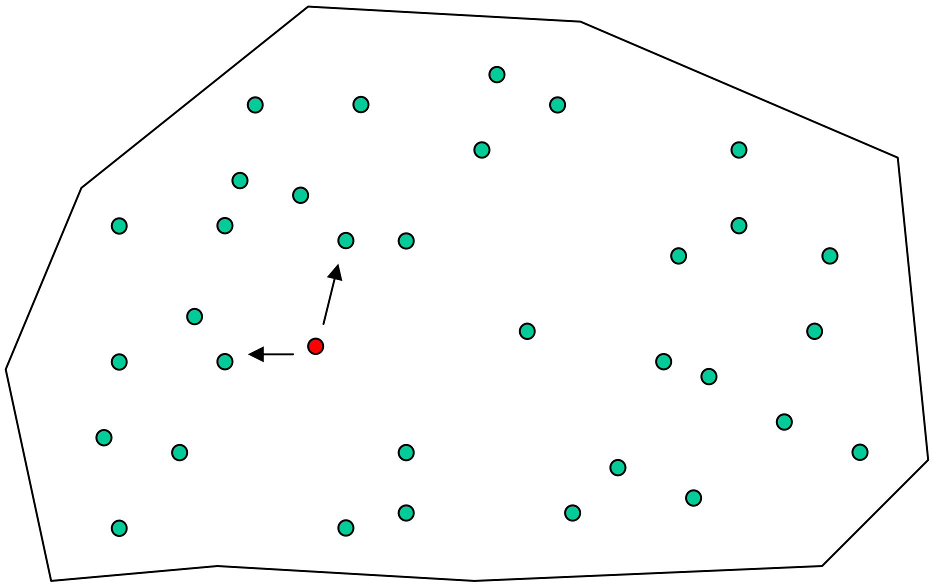


What does it aim to
Examples

How many nodes are “heard” (i.e. a packet is captured) by a given node when nodes are distributed in the region according to a PPP with intensity ρ.

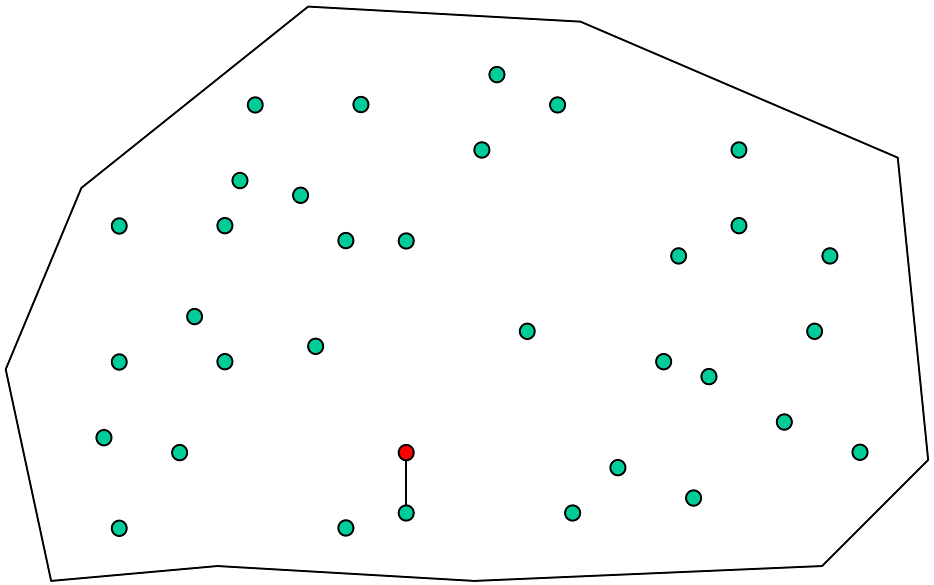
Does a “Magic Number” exist to have a fully connected network?

Overhearing can be controlled.



What does it aim to
Examples

What are the statistics of the distance between a node and the k-th closest node when nodes are distributed in the region according to a PPP with intensity ρ.



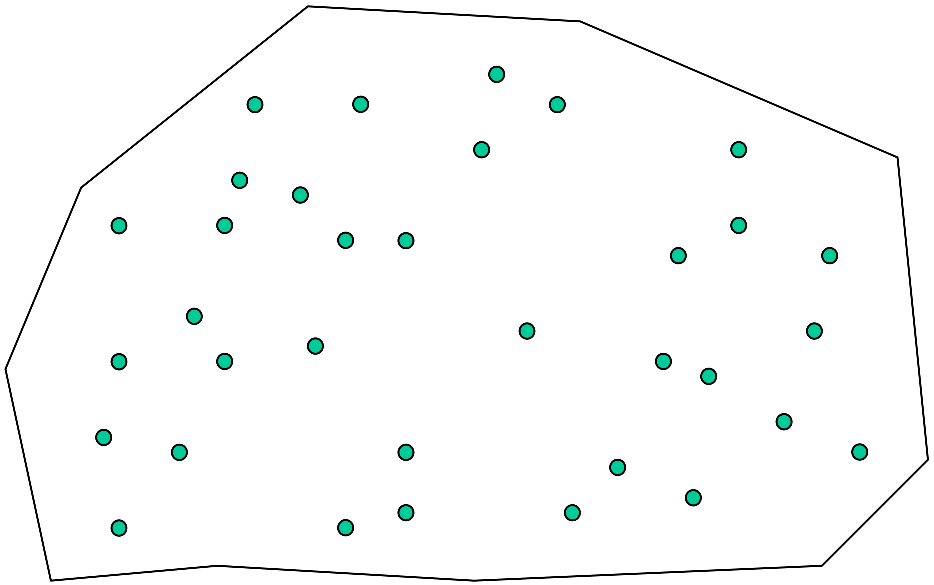
What does it aim to
Examples

Connectivity Theory can be useful to the study of

Network Lifetime

Network Capacity

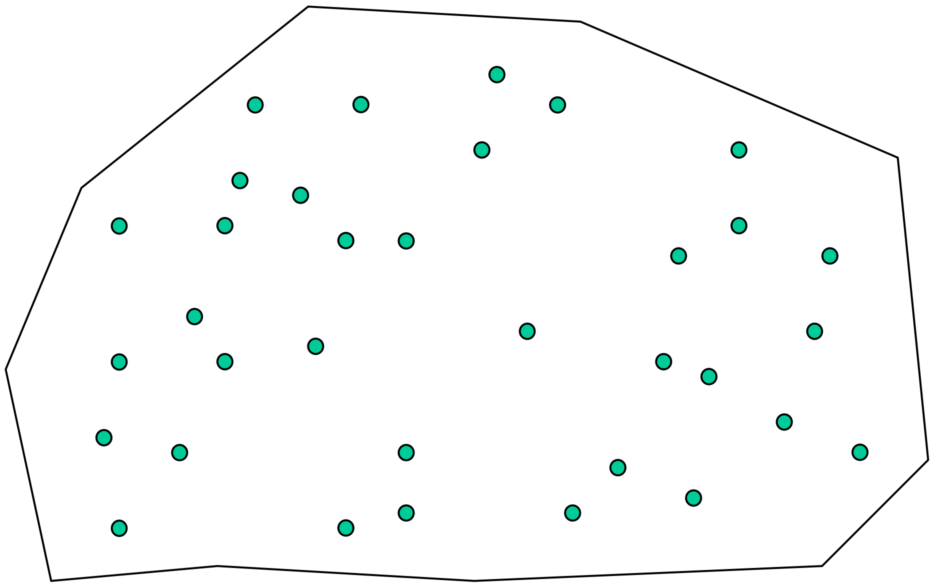
(see Gupta and Kumar, IEEE Trans. - IT, March 2000)



Regions

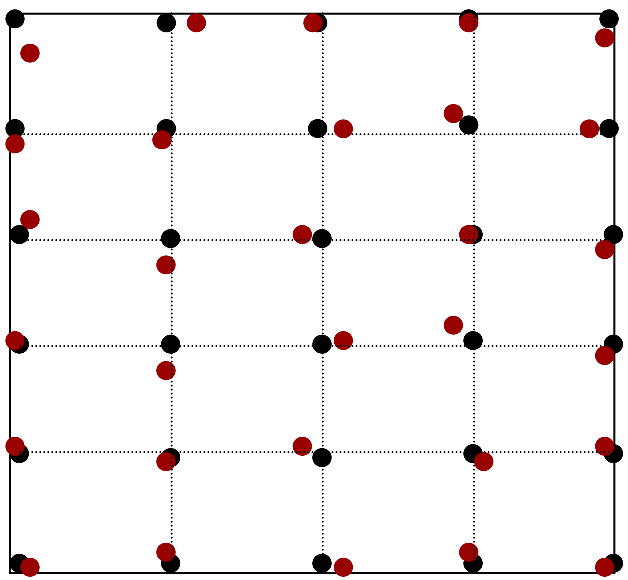
In the literature, typically three types of statistical scenarios are considered:

- Square of unit side (or disk of unit radius) with nodes distributed in the region according to a PPP with intensity ρ;
- A PPP with intensity ρ over an unbounded region;
- Square of unit side (or disk of unit radius) with N (N is given) nodes uniformly distributed at random in the region.



Regular or Quasi-Regular Scenarios

- PPP and regular topologies are two extreme cases
- Sensors on a regular square grid turn out to be more efficient but are of no practical interest
- Q-Regular networks are based on a Gaussian deviation about an ideal grid point (type A)
- **Tradeoff between performance and deployment cost**

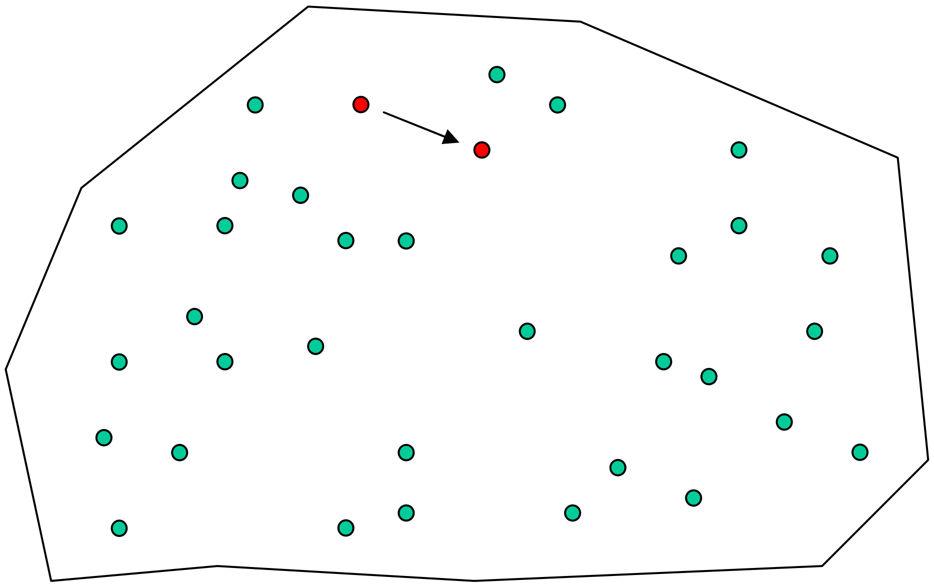


Link Connectivity

To study (network) connectivity, one has to define link connectivity properties.

Different models in the literature, all considering narrowband systems.

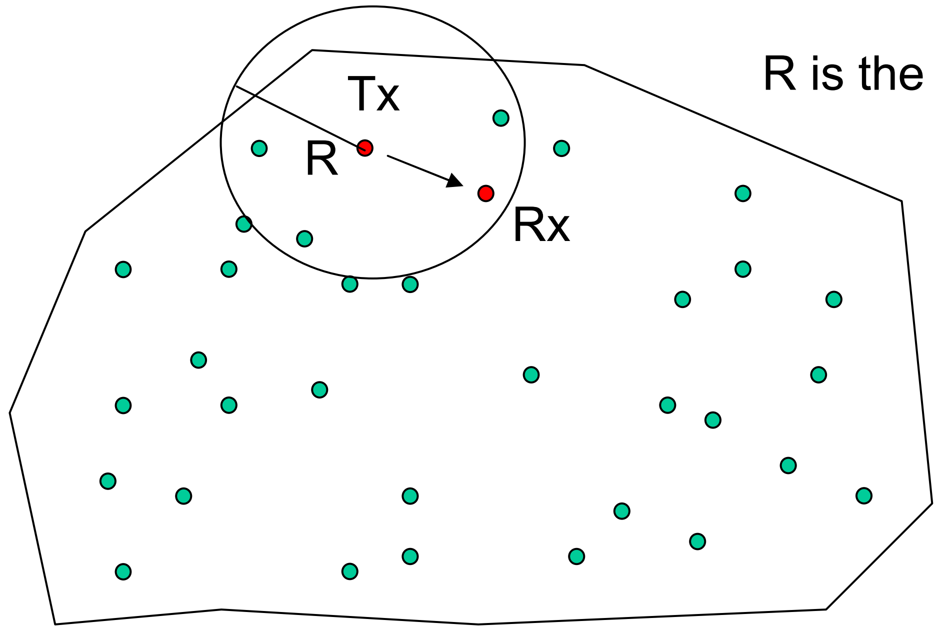
Can be used for 802.15.4, Bluetooth.



Link Connectivity

Model 1 (deterministic distance - dependent model). The most widely used.

Rx is connected to Tx
- if SNR is above minimum threshold α, i.e.
- if received power P_r is above minimum threshold P_{rmin}, i.e.
- if power loss L is smaller than maximum value L_{th} and $L = k_0 + k_d \log r$ i.e.
- if distance r is below a given maximum value R

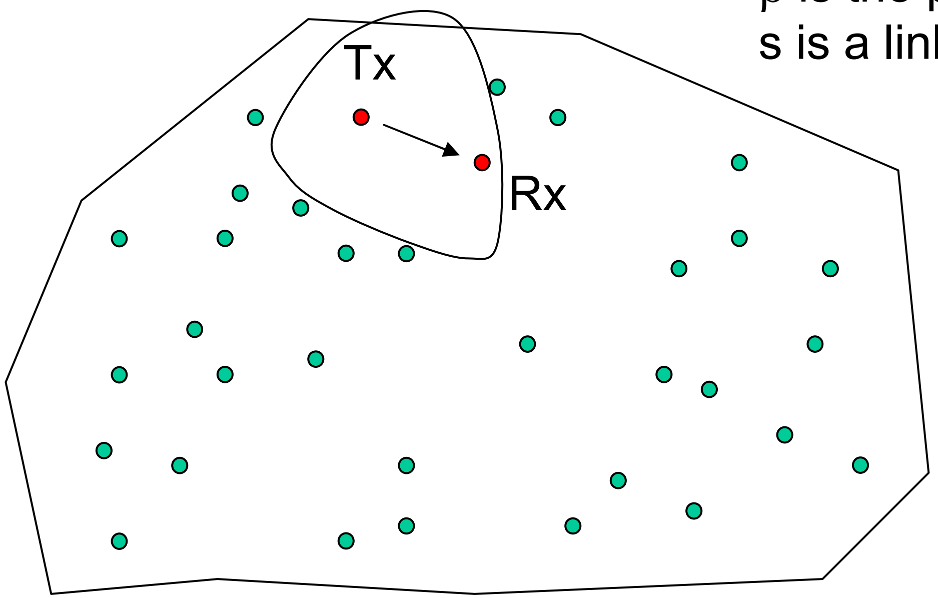


R is the transmitting range.

Link Connectivity

Model 2 (random distance - dependent model). Used in the following.

Rx is connected to Tx if SNR is above minimum threshold α, i.e.
if received power P_r is above minimum threshold P_{rmin}, i.e.
if power loss L is smaller than maximum value L_{th}
and $L = k_0 + k_d \log r + \mathbf{s}$
$k_d = 10\ \beta$
β is the propagation exponent
s is a link-dependent r.v.

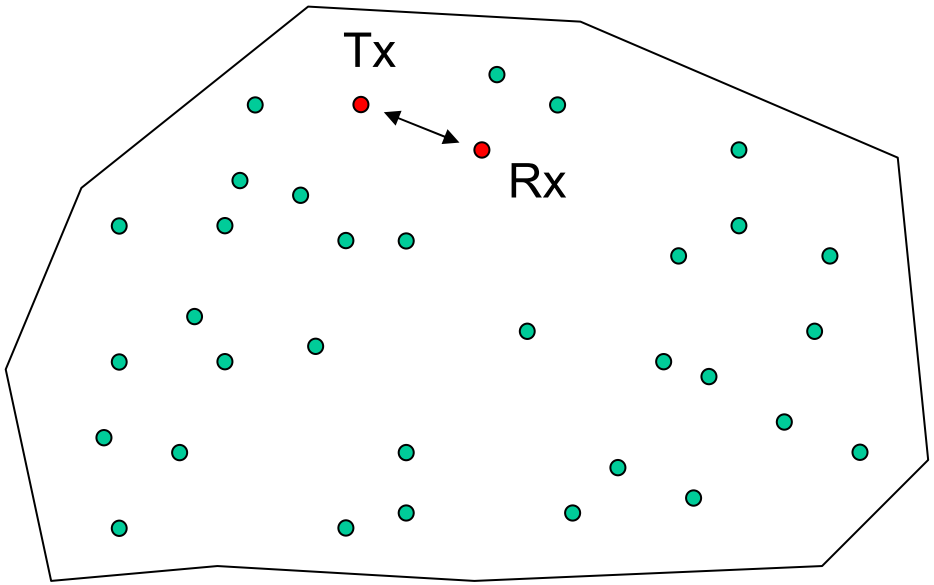


Link Connectivity

We adopt **Model 2**, with s modelled as zero mean Gaussian r.v. with variance σ^2.

$L = k_0 + k_1 \ln r + s$ $\qquad$ $k_1 = k_d / \ln 10$

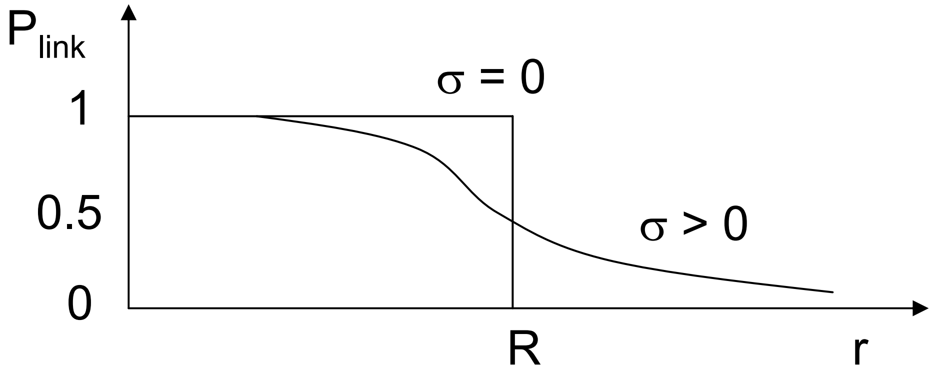
When variance of s is zero, Model 2 converges to Model 1.

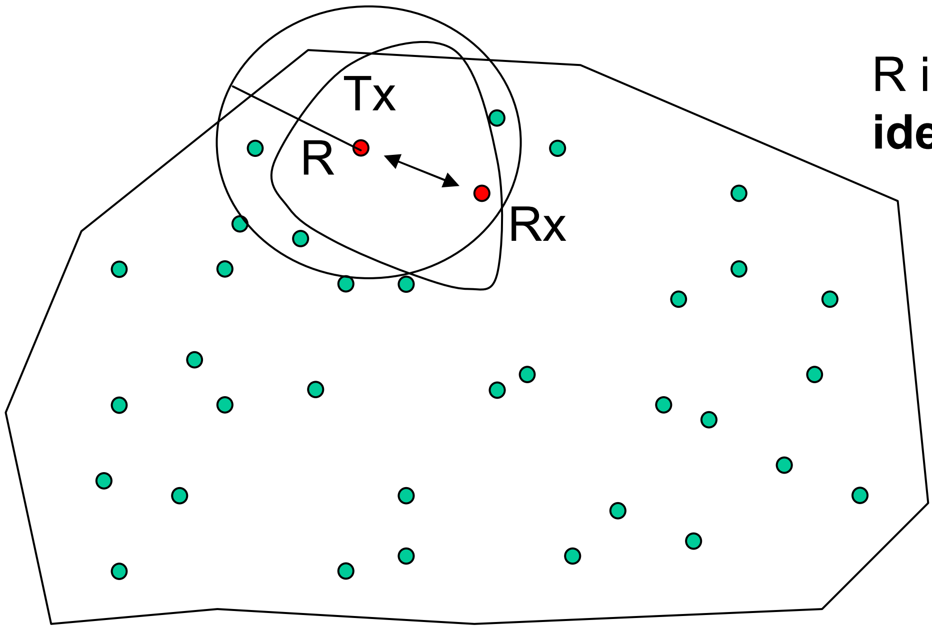


Link Connectivity

$$P_{link} = \Pr\{L(dB) < L_{th}\} = \Pr\{s < L_{th} - k_0 - k_1 \ln(r)\} =$$

$$= \frac{1}{2} + \frac{1}{2} erf\left(\frac{L_{th} - k_0 - k_1 \ln(r)}{\sqrt{2}\sigma}\right)$$





R is the
ideal transmission range

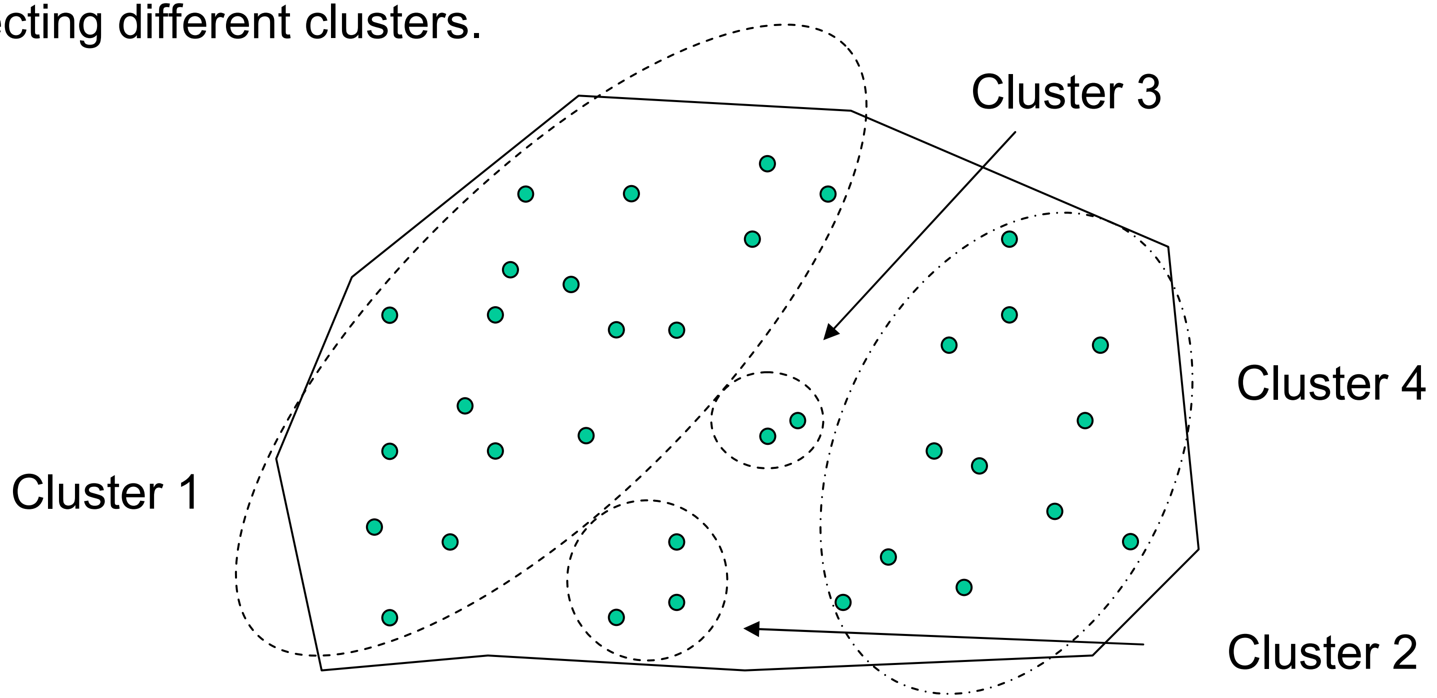
$R = \exp((L_{th} - k_0)/k_1)$

Full Connectivity

Traditional definition:

A network is fully connected if there exists any path (sequence of hops) between every pair of nodes.

Note that a clustered network is not fully connected if there are no gateways connecting different clusters.



On the definitions of full connectivity

Traditional definition:

A network is fully connected if there exists any path (sequence of hops) between every pair of nodes.

This definition is compliant with the objective of ad hoc networks, i.e. to allow every node being in contact with any other node.

But this is not the goal of a WSN.

In WSNs, nodes (sensors) want to transmit their samples to a given node, namely, the sink (or any node in a given set, in the case of multi-sink networks).

Definition more suitable for WSNs:

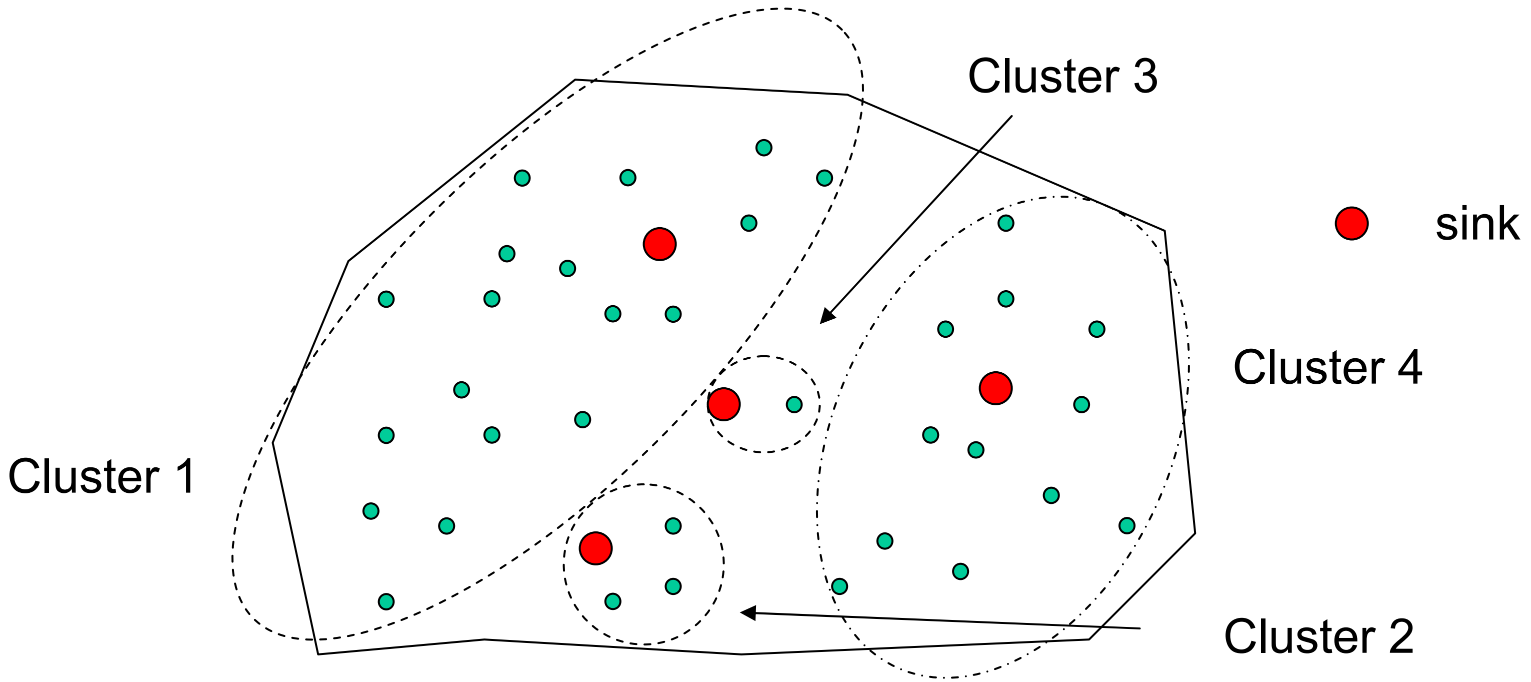
A WSN is fully connected if all nodes can report their samples to a sink through any path.

On the definitions of full connectivity

Note that under such new definition, a WSN can be fully connected even if some nodes can not reach other nodes, in a multi-sink scenario.

In other words, a WSN can be fully connected even if clustered.

The difference is relevant for finite (and small) densities.



Related Work

B. Bollobs, Random Graphs, *s. e. Cambridge University Press, Ed.,* 2001.

R. Meester and R. Roy, Continuum Percolation, *C. U. Cambridge University Press*, Ed., 1996.

M. D. Penrose and A. Pistztora, "Large deviations for discrete and continuous percolation," *Advances in Applied Probability*, vol. 28, pp. 29–52, 1996.

M. D. Penrose, "On the spread-out limit for bond and continuum percolation," *Annals of Applied Probability*, vol. 3, pp. 253–276, 1993.

M. D. Penrose, "On k-connectivity for a geometric random graph," *Random Structures and Algorithms*, vol. 15, pp. 145–164, 1999.

L. Booth, J. Bruck, M. Cook, and M. Franceschetti, "Ad hoc wireless networks with noisy links," in *Information Theory, 2003. Proceedings. IEEE International Symposium on*, Jun. 2003, pp. 386–386.

L. Booth, J. Bruck, M. Franceschetti, and R. Meester, "Power requirements for connectivity in clustered wireless networks," in *Information Theory, 2002. Proceedings. 2002 IEEE International Symposium on*, 2002, p. 353.

Related Work

C. Bettstetter and J. Zangl, "How to achieve a connected ad hoc network with homogeneous range assignment: an analytical study with consideration of border effects," in *Mobile and Wireless Communications Network*, 2002 4th International Workshop on, Sep. 2002, pp. 125–129.

C. Bettstetter, "On the minimum node degree and connectivity of a wireless multihop network," in *Mobile Ad Hoc Networks and Comp.(Mobihoc)*, Proc. ACM Symp. on, Jun. 2002.

P. Santi and D. M. Blough, "The critical transmitting range for connectivity in sparse wireless ad hoc networks," *IEEE Trans. Mobile Comput.*, vol. 2, no. 1, pp. 25–39, 2003.

Z. Vincze, R. Vida, and A. Vidacs, "Deploying multiple sinks in multihop wireless sensor networks," in *Pervasive Services, IEEE International Conference on*, 15-20 July 2007, pp. 55–63.

Section 3
Small Worlds

Small Worlds

Sets of nodes geometrically distributed in a region, connected through a set of links, can sometime behave like small worlds:

the mean number of hops (“separation degree”) needed to reach a destination node from any source node is finite and low, regardless of the number of nodes, even if tending to infinity

S. Milgram *“The Small world problem”.* **Psycology Today, 1967**
M. Buchanan *“Nexus”.* **Mondadori, 2004 [IT]**
D.J. Watts and S.H. Strogatz *“Collective Dynamics of Small Word Networks”.* **Nature vol. 393, 1998**

Small Worlds

Sets of nodes geometrically distributed in a region, connected through a set of links, can sometime behave like small worlds:

the mean number of hops (“separation degree”) needed to reach a destination node from any source node is finite and low, regardless of the number of nodes, even if tending to infinity

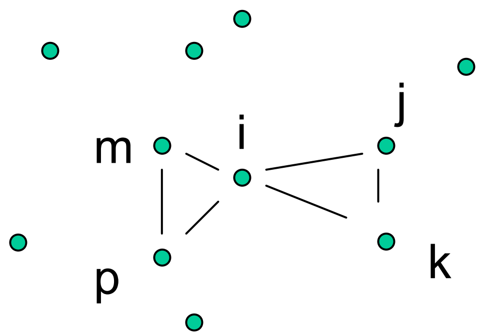
Keeping the mean number of hops low whatever the size of a network, is a relevant property for a large network (scalability)

Do all small worlds have the same type of topology?

No. There exists several types of small worlds.

Small Worlds

The **mean clustering coefficient** (measuring the ratio between the mean number of links between pairs of neighbors to a node, and its maximum value, averaged over all nodes) can be high or low depending on the type of small world.



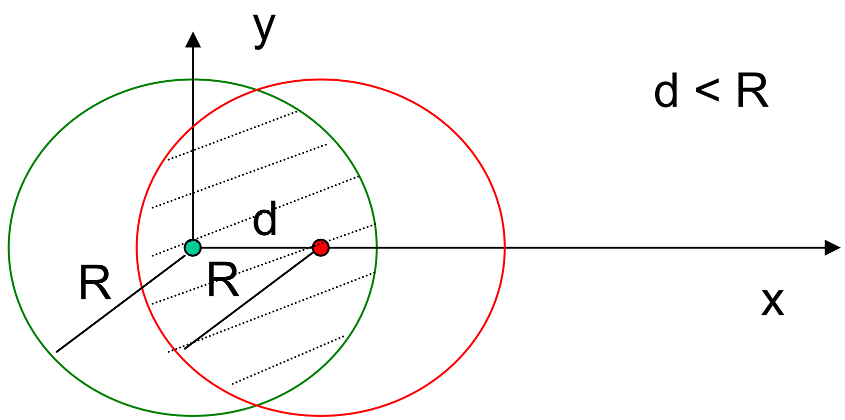
cl. coeff for node i = (1+1) / (4*3 / 2) = 0,33
cl. coeff for node j = (1) / (2*1 / 2) = 1

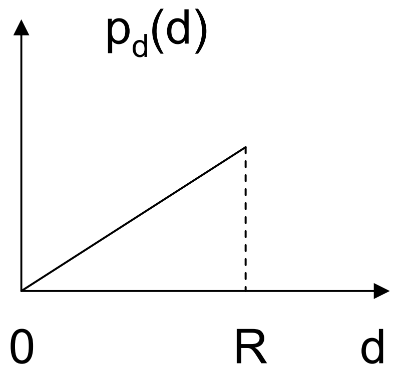
Understanding the properties of small worlds can help understanding the way to build Topologies.

A small value of the mean clustering coefficient represents a more efficient topology (e.g. broadcasting makes more sense).

Small Worlds

Numerical evaluation of the **mean clustering coefficient (with Model 1)**





The dashed area should be averaged over the distribution of d:

mcc (b) = int(d/2, R) sqrt(R^2 - x^2) dx / 0.25 π R^2 =
= 1 – (2 / π)(b sqrt(1 - b^2) + arcsinb)   b = d / 2R

mcc = int(0, 0.5) mcc(b) p_b(b) db =
= 1 – 3 sqrt(3) / 4 π ~ 0.59

It does not depend on R

Small Worlds
Example

A square Q of side L = 1000 m is given
and N = 1000 nodes uniformly and randomly distributed in Q,
and a transmission range R = 90 m.
Link connectivity is based on Model 1.
Links between far nodes are impossible.

Now, a given percentage x of links is modified,
choosing randomly destination and source regardless of distances (“rewiring”).
The graph tends to be more random and links between far nodes are possible.

x	mean number of hops		mean clustering coefficient	
x = 0 %	**7,26**		**0,62**	
x = 9 %	4,16		0,61	
x = 20 %	3,70	→ halved!	0,60	→ unchanged
x = 40 %	3,36		0,58	

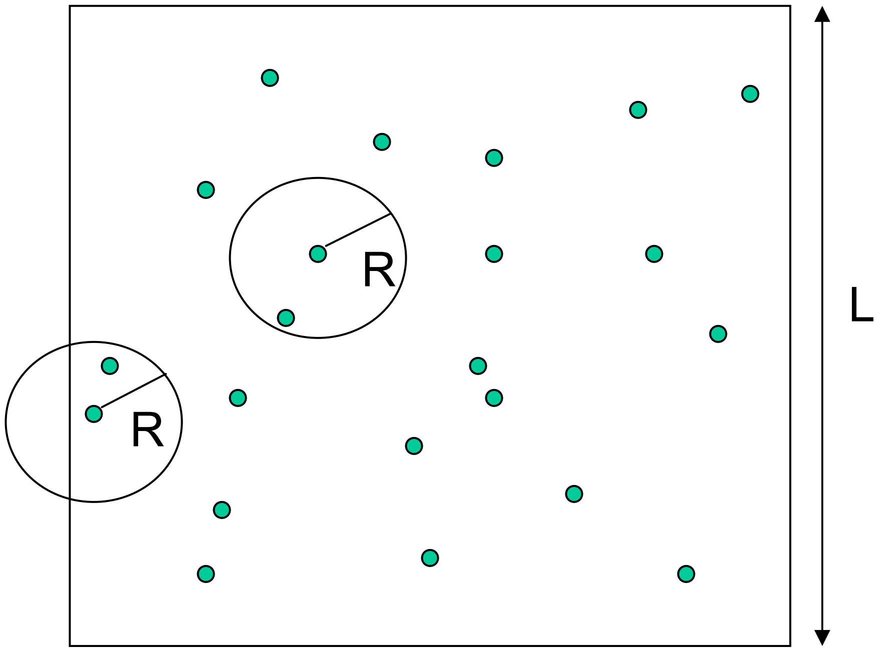
Small Worlds

Example

Model 2

N = 1000

x = 0



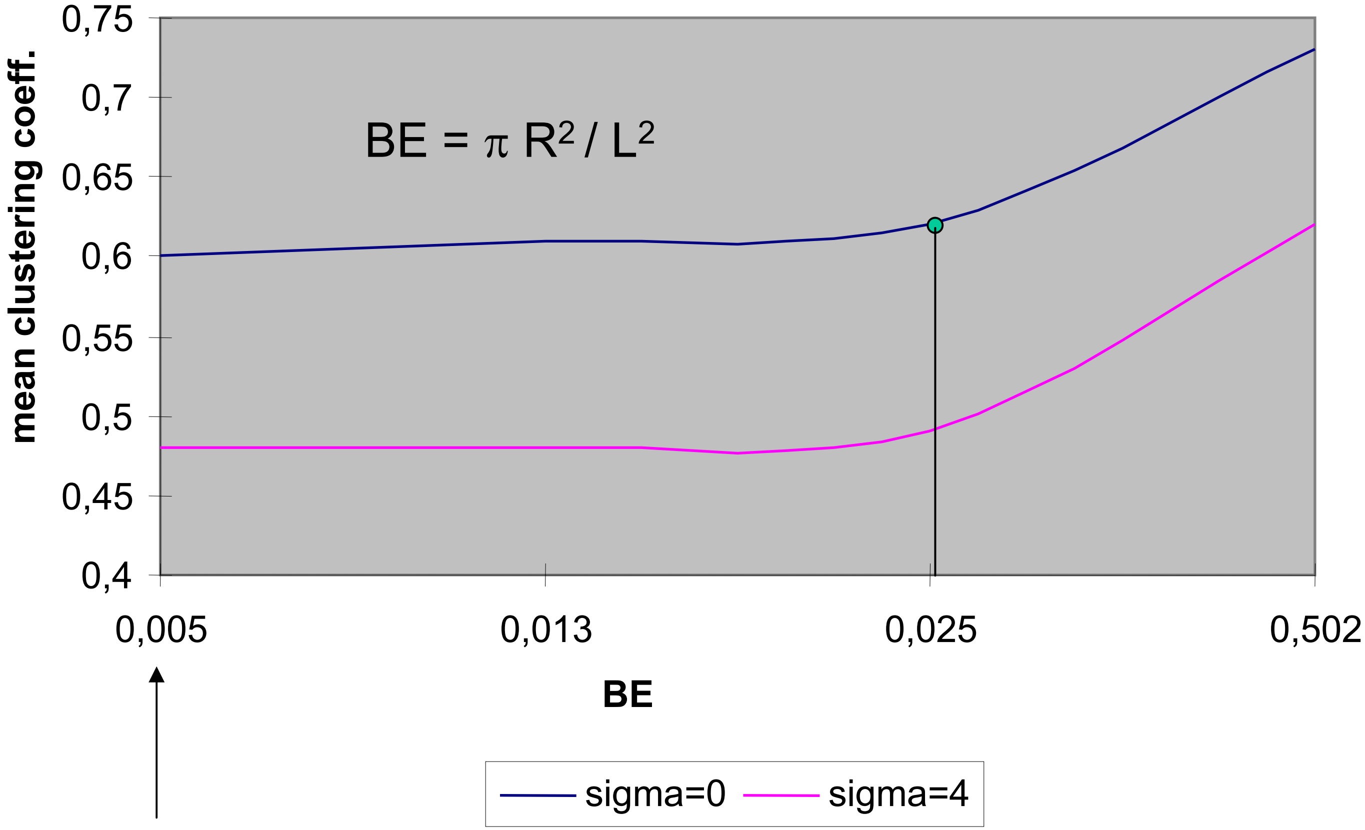
$BE = \pi R^2 / L^2$

Small Worlds
Example

Model 2

N = 1000

x = 0



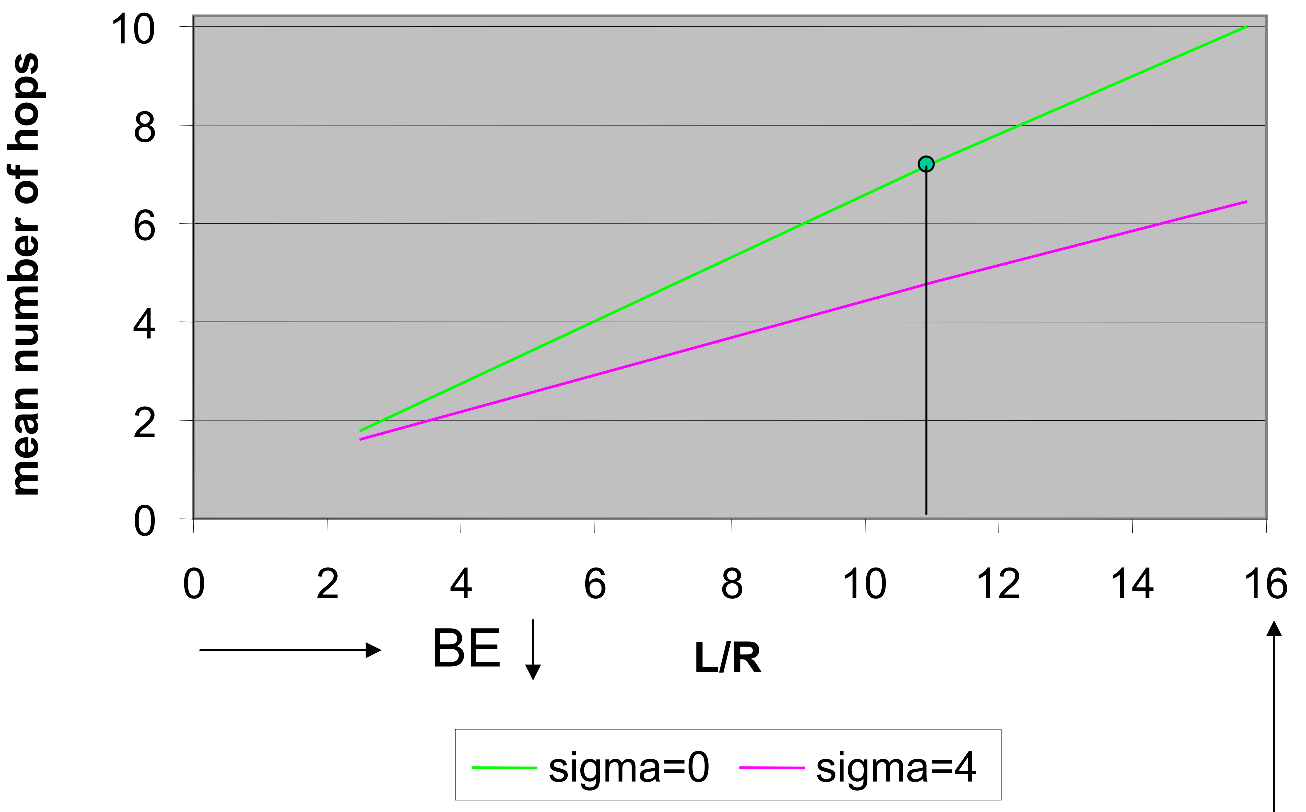
Still fully connected

Small Worlds
Example

Model 2

N = 1000

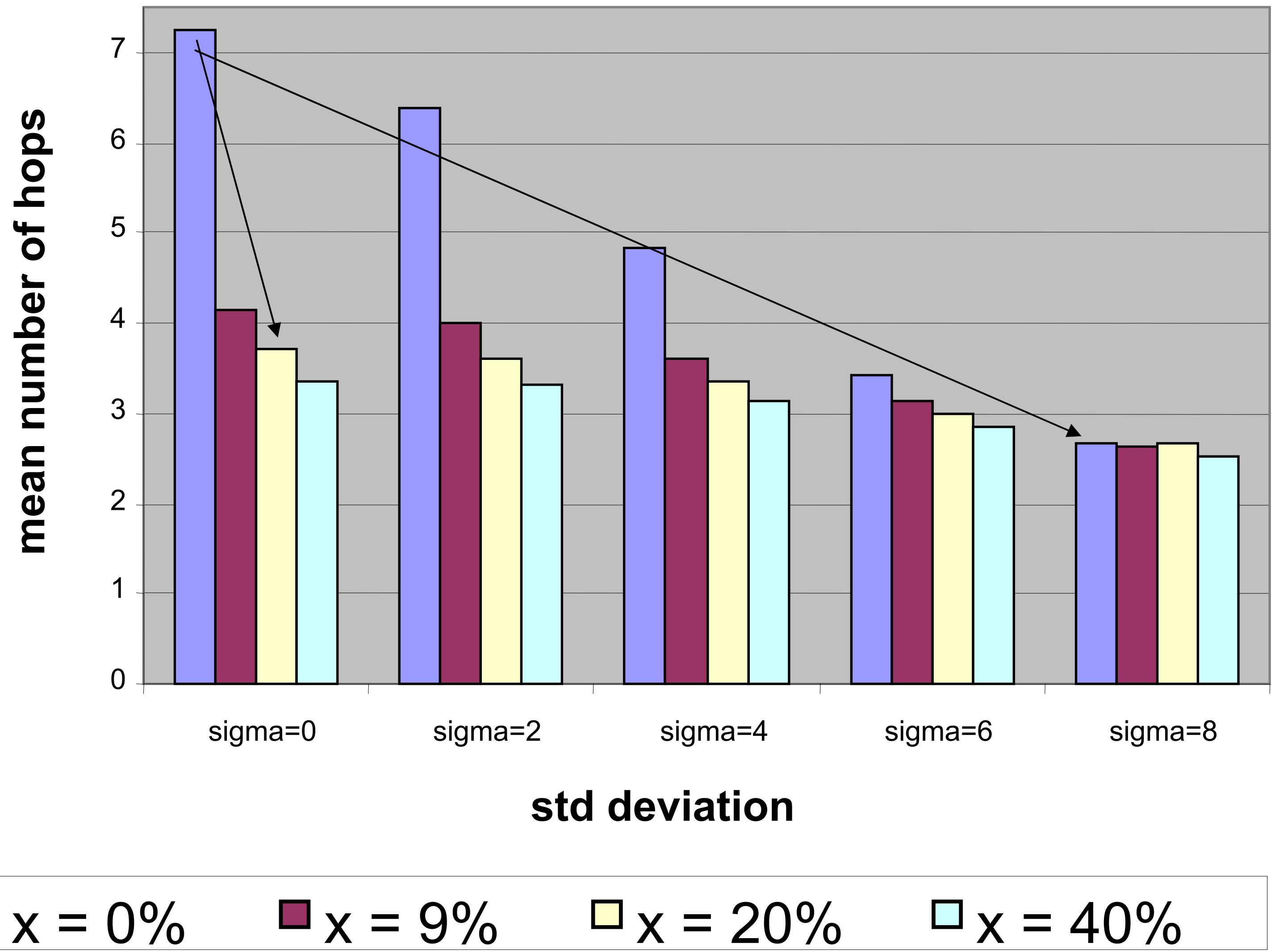
x = 0



Still fully connected

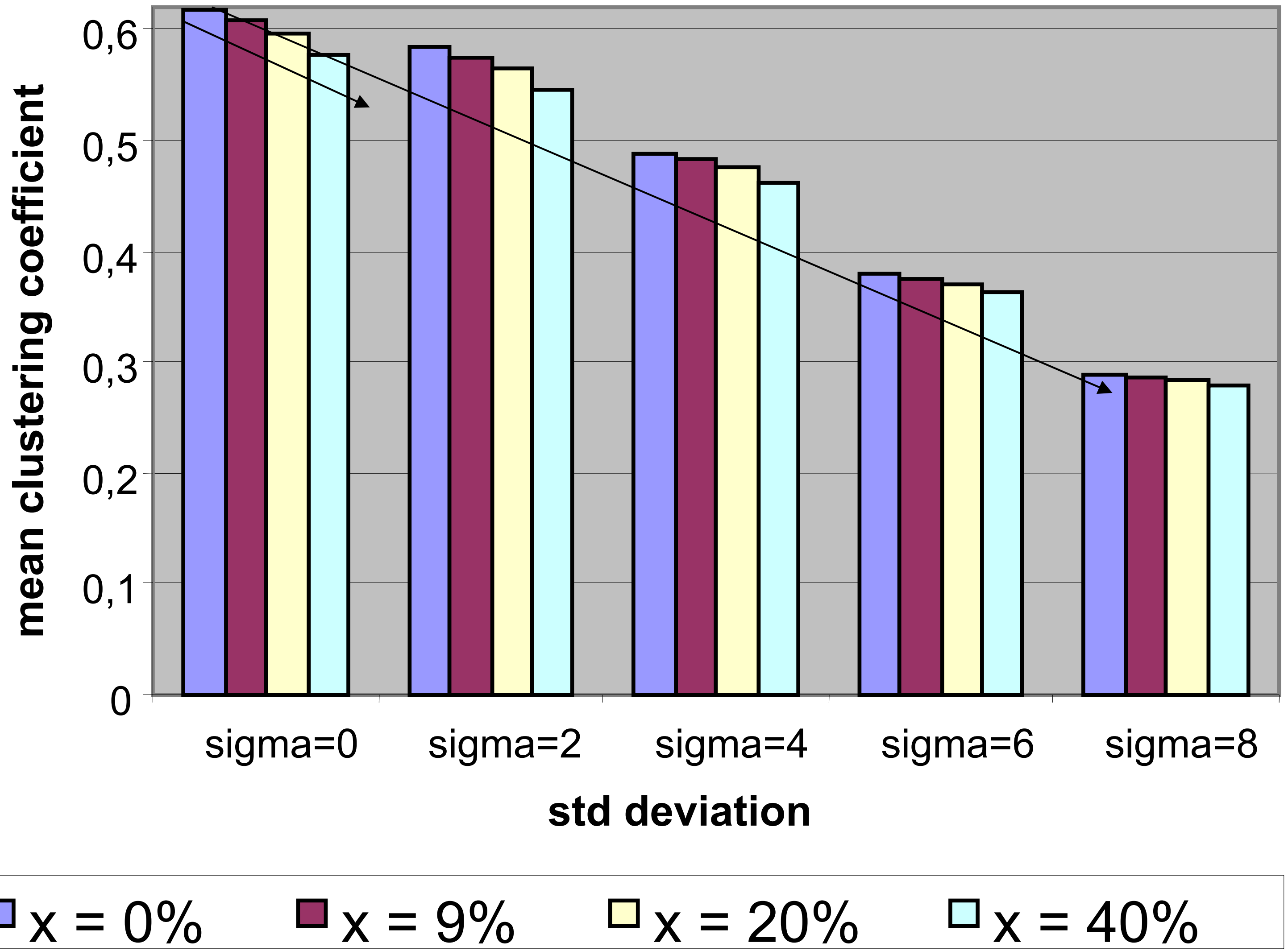
Small Worlds
Example

Model 2
ideal
transm. range
set to 90 m.
L = 1000.
N = 1000.



Small Worlds
Example

Model 2
ideal
transm. range
set to 90 m.
L = 1000.
N = 1000.



Small Worlds

So, random rewiring is an efficient way to reduce the mean number of hops.

However, having a random transmission range is even more efficient.

Small Worlds

A different way to achieve a small world is through the concept of **Hub**.

Hubs are nodes with a much larger number of neighbours than the average (think at the concept of Hub in airway systems).

Consider a square Q of side L = 1000 m,
and N = 1000 nodes uniformly and randomly distributed in Q,
and a ideal transmission range R = 90 m.
Model 2 is used.

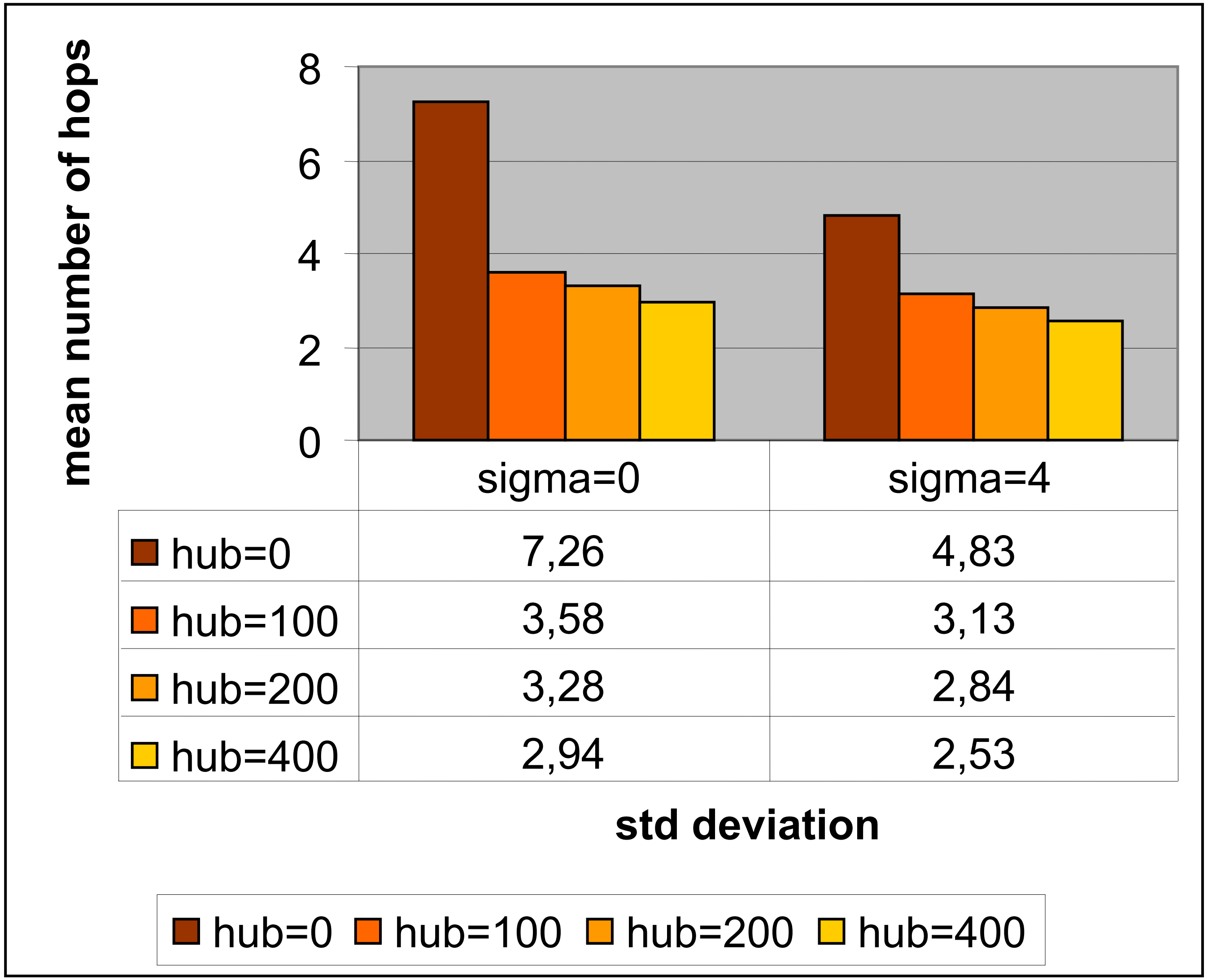
Assume a percentage x of nodes has ideal transmission range of 400 meters.

Small Worlds

Example

Model 2

L = 1000.
N = 1000.



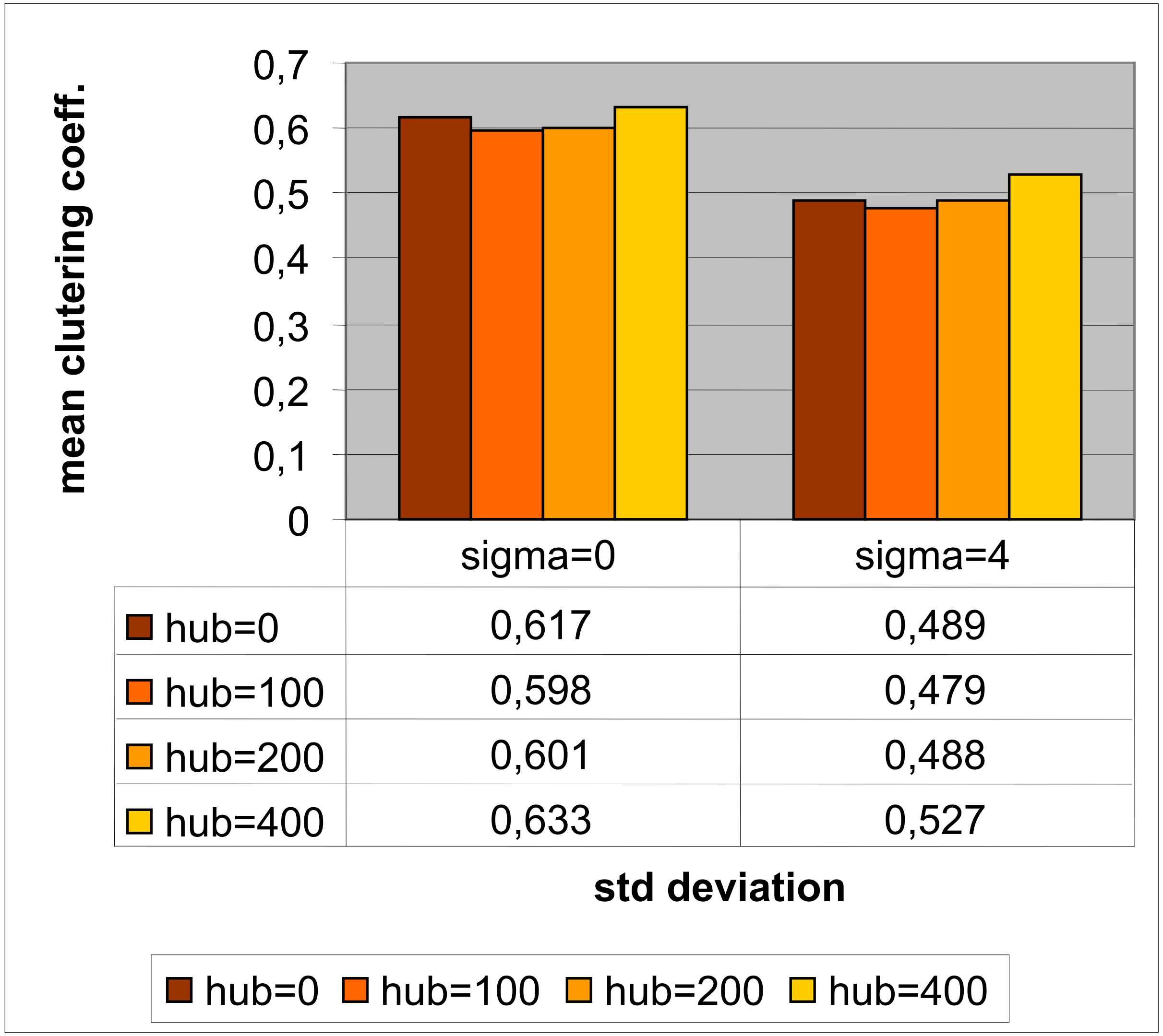
	sigma=0	sigma=4
hub=0	7,26	4,83
hub=100	3,58	3,13
hub=200	3,28	2,84
hub=400	2,94	2,53

Small Worlds

Example

Model 2

L = 1000.
N = 1000.

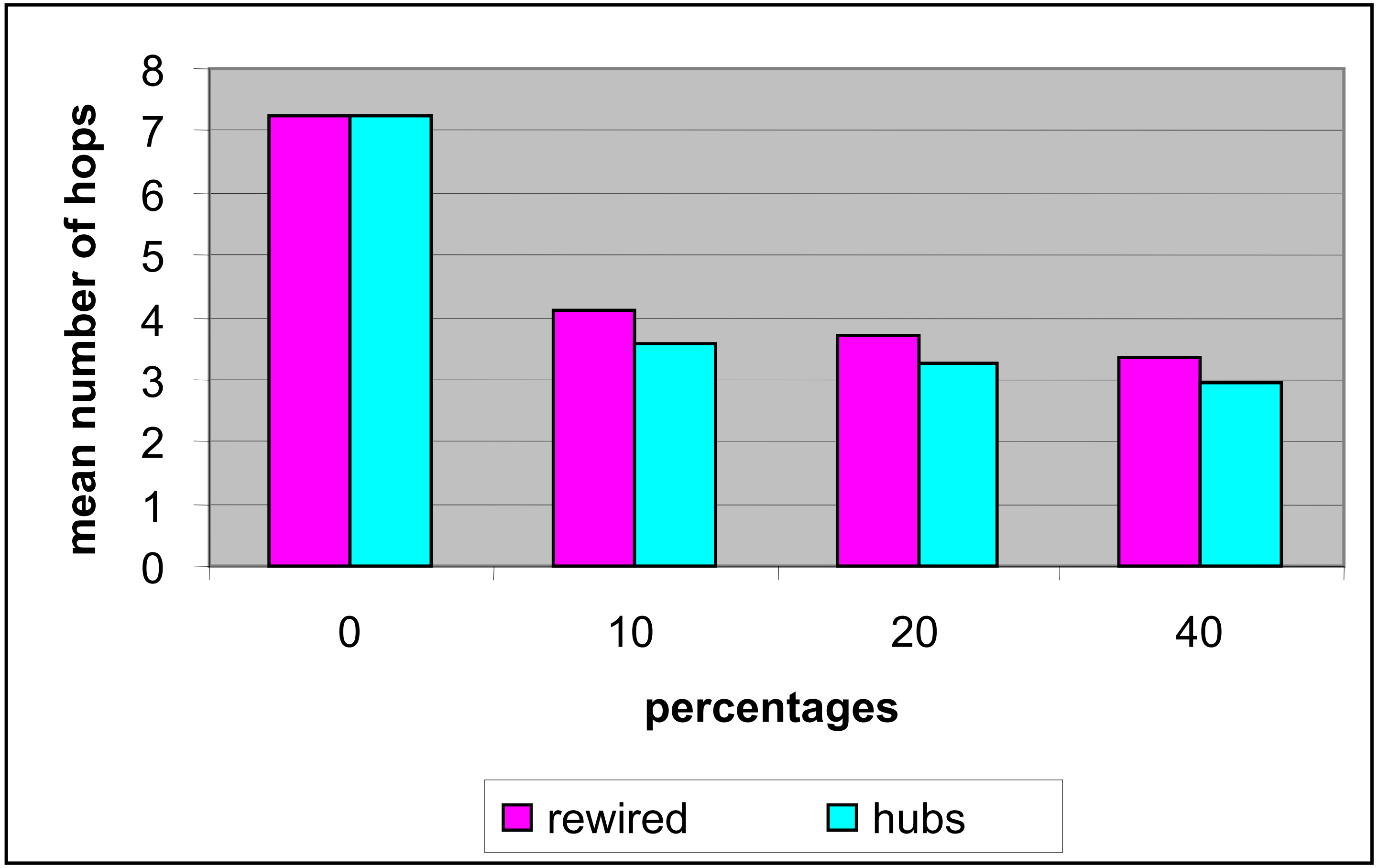


	sigma=0	sigma=4
hub=0	0,617	0,489
hub=100	0,598	0,479
hub=200	0,601	0,488
hub=400	0,633	0,527

Small Worlds
Example

Model 2

L = 1000.
N = 1000.



Small Worlds

The same mean number of hops can be obtained from an ordered graph,
as rewiring a percentage x of links,
by giving to a percentage x of nodes a transmission range of 400 meters.

The first option (rewiring) involves twice as much nodes wrt the latter (hubs).
In the latter case, the clustering coefficient remains constant for sigma zero
as it does not depend on the transmitting range (apart from border effects).

To Sum Up

Hubs are better than long, random links.

Random fluctuations of received power
decrease the mean clustering coefficient.

When link loss variance is low,
the mean clustering coefficient is large under PPP assumptions

Section 3
Critical Transmission Range

Critical Transmission Range
The Giant Component

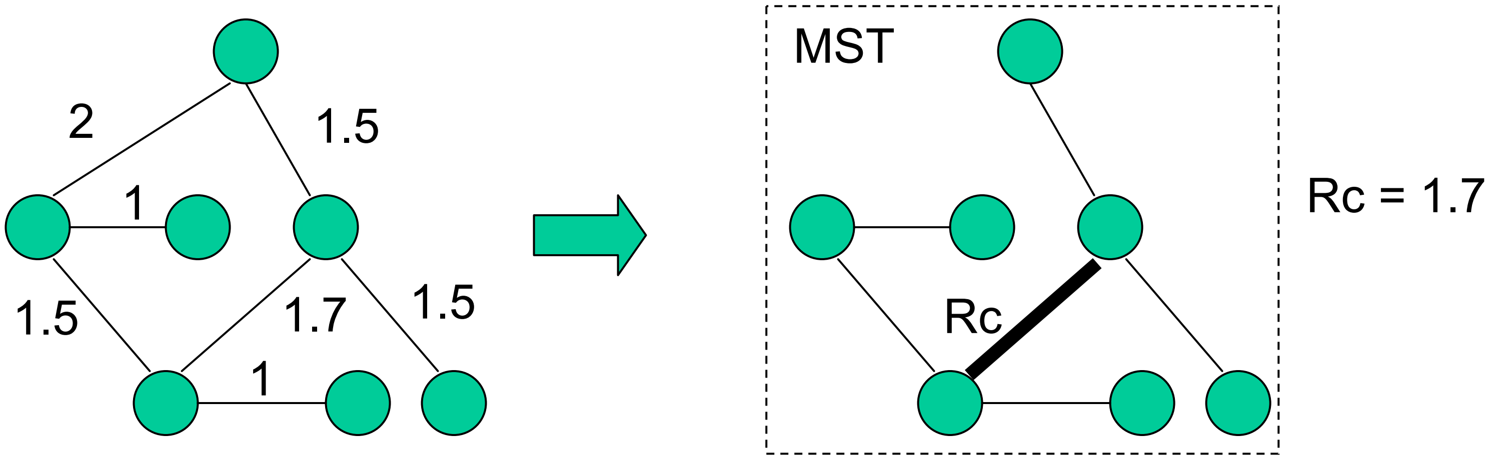
Critical Transmission Range

Critical Transmission Range:
The minimum value of R for a Network s. t. the network is fully connected.

If the nodes in the Network are randomly distributed, the CTR is a random variable.

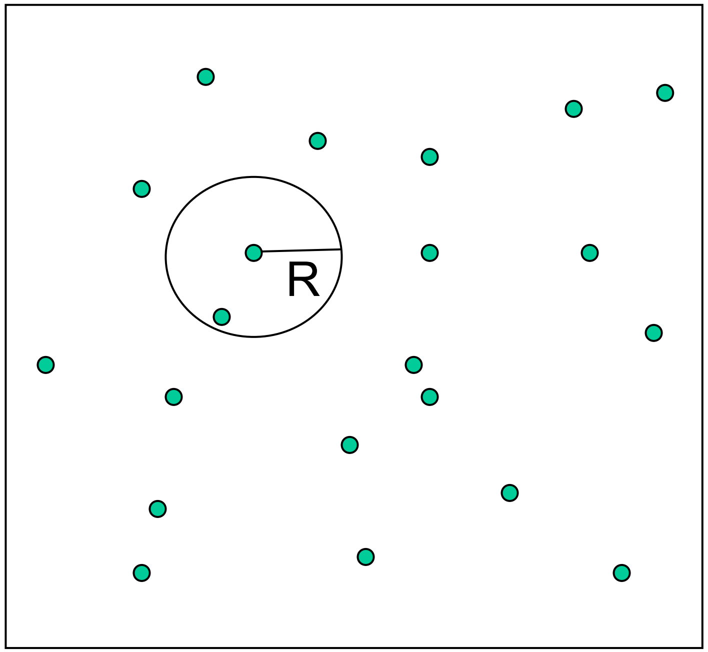
Theorem:

The CTR for connectivity Rc of a Network equals the length of the longest edge of the Euclidean MST of the corresponding Communication Graph



Critical Transmission Range

Unit square



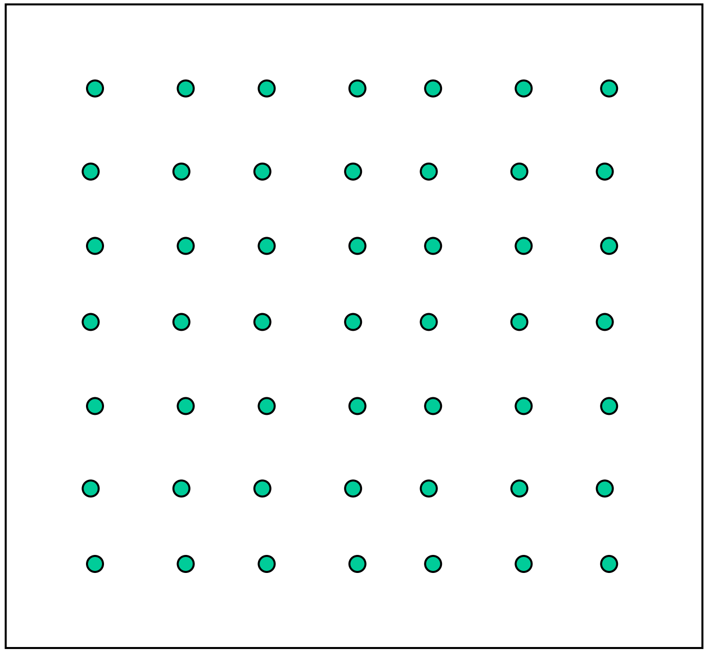
n number of nodes

→ node density is n / 1 = n

R transmission range

Critical Transmission Range

Unit square

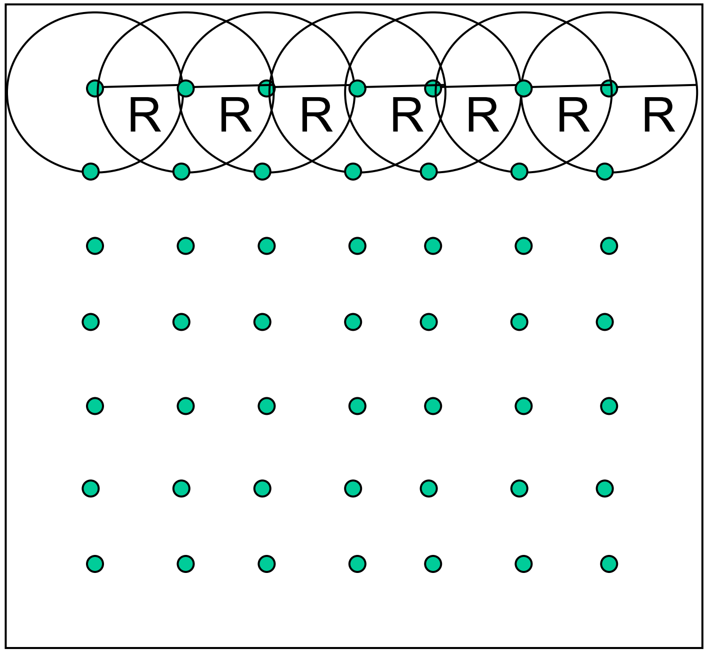


n = 49

Regular grid as reference

Critical Transmission Range

Unit square



CTR:

R s.t. R [sqrt(n) + 1] = 1

→ CTR = 1 / [sqrt(n) + 1]

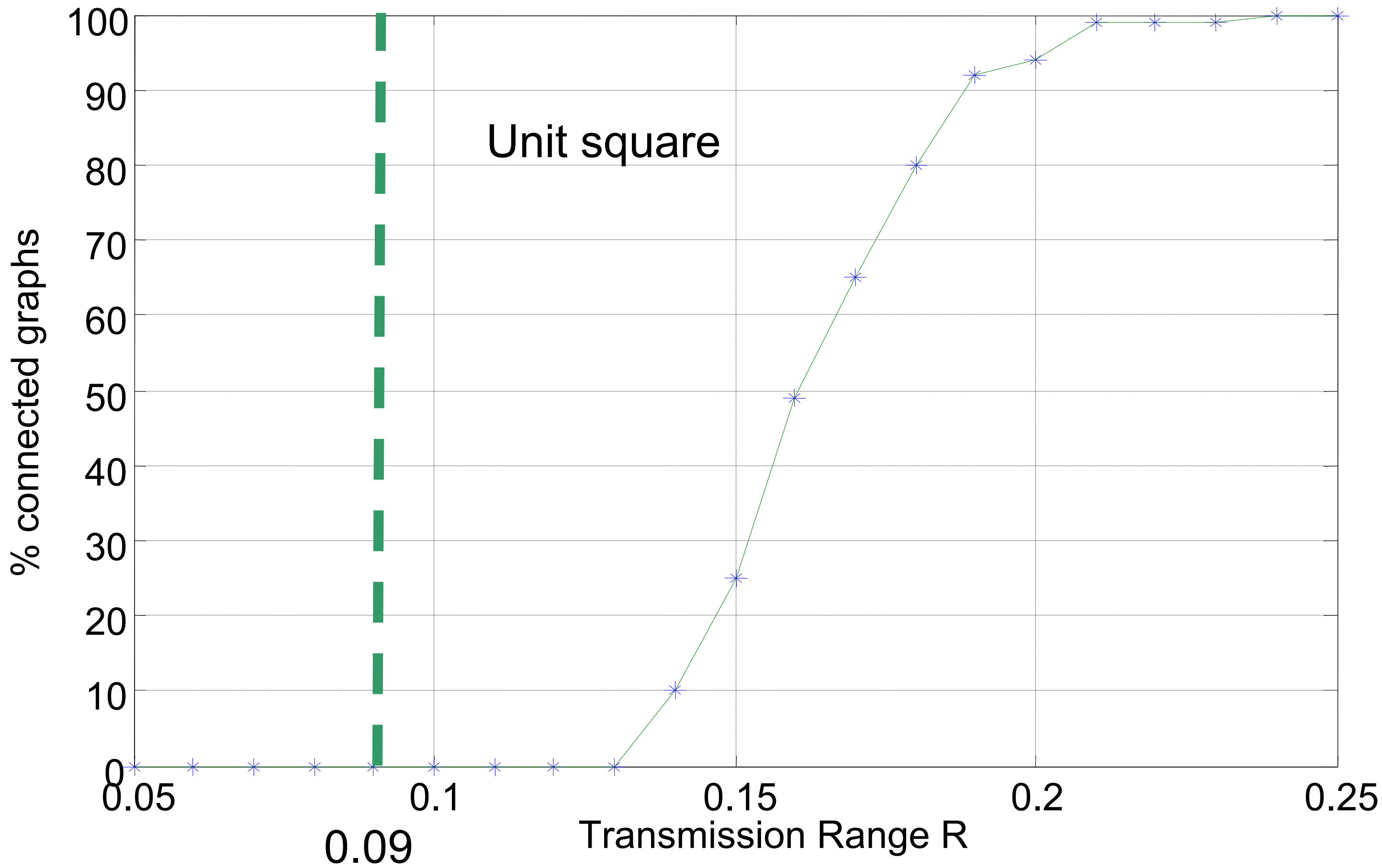
In a square of side L

→ CTR = L / [sqrt(n) + 1]

Critical Transmission Range

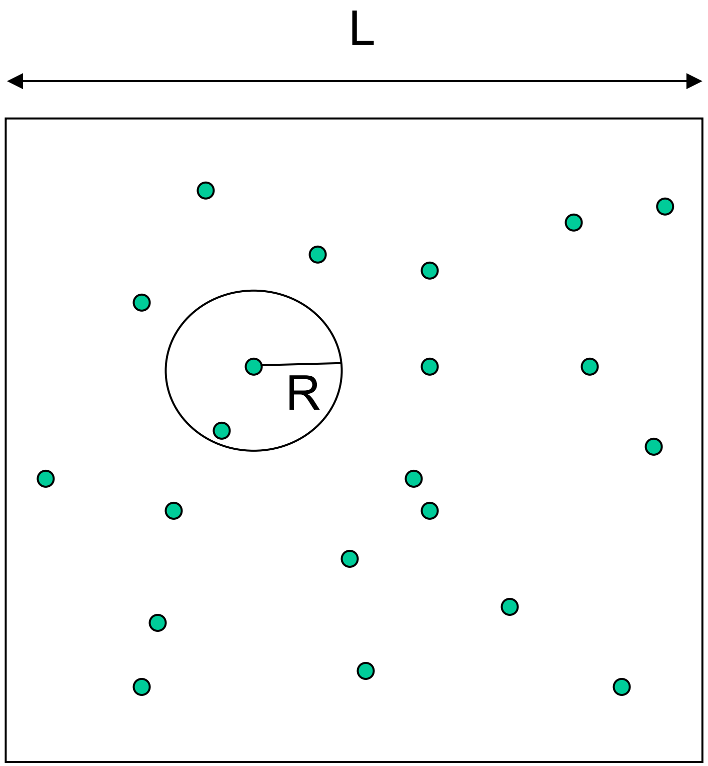
n = 100 → CTR = 0.09 in a regular grid

100 graphs



Randomness of node position has a significant impact on connectivity issues

Critical Transmission Range



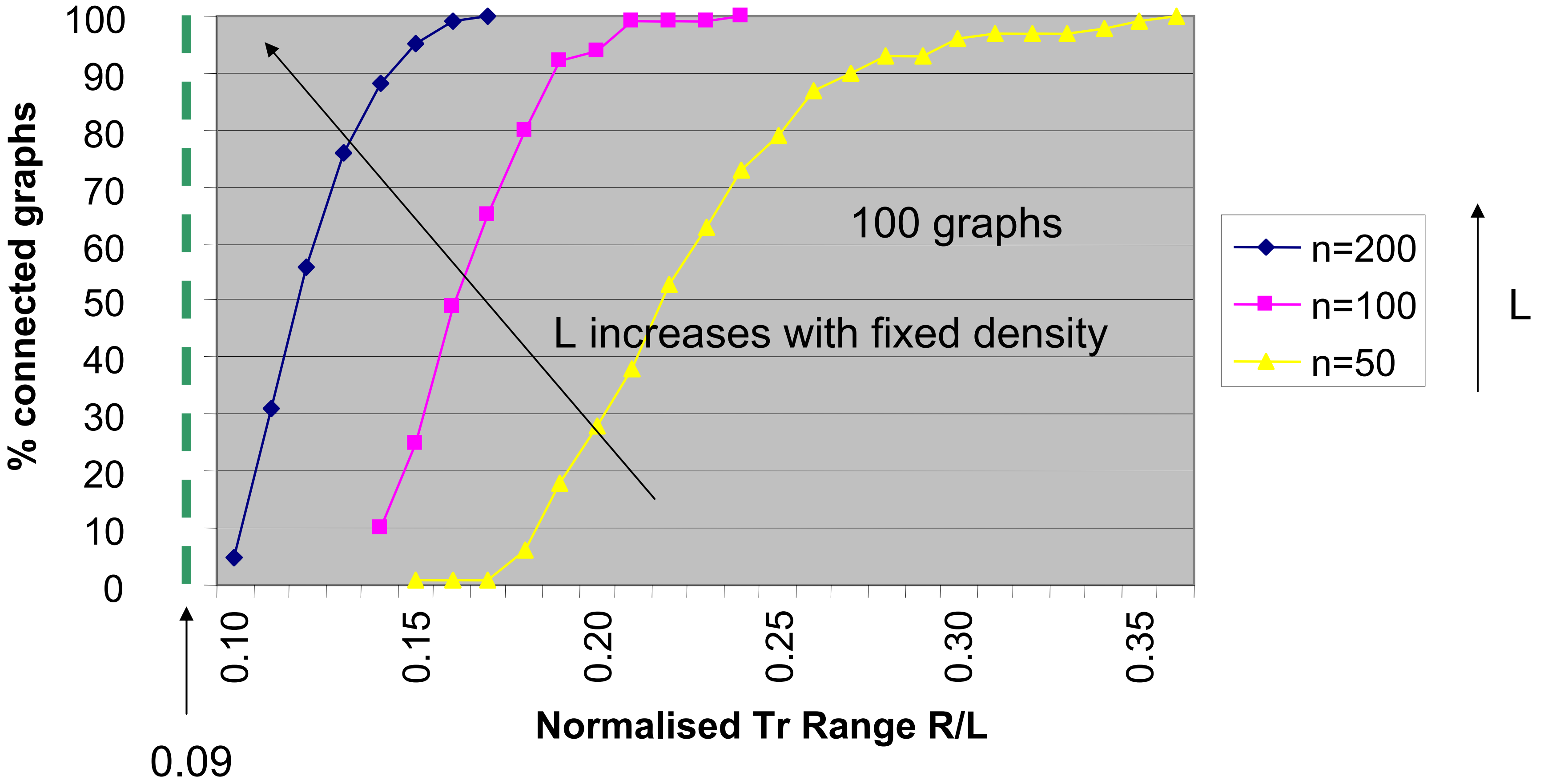
n number of nodes

→ node density is n / L^2

→ $n = \rho L^2$

R transmission range

Critical Transmission Range



Size of scenario has a significant impact on connectivity issues

Critical Transmission Range

Theorem (Penrose 1997):

Given the unit square and n nodes distributed randomly and uniformly, then the limit for n tending to infinity of
Prob [$n\pi \, (Rc)^2 - \log n \leq b$]
is
$1 / \exp(\exp(-b))$
for any b in $\mathbb{R}$ where Rc is the CTR.

As a corollary, for n tending to infinity we have (choose b tending to infinity)
Prob [Rc $\leq$ sqrt ((b + logn) / $n\pi$)] = 1.

In other words, **sqrt ((b + logn) / $n\pi$)]** is an upper bound to Rc for n tending to infinity and since it tends to zero for proper selections of b, it is a very tight upper bound.

For finite values of n, this expression does not necessarily represent an upper bound.

Critical Transmission Range

For which values of n are the predicted values of Rc accurate?

For instance, with b = loglogn

n	Rc (Penrose)	Rc (sim., 99% conf)	Rc=1 / [sqrt(n) + 1]
10	0.32	**0.66**	0.24
100	0.14	**0,23**	0.09
1000	0.05	**0.08**	0.03

Not very accurate for practical values of n.

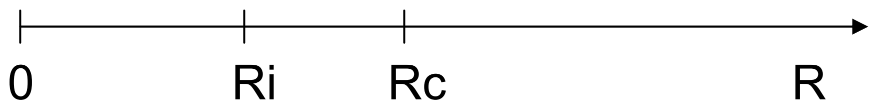
Considerations for n tending to infinite are often unrealistic for practical values of node densities

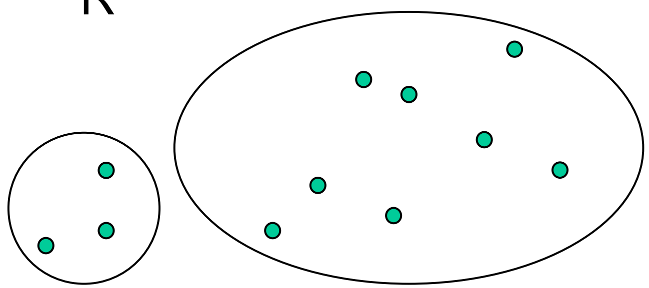
The Giant Component

Consider a given graph, and let R increase starting from 0.

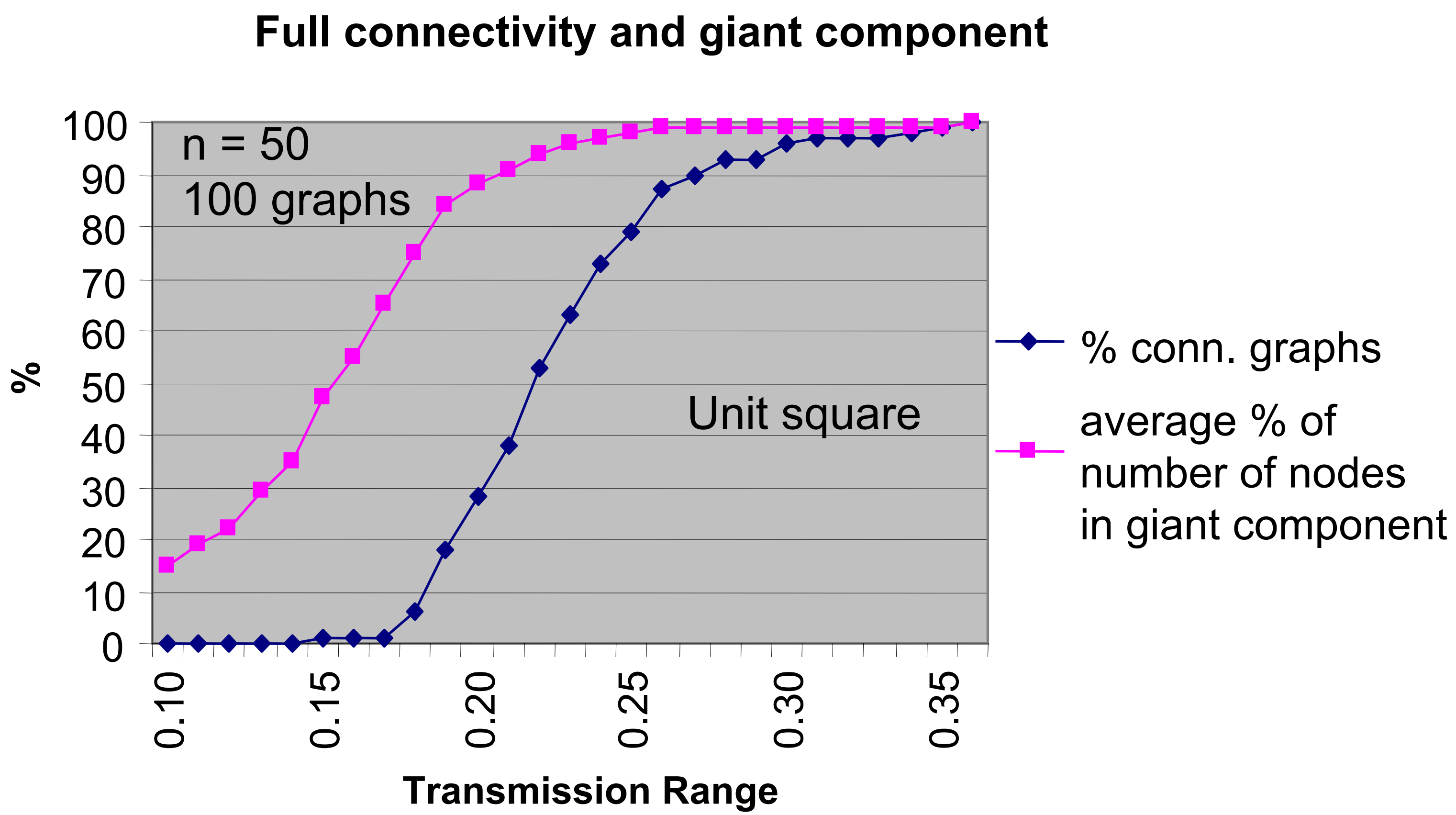
For R = 0 the graph is not connected.
When R increases, nodes group together in clusters.
When R = Ri the last isolated node disappears.
When R = Rc the graph becomes connected.

Clearly, Ri < Rc as for some Ri < RA < Rc there might be a communication graph not connected even in the absence of isolated nodes (a “clustered” network with isolated clusters).

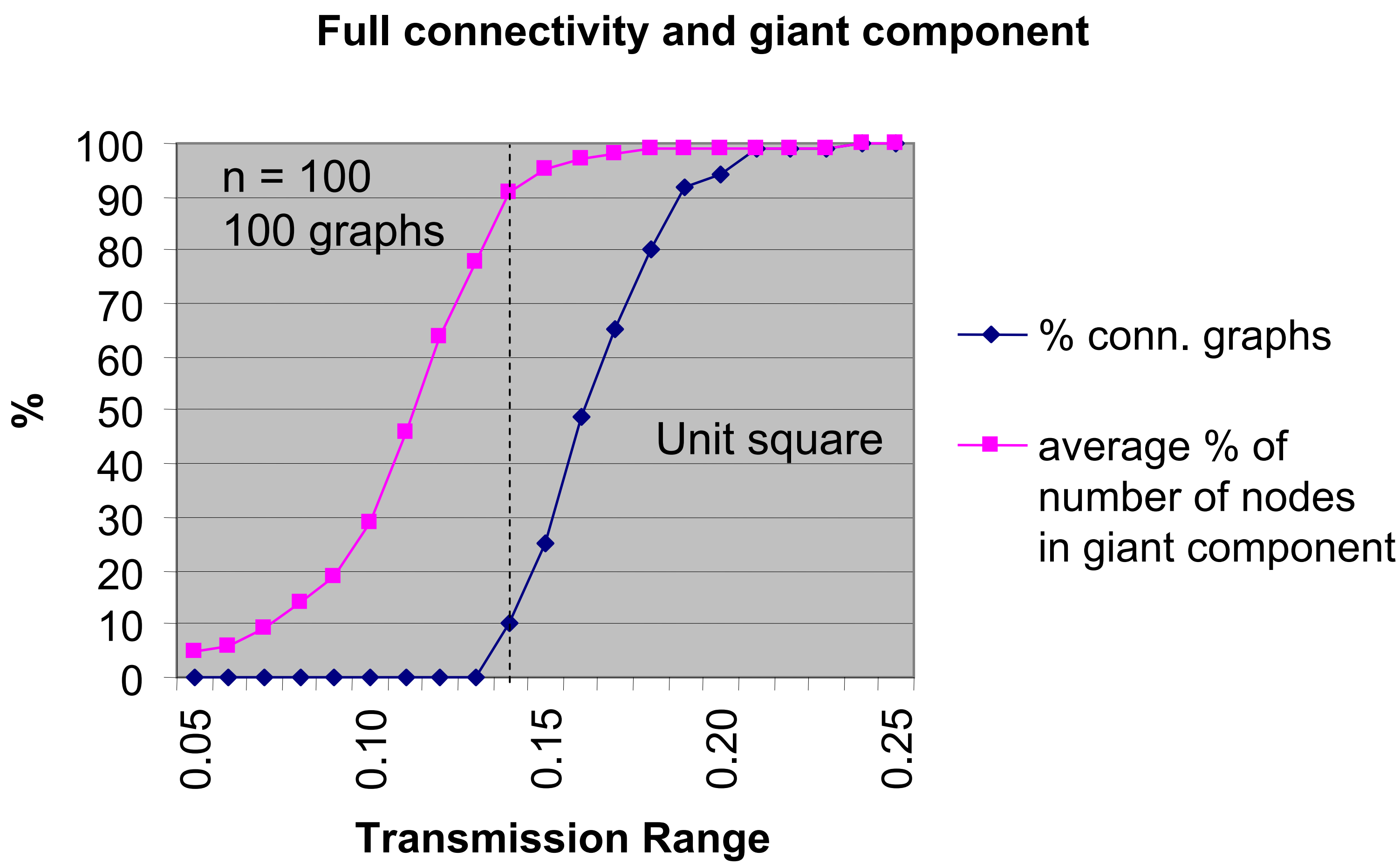




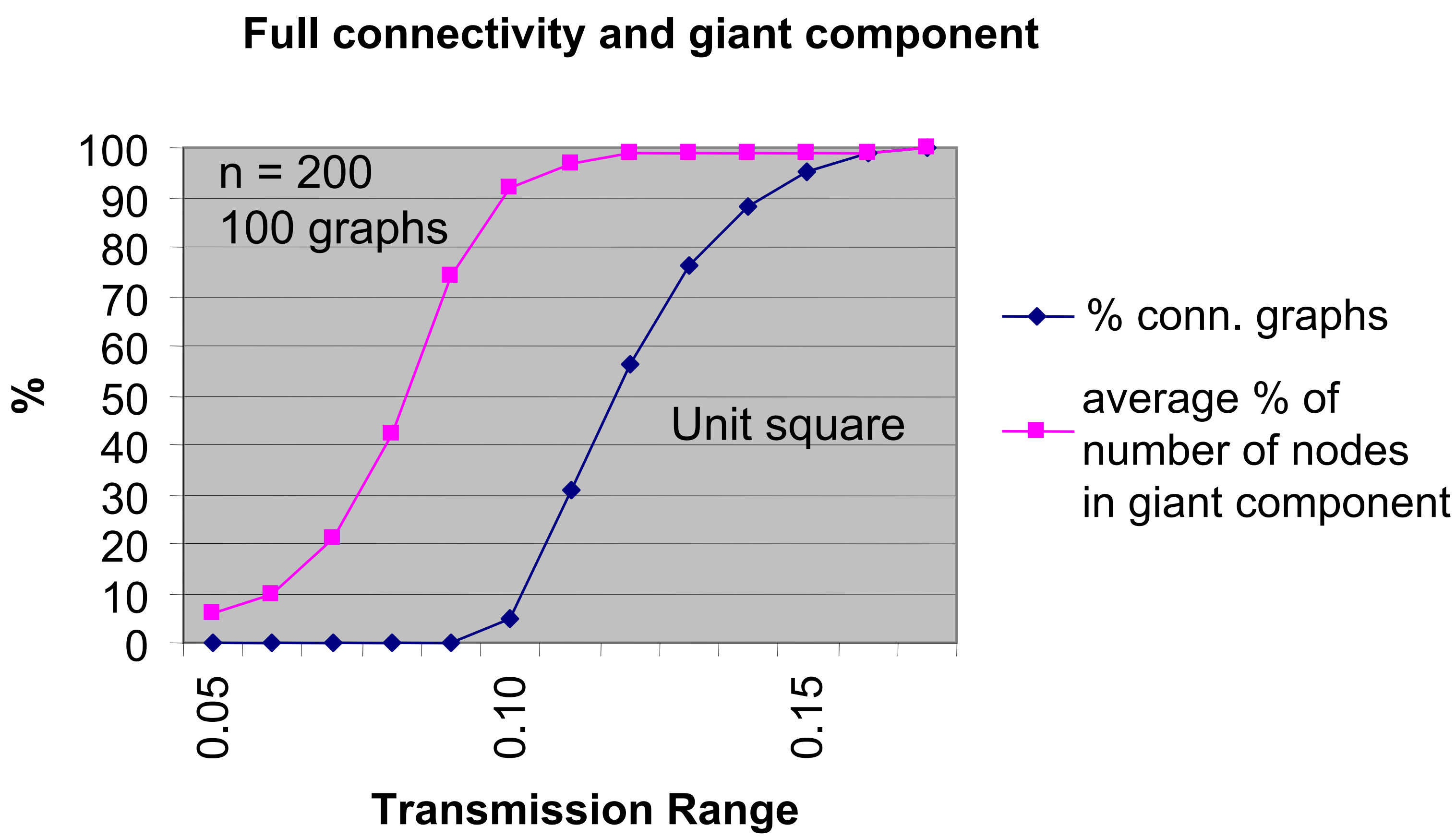
The Giant Component



The Giant Component



The Giant Component



The Giant Component

The giant component is a phenomenon appearing also for practical values of n.

Therefore, it can be claimed that, approximately, for any value of n

**the probability of a Communication Graph to be connected equals
the probability of no isolated nodes**.

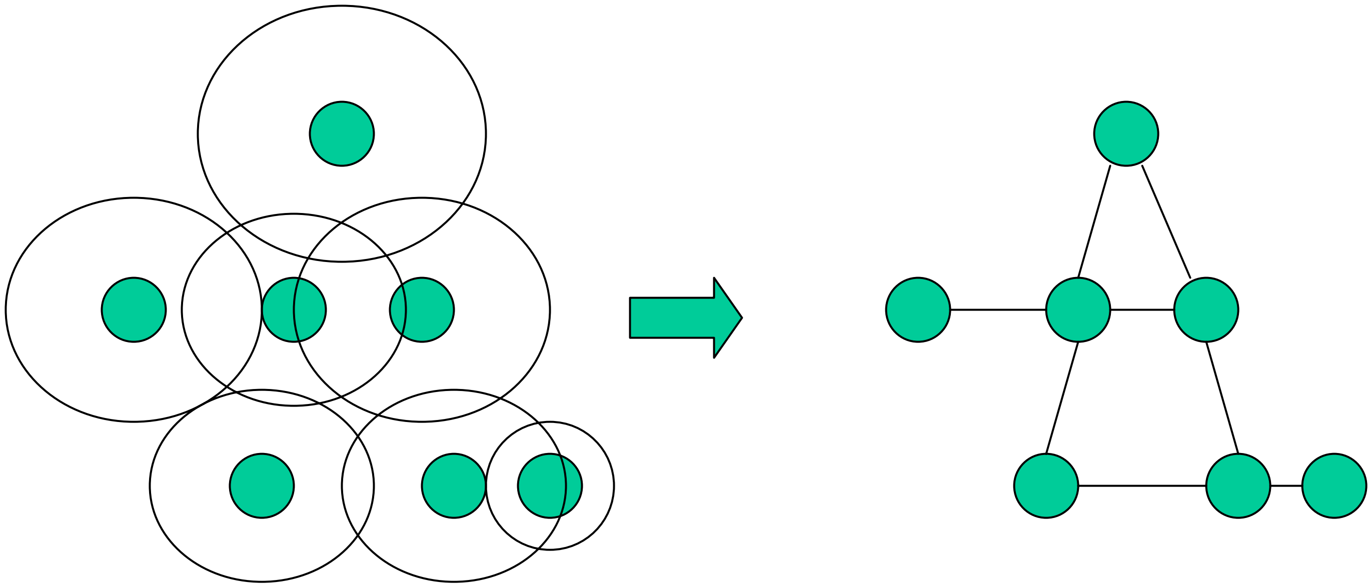
However, this approximation has not been investigated mathematically for finite n.

Continuum Percolation Theory

Given a realisation of a PPP, a disk of radius R_n is centered at each node n, and a graph is formed where an edge exists between two nodes if the corresponding disks intersect. The radii can be different, and random.

It can be seen as a special case of Stochastic Geometry, which deals with geometrical objects of given form, whose positions are random.

With respect to GRG Theory, the number of nodes is not deterministically known.



Limitations of these Approaches

GRGs and Continuum Percolation:

Disks, possibly of random radius
Normally used with Link Connectivity Model 1.

GRGs:

Node densities tending to infinity in finite regions (dense networks)
Properties of networks with finite densities not perfectly understood.

Disk Model widely used
Approaches with density tending to infinity
Relevance of border effects

Section 4
Connectivity Over an Unlimited Region

John Orriss
Orriss' First Result
Modification
Orriss' Second Result
Corollary

John Orriss

IEEE TRANSACTIONS ON COMMUNICATIONS, VOL. 51, NO. 4, APRIL 2003

Probability Distributions for the Number of Radio Transceivers Which Can Communicate With One Another

[1]

John Orriss and Stephen K. Barton, *Senior Member, IEEE*

PPP over unbounded region
omnidirectional antennas

$$L = k_0 + k_1 \ln R + S \leq l_1 .$$

II. PROBABILITY DISTRIBUTION OF THE DISTANCE SUBJECT TO A MAXIMUM LOSS

III. NUMBER OF AUDIBLE BASE STATIONS

Orriss' First Result

II. Probability Distribution of the Distance Subject to a Maximum Loss

Let mobile M and base station B be within communication range. This is possible if the loss from M to B does not exceed some specified value l_1. Suppose, therefore, that

$$L = k_0 + k_1 \ln R + S \leq l_1. \qquad (2)$$

If this condition is satisfied, then M and B will be referred to as l_1 audible or, when there is no ambiguity, simply as audible to one another.

Take $S = s$: from (2)

$$R \leq e^{\frac{l_1 - k_0 - s}{k_1}} \qquad (3)$$

and hence

$$P(R \leq r | S = s) = kr^2, \qquad 0 \leq r \leq e^{\frac{l_1 - k_0 - s}{k_1}} \qquad (4)$$

where

$$k = e^{-\frac{2(l_1 - k_0 - s)}{k_1}} \qquad (5)$$

and so the conditional density for R, given $S = s$, is

$$f_{R|S}(r|s) = 2re^{-\frac{2(l_1 - k_0 - s)}{k_1}}, \qquad 0 \leq r \leq e^{\frac{l_1 - k_0 - s}{k_1}} \qquad (6)$$

by differentiation of (4) and substitution of (5). Now S is normally distributed, mean zero, variance σ^2, so the joint density of R and S is

$$f_{RS}(r, s) = f_{R|S}(r|s) f_S(s) = \frac{2re^{-\frac{2(l_1 - k_0 - s)}{k_1}} e^{-\frac{s^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}$$

for

$$-\infty < s < \infty, \qquad 0 \leq r \leq e^{\frac{l_1 - k_0 - s}{k_1}}$$

or, equivalently

$$0 \leq r < \infty, \qquad -\infty < s \leq l_1 - k_0 - k_1 \ln r.$$

Hence, the density of R, subject to (2), is

$$f_R(r) = \int_{-\infty}^{l_1 - k_0 - k_1 \ln r} f_{RS}(r, s) ds$$

$$= 2re^{-\left(\frac{2}{k_1}\right)\left[l_1 - k_0 - \frac{\sigma^2}{k_1}\right]} \int_{-\infty}^{l_1 - k_0 - k_1 \ln r} \frac{e^{\frac{-\left[s - \frac{2\sigma^2}{k_1}\right]^2}{(2\sigma^2)}}}{(\sigma\sqrt{2\pi})} ds$$

after some algebra. This can be written

Orriss' First Result

$$\boxed{f_R(r) = 2Kr\Phi(a - b\ln r)} \quad (7)$$

where $a = [k_1 - k_0 - 2\sigma^2/k_1]/\sigma$, $b = k_1/\sigma$, $K = \exp[-(2/k_1)][k_1 - k_0 - \sigma^2/k_1]]$, and $\Phi(x) = \int_{-\infty}^{x}(1/\sqrt{2\pi})e^{-u^2/2}du$.

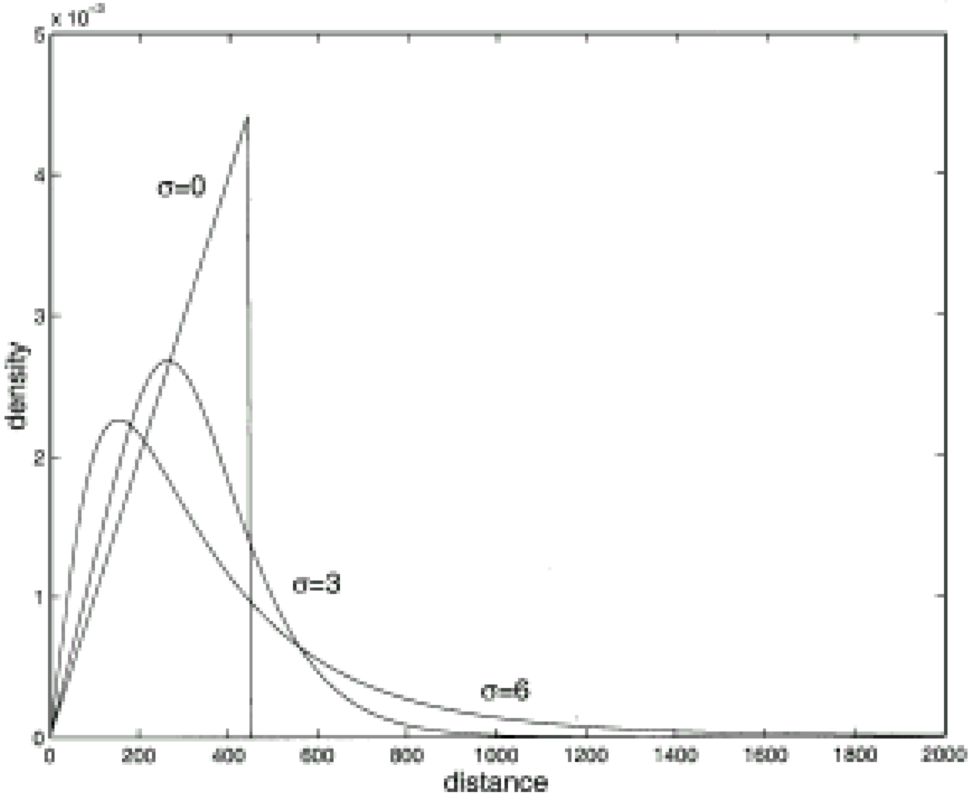


Fig. 1. PDF of distance between communicating nodes.

Modification

Orriss: $$f_R(r) = 2Kr\Phi(a - b\ln r)$$ where

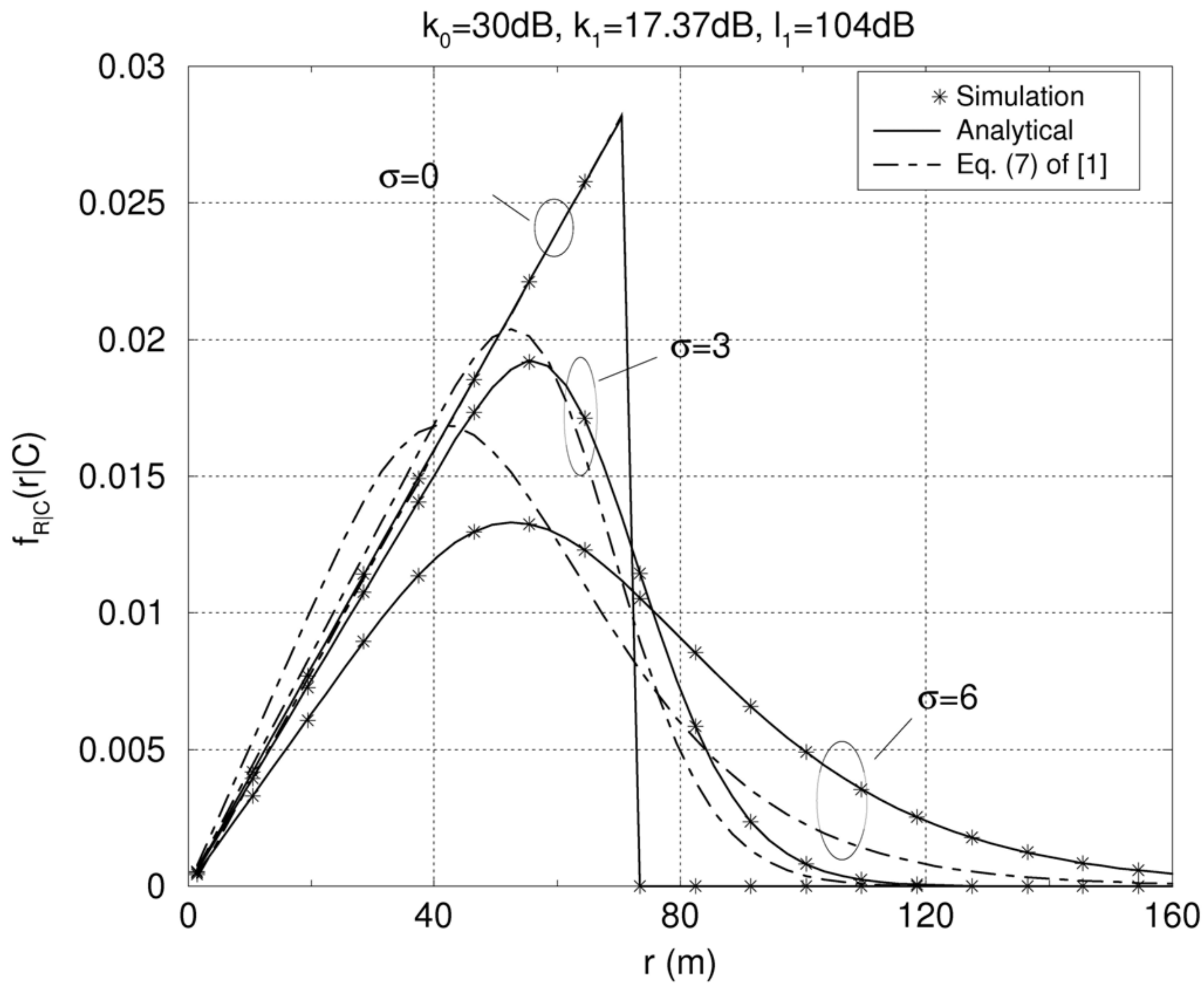
$$a = [L_{th} - k_0 - 2\sigma^2/k_1]/\sigma,$$
$$b = k_1/\sigma$$
$$K = \exp[-(2/k_1)[L_{th} - k_0 - \sigma^2/k_1]],$$
$$\Phi(x) = \int_{-\infty}^{x} (1/\sqrt{2\pi}) e^{-u^2/2} du.$$

Correct expression (still unpublished):

$$f_{R|\mathcal{C}}(r|\mathcal{C}) = r\, e^{-\frac{2}{k_1}\left(l_1 - k_0 - \sigma^2/k_1\right)} \operatorname{erfc}\left(\frac{k_0 - l_1 + k_1 \ln r}{\sqrt{2}\sigma}\right)$$

where $\operatorname{erfc}(\cdot)$ denotes the complementary error function.

Modification



Orriss' Second Result

III. Number of Audible Base Stations Within a Range of Distances

Define the random variable $N_{r_1,r}$ to be the number of station between distances r_1 and r from the mobile $(r \geq r_1)$ and wit loss $\leq l_1$, and define $P_n(r)$ to be the probability that this numbe is exactly n. Then, for $n \geq 1$

$$\begin{aligned} P_n(r+\delta r) &= P_n(r)[1-2\pi\rho r\delta r] \\ &\quad + P_{n-1}(r)2\pi\rho r\delta r\Phi\left(\frac{l_1-k_0-k_1\ln r}{\sigma}\right) \\ &\quad + P_n(r)2\pi\rho r\delta r\left[1-\Phi\left(\frac{l_1-k_0-k_1\ln r}{\sigma}\right)\right] \\ &\quad + o(\delta r). \end{aligned} \tag{8}$$

The three products on the right correspond to the three possible cases:

first, exactly n stations between r_1 and r, none between r and $r+\delta r$;
second, exactly $n-1$ stations between r_1 and r, and one, whose loss does not exceed l_1, between r and $r+\delta r$; and
third, exactly n stations between r_1 and r, and one, whose loss does exceed l_1, between r and $r+\delta r$.

Writing $a_1 = (l_1-k_0)/\sigma$, $b_1 = k_1/\sigma$, (8) becomes, after rearrangement in terms of the incremental ratio of $P_n(r)$ and letting $\delta r \to 0$

$$P_n'(r) = 2\pi\rho r\Phi(a_1-b_1\ln r)[P_{n-1}(r)-P_n(r)], \quad n\geq 1. \tag{9}$$

Similarly

$$P_0'(r) = -2\pi\rho r\Phi(a_1-b_1\ln r)P_0(r). \tag{10}$$

Define a probability generating function

$$\Pi(r,z) = \sum_{n=0}^{\infty} P_n(r)z^n. \tag{11}$$

From (9), (10), and (11)

$$\frac{\partial\Pi}{\partial r} = 2\pi\rho r\Phi(a_1-b_1\ln r)(z-1)\Pi$$

and so

$$\frac{\partial}{\partial r}(\ln\Pi) = 2\pi(z-1)\rho r\Phi(a_1-b_1\ln r). \tag{12}$$

Define

$$\begin{aligned} \Psi(a_1,b_1;r) &= r^2\Phi(a_1-b_1\ln r) - e^{\frac{2a_1}{b_1}+\frac{2}{b_1^2}} \\ &\quad \times\Phi\left(a_1-b_1\ln r+\frac{2}{b_1}\right) \end{aligned} \tag{13}$$

which is an indefinite integral of $2r\Phi(a_1-b_1\ln r)$. Hence

Orriss' Second Result

$$\ln \Pi = \pi(z-1)\rho\,[\Psi(a_1, b_1; r) + G(z)] \tag{14}$$

where $G(z)$ is an arbitrary function of z. When $r = r_1$, $\Pi(r_1, z) \equiv 1$, since, with probability 1, no station is within a zero range of distances. Hence

$$G(z) \equiv -\Psi(a_1, b_1; r_1). \tag{15}$$

The solution of the equation is, therefore

$$\ln \Pi = \pi(z-1)\rho\,[\Psi(a_1, b_1; r) - \Psi(a_1, b_1; r_1)] \tag{16}$$

so the distribution is Poisson with mean

$$\mu_{r_1, r} = \pi\rho\,[\Psi(a_1, b_1; r) - \Psi(a_1, b_1; r_1)]\,. \tag{17}$$

Some special cases are of interest. The distribution of the number of audible base stations within range r of the mobile is given by setting $r_1 = 0$. Since $\Psi(a_1, b_1; 0) = -\exp[(2a_1/b_1) + (2/b_1^2)]$, the distribution has mean $\mu_{0,r} = \pi\rho[\exp[(2a_1/b_1) + (2/b_1^2)] + \Psi(a_1, b_1; r)]$.

The distribution of the total number of audible base stations is given by letting $r \to \infty$. Then $\Psi(a_1, b_1; r) \to 0$, so the mean is $\mu_{r_1,\infty} = -\pi\rho\Psi(a_1, b_1; r_1)$ for the number at distance greater than r_1 and $\mu_{0,\infty} = \pi\rho \exp[(2a_1/b_1) + (2/b_1^2)] = \pi\rho \exp[(2(l_1 - k_0)/k_1) + (2\sigma^2/k_1^2)]$ for the total number.

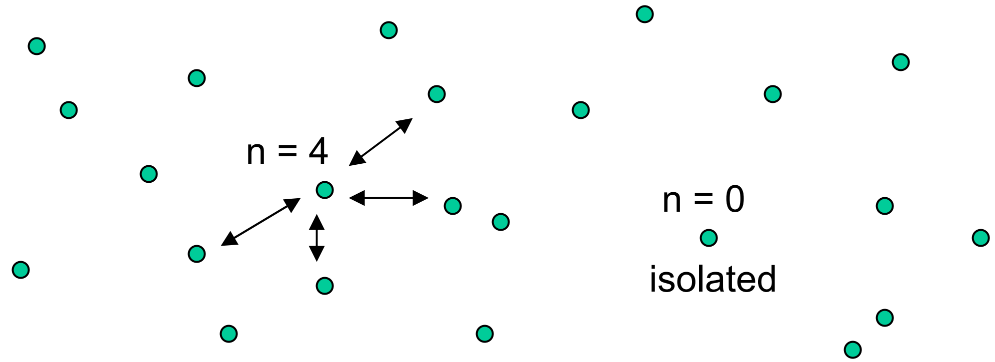
This mean may be related to the number of base stations audible in the absence of shadowing. In this case, from (3) with $s = 0$, all stations within radius $\exp[(l_1 - k_0)/(k_1)]$ are audible. The area within this radius is $A = \pi \exp[(2(l_1 - k_0))/(k_1)]$, so the expected number is $n_0 = \rho A$. Hence, $\mu_{0,\infty} = n_0 e^{2\sigma^2/k_1^2}$. This is equal to n_0 when $\sigma = 0$, but for any nonzero value of σ it is larger than n_0. As already noted in the last section, the introduction of shadowing results in the gain of a number of more distant, and hence, previously inaudible base stations partially counterbalanced by the loss of some nearer and previously audible ones. The result shows that (on average) the gain is greater than the loss. In effect, in the absence of shadowing, the mean number audible is equal to the expected number of base stations within a circle of radius $\exp[(l_1 - k_0)/(k_1)]$, whereas, in the presence of shadowing, the mean number audible is equal to the expected number of base stations within a circle of larger radius $\exp[((l_1 - k_0)/(k_1)) + (\sigma^2/k_1^2)]$.

Orriss' Second Result

The number of nodes heard by a given node is Poisson, with mean

$$Nm = \rho\, \pi\, \exp(2\, (l_1 - k_0) / k_1)\, \exp(2\sigma^2 / k_1^2)$$

Then, the probability of a node to be isolated is P(iso) = exp (-Nm).



Orriss' Second Result

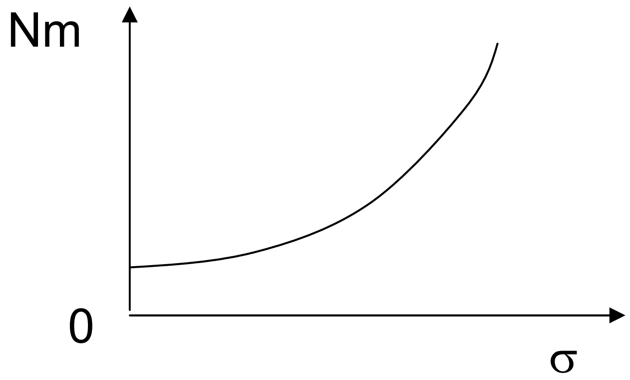
Note: Nm is linearly proportional to node density

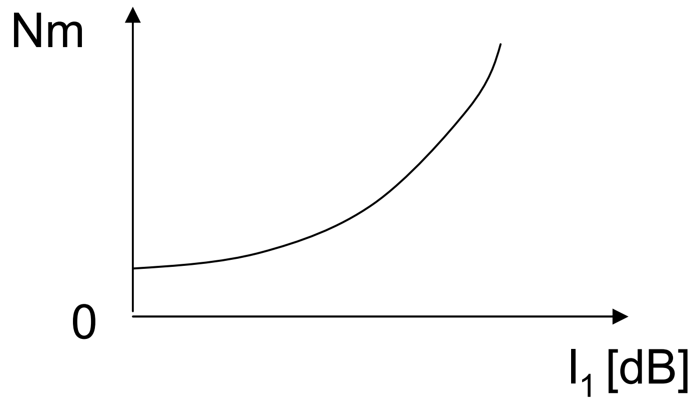
Nm is quadratically proportional to the ideal transmission range R:

$Nm = \rho \pi R^2 \exp(2\sigma^2 / k_1^2)$

Nm is exponentially proportional to the variance of shadowing

Nm is exponentially proportional to transmit power: $I_1 = P_t - P_{rmin}$





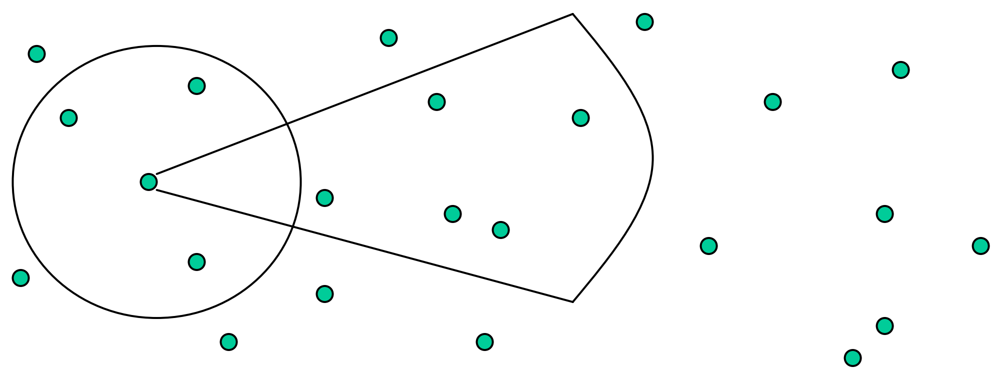
Channel fluctuations may significantly increase network connectivity

Corollary

The number of nodes heard by a given node is Poisson, with mean

$$Nm = \rho\, \pi \exp\left(2\,(l_1 - k_0) / k_1\right) \exp(2\sigma^2 / k_1^2)$$

Do randomly directed directional antennas help increasing connectivity?



Corollary

The number of nodes heard by a given node is Poisson, with mean

$$Nm = \rho\, \pi \exp\left(2\,(l_1 - k_0) / k_1\right) \exp(2\sigma^2 / k_1^2)$$

Do randomly directed directional antennas help increasing connectivity?

Denoting Nm = Nm (G) the mean as a function of antenna gain,
implicitly included in k_o, then with directional antennas

$$Nm(G) = (1 / G)\, \rho\, \pi \exp\left(2\,(l_1 - k_0 + 10\log G) / k_1\right) \exp(2\sigma^2 / k_1^2)$$

$$= Nm(1) \exp\left(20\log G) / k_1\right) / G$$

Therefore the ratio $Nm(G) / Nm(1) = \exp\left(20\log G) / k_1\right) / G$ measures the advantage of using directional antennas.

For example:
$G = 4$ [6 dB], $k_1 = 13.3 \rightarrow Nm(G) / Nm(1) = 0.62 \rightarrow$ **connectivity is decreased!**

Section 5
Connectivity for WSNs

Full Connectivity

Traditional definition:

A network is fully connected if there exists any path (sequence of hops) between every pair of nodes.

This definition is compliant with the objective of ad hoc networks, i.e. to allow every node being in contact with any other node.

But this is not the goal of a WSN.

In WSNs, nodes (sensors) want to transmit their samples to a given node, namely, the sink (or any node in a given set, in the case of multi-sink networks).

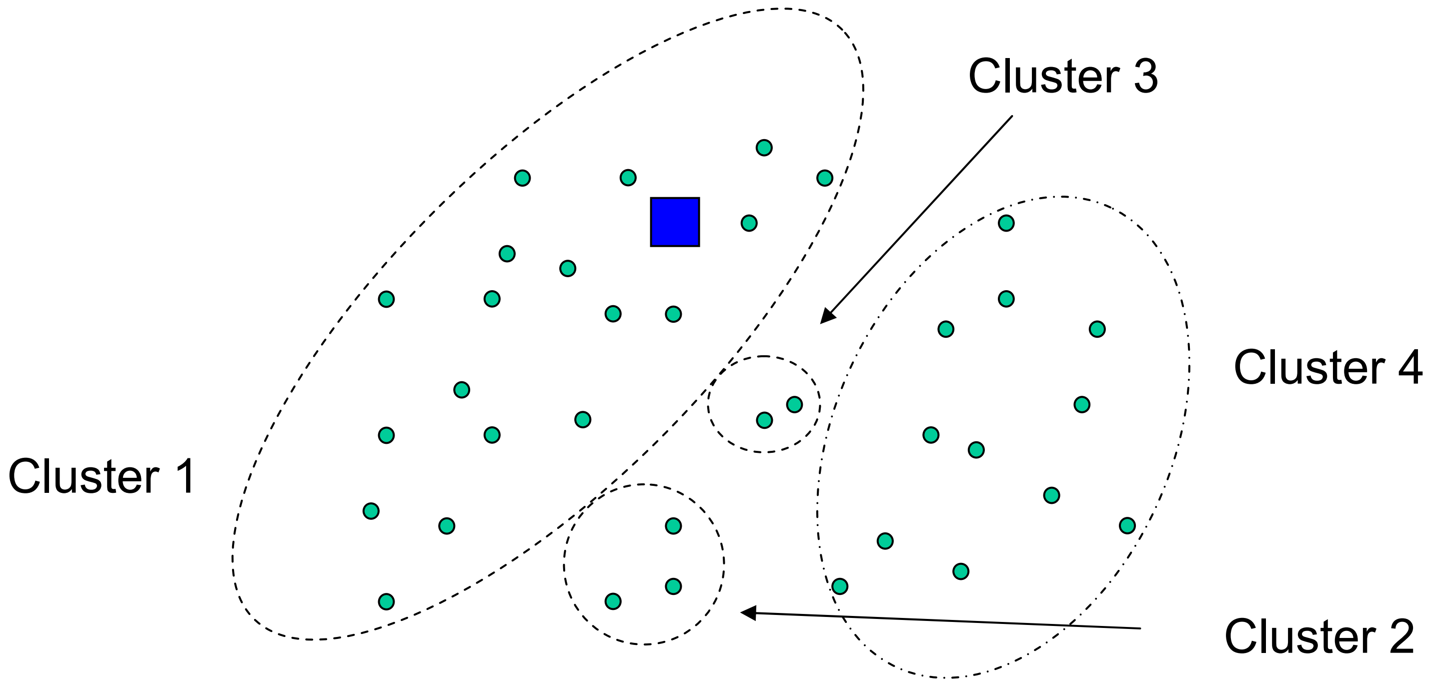
Definition more suitable for WSNs:

A WSN is fully connected if all nodes can report their samples to a sink through any path.

Full Connectivity

A WSN is fully connected if all nodes can report their samples to a sink through any path.

No difference in a single-sink scenario



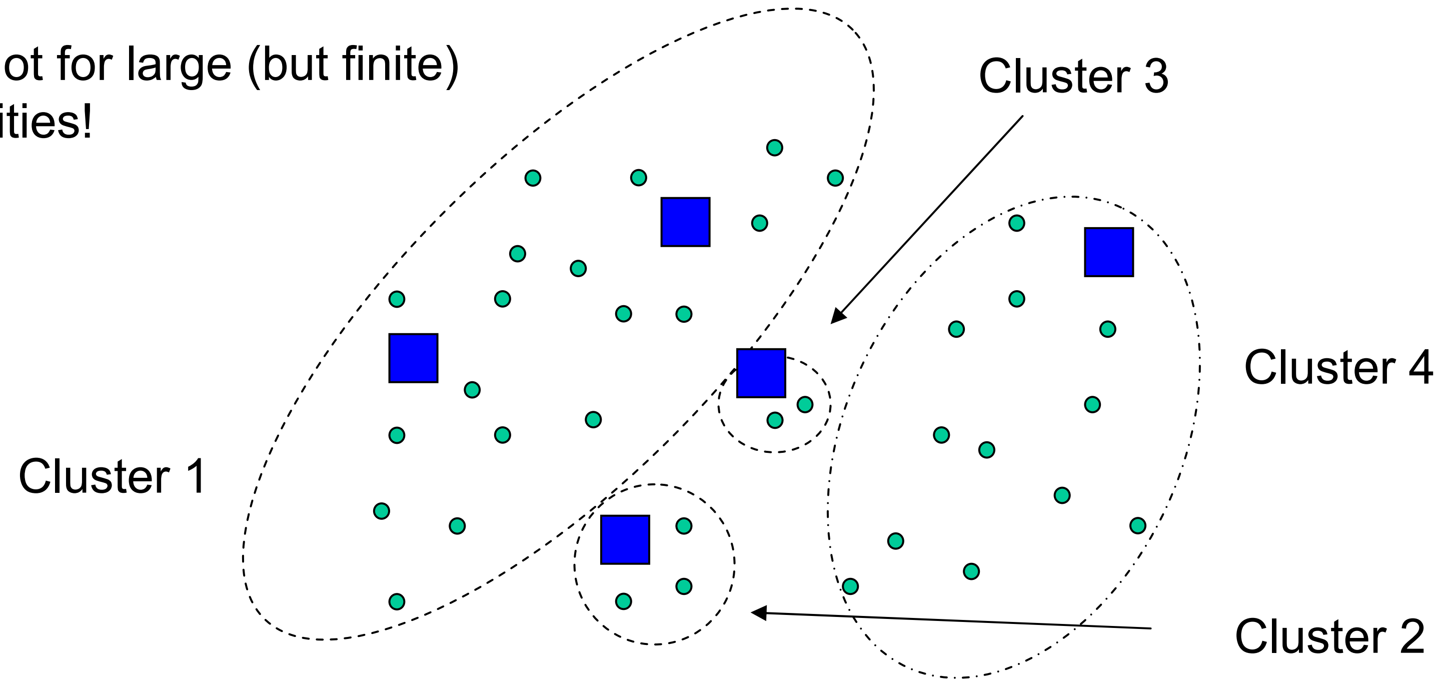
Full Connectivity

A WSN is fully connected if all nodes can report their samples to a sink through any path.

No difference in a single-sink scenario

Difference in a multi-sink scenario

But not for large (but finite) densities!



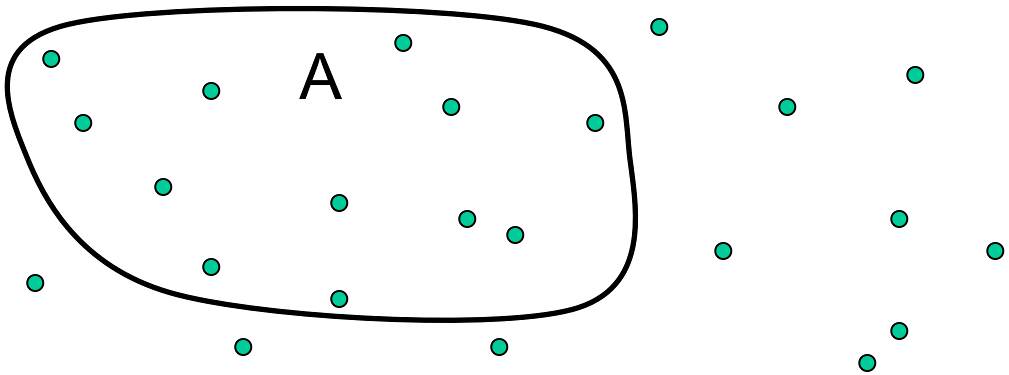
Full connectivity: approximated analysis

Probability of a network of area A, given n_o nodes in A, being fully connected:

$$P(\text{con} \mid n_o) \underset{(1)}{\approx} P(\text{no isolated nodes in A} \mid n_o) \underset{(2)}{\approx} (1 - P(\text{iso}))^{n_o}$$

(1) If network is dense: indeed, P(con) $\leq$ P(no isolated nodes in A)
(2) If P(iso) is small and network is dense

Dense network:	ρ A >> 1
P(iso) small:	P(iso) << 0.001



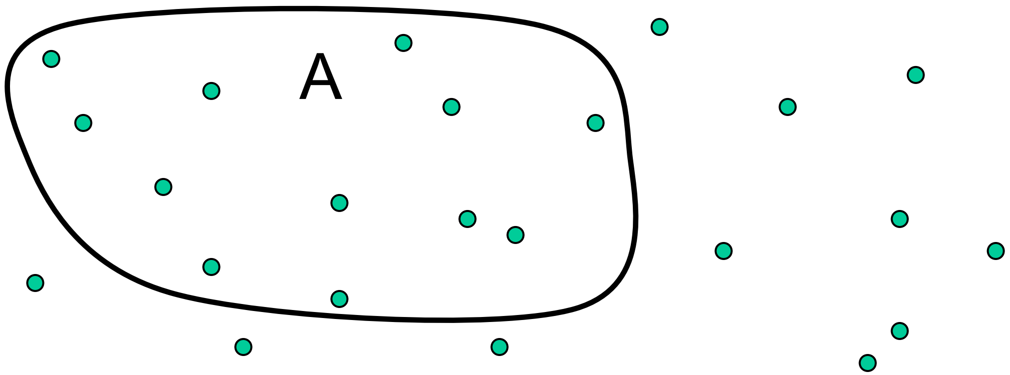
Full connectivity: approximated analysis

Probability of a network of area A being fully connected:

$P(con) \quad = E_{n_o}[\, P(con \mid n_o) \,] = E_{n_o}[(1 - P(iso))^{n_o}] = \ldots$

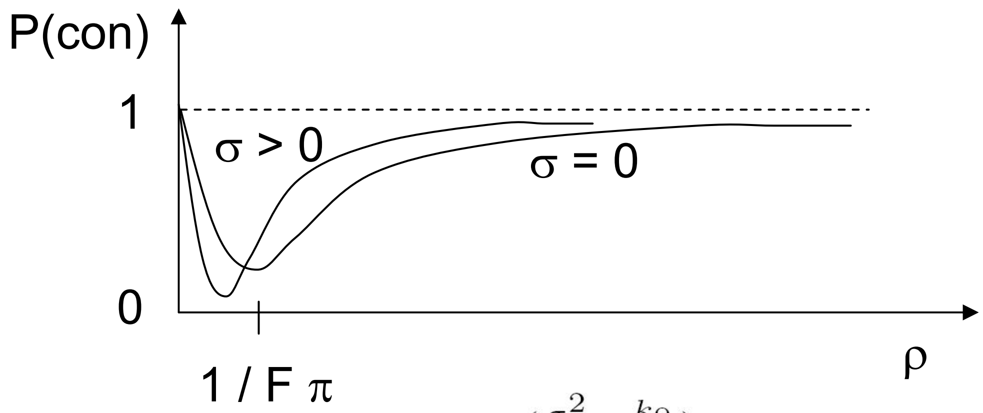
$= \exp[\, -\rho\, A \exp(-\, Nm) \,]$ $\qquad Nm = \pi \rho e^{2(\frac{\sigma^2}{k_1^2} - \frac{k_0}{k_1})} e^{2(\frac{Lth}{k_1})}$ $\qquad [L_{th} = l1]$

Note: A must be large → $A \gg \pi R^2$



Full connectivity: approximated analysis

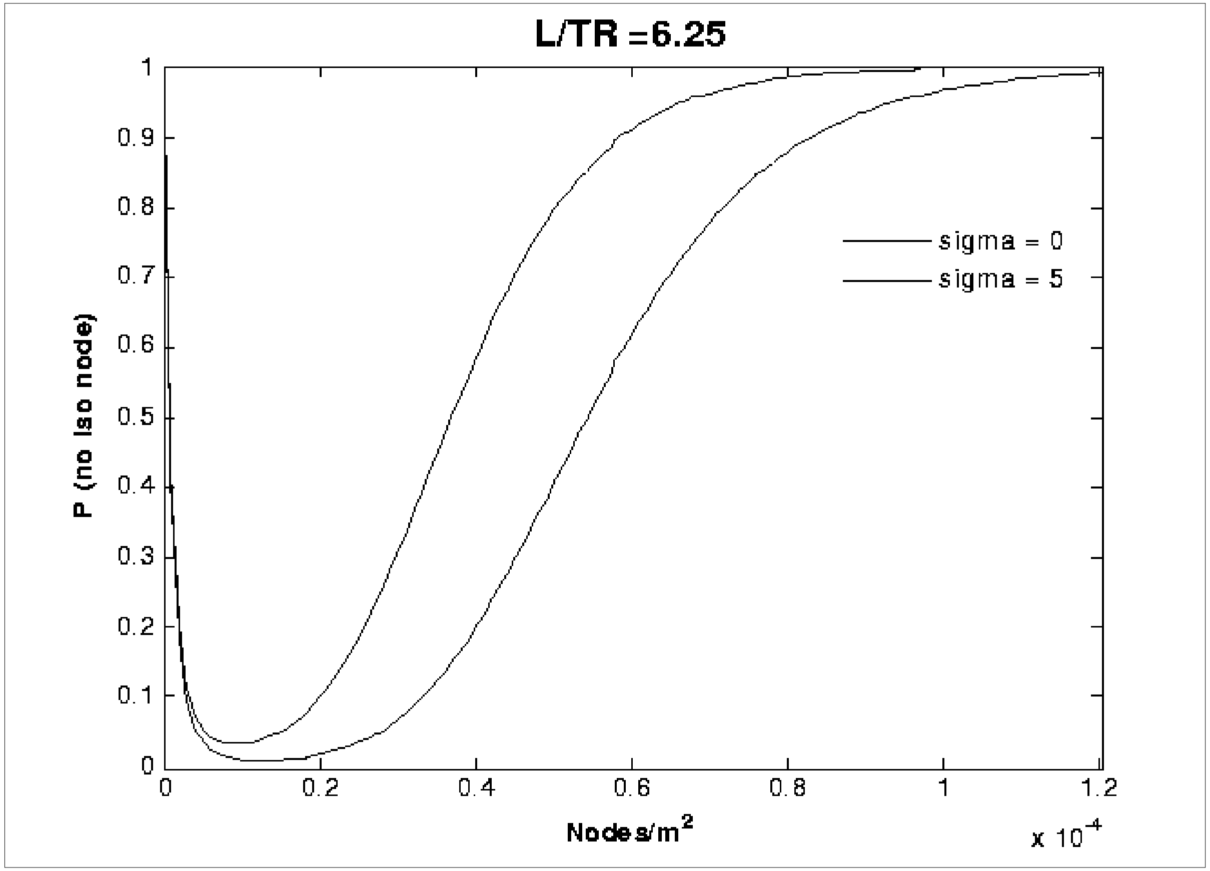
Probability of a network of area $A = L^2$ being fully connected



$$F = e^{2\left(\frac{\sigma^2}{k_1^2} - \frac{k_0}{k_1}\right)} e^{2\left(\frac{Lth}{k_1}\right)}$$

Full connectivity: approximated analysis

Probability of a network of area $A = L^2$ being fully connected. TR is ideal trans. range



Channel fluctuations significantly reduce the number of required nodes

Full Connectivity

In multi-sink scenarios both sinks and sensors can be considered distributed according to two independent PPPs with different densities.

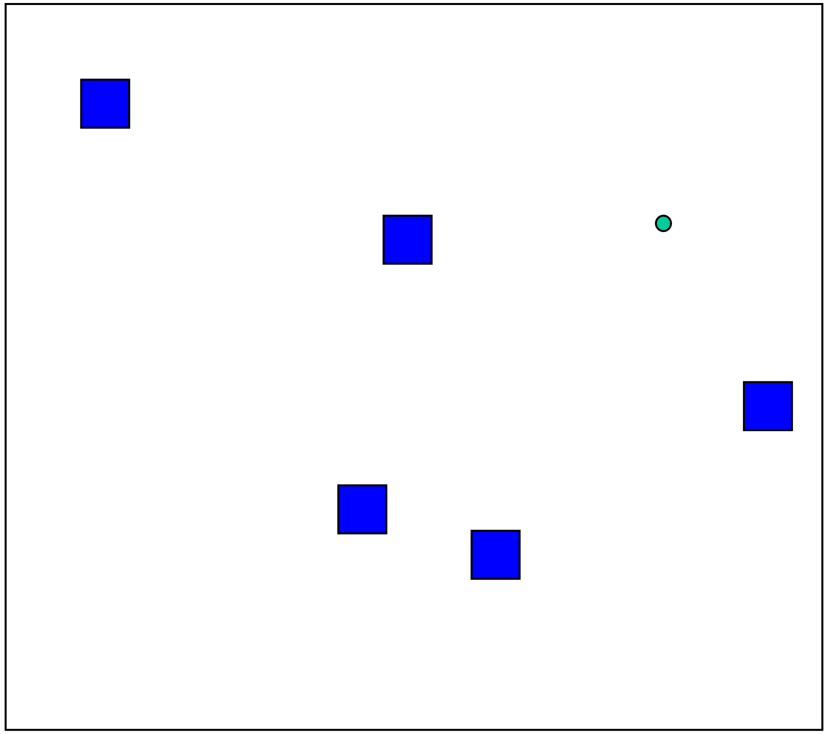
Sinks density ρ_0

Sensors density ρ

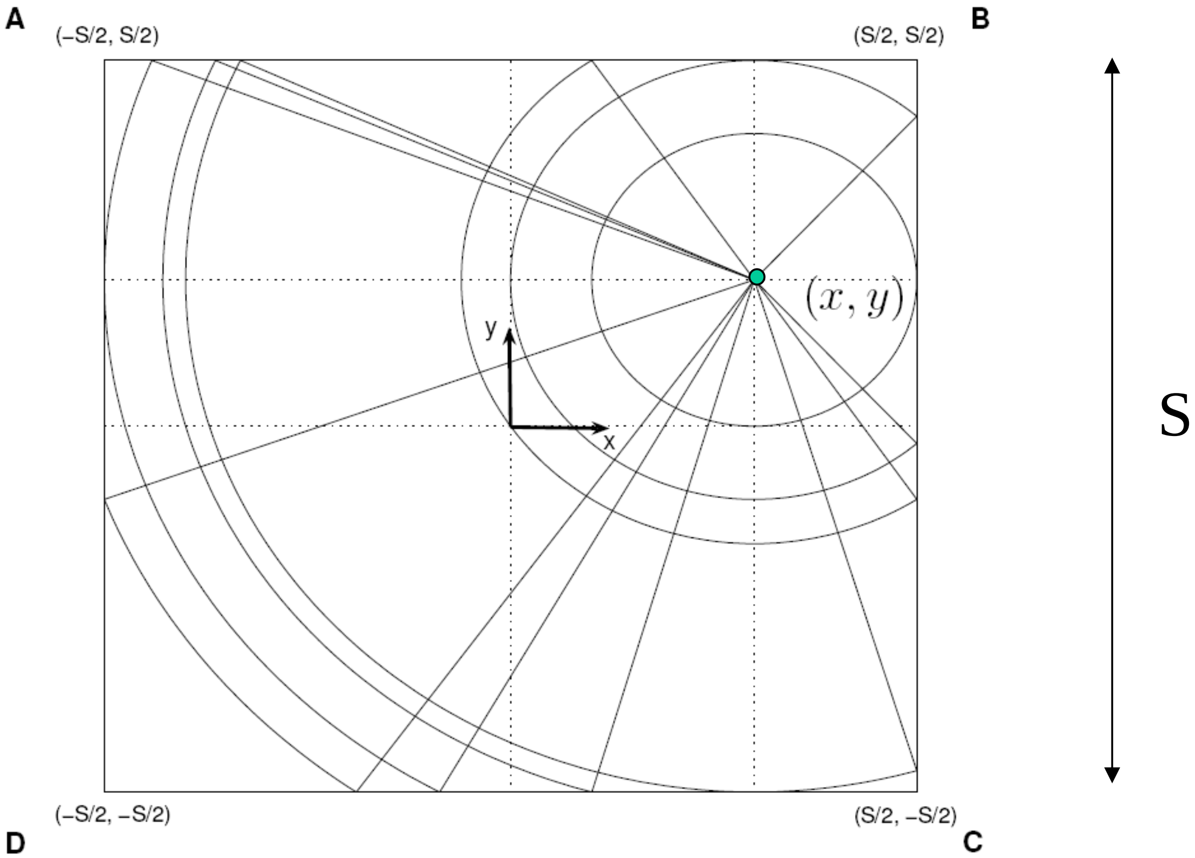
Section 6
Connectivity Over Limited Regions for WSNs

Connectivity in Squares
Full Connectivity in Squares – Single Hop
Reachability in Squares – Single Hop
Reachability in Squares – Multiple Hops, Tree Topologies

Connectivity in Squares



Connectivity in Squares



Connectivity in Squares

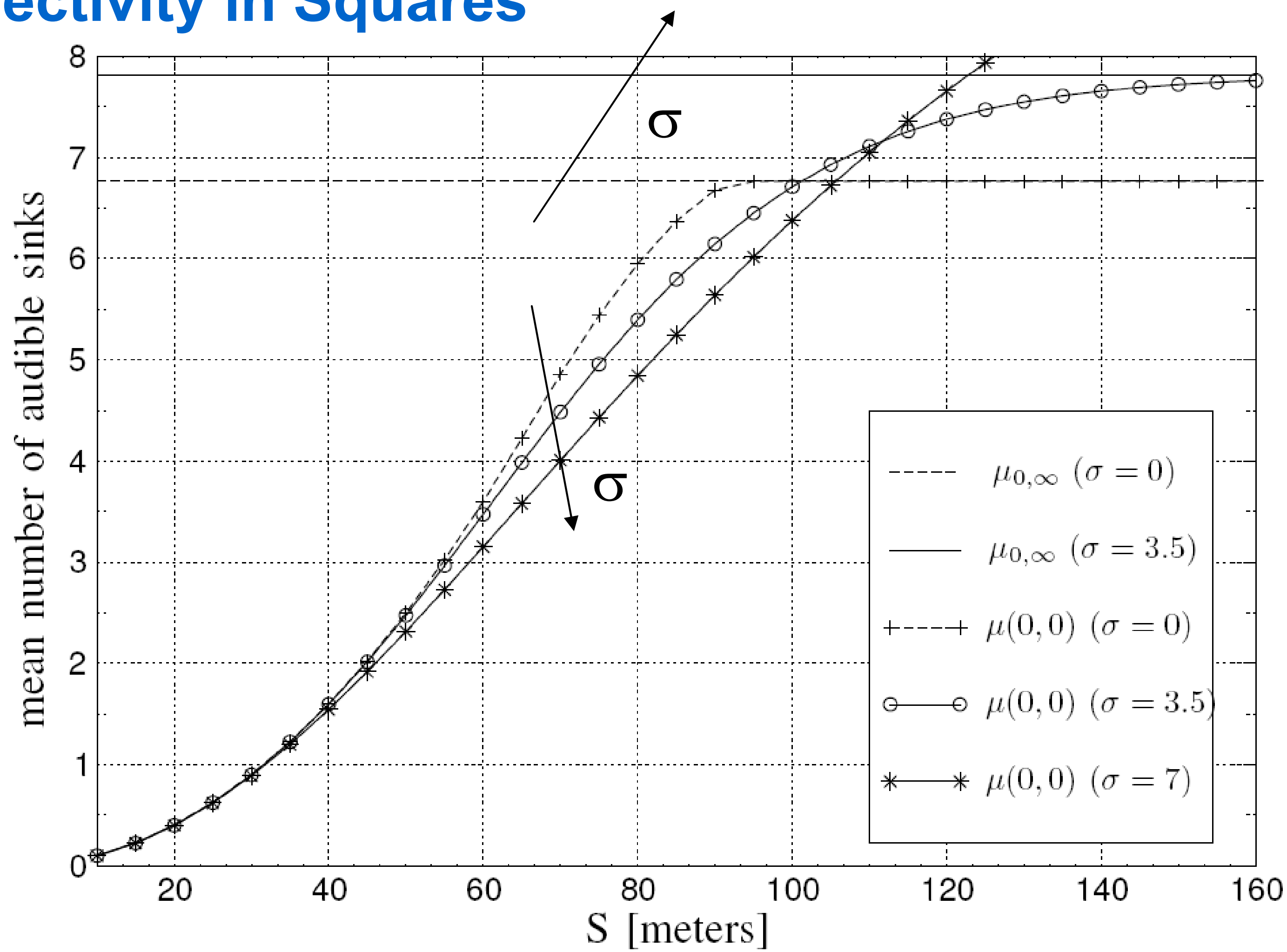
Number of sinks heard from a node in (x,y) is Poisson with mean

$$\mu(x,y) = \sum_{i=1}^{8} \int_{r_{1,i}}^{r_{2,i}} 2\theta_i(r) \cdot \rho_0 \cdot r \cdot \Phi(a_1 - b_1 \ln r) dr$$

Region	Range: $r_1 \leq r \leq r_2$
1	$0 \leq r \leq \frac{S}{2} - x$
2	$\frac{S}{2} - x \leq r \leq \frac{S}{2} - y$
3	$\frac{S}{2} - y \leq r \leq \sqrt{(\frac{S}{2}-x)^2 + (\frac{S}{2}-y)^2}$
4	$\sqrt{(\frac{S}{2}-x)^2 + (\frac{S}{2}-y)^2} \leq r \leq \frac{S}{2} + y$
5	$\frac{S}{2} + y \leq r \leq \sqrt{(\frac{S}{2}-x)^2 + (\frac{S}{2}+y)^2}$
6	$\sqrt{(\frac{S}{2}-x)^2 + (\frac{S}{2}+y)^2} \leq r \leq \frac{S}{2} + x$
7	$\frac{S}{2} + x \leq r \leq \sqrt{(\frac{S}{2}+x)^2 + (\frac{S}{2}-y)^2}$
8	$\sqrt{(\frac{S}{2}+x)^2 + (\frac{S}{2}-y)^2} \leq r \leq \sqrt{(\frac{S}{2}+x)^2 + (\frac{S}{2}+y)^2}$

Region	$\theta(r)$
1	π
2	$\frac{\pi}{2} + \arcsin \frac{\frac{S}{2}-x}{r}$
3	$\frac{\pi}{2} + \arcsin \frac{\frac{S}{2}-x}{r} - \arccos \frac{\frac{S}{2}-y}{r}$
4	$\frac{\pi}{2} + \frac{1}{2}(\arcsin \frac{\frac{S}{2}-x}{r} - \arccos \frac{\frac{S}{2}-y}{r})$
5	$\frac{\pi}{2} - \arccos \frac{\frac{S}{2}+y}{r} + \frac{1}{2}(\arcsin \frac{\frac{S}{2}-x}{r} - \arccos \frac{\frac{S}{2}-y}{r})$
6	$\frac{\pi}{2} - \frac{1}{2}(\arccos \frac{\frac{S}{2}+y}{r} + \arccos \frac{\frac{S}{2}-y}{r})$
7	$\frac{1}{2}(\arcsin \frac{\frac{S}{2}-y}{r} + \arcsin \frac{\frac{S}{2}+y}{r}) - \arccos \frac{\frac{S}{2}+x}{r}$
8	$\frac{1}{2}(\arcsin \frac{\frac{S}{2}+y}{r} - \arccos \frac{\frac{S}{2}+x}{r})$

Connectivity in Squares



Channel fluctuations increase network connectivity only in unlimited region

Full Connectivity in Squares – Single Hop

$$q(x,y) = 1 - exp(- \mu(x,y))$$

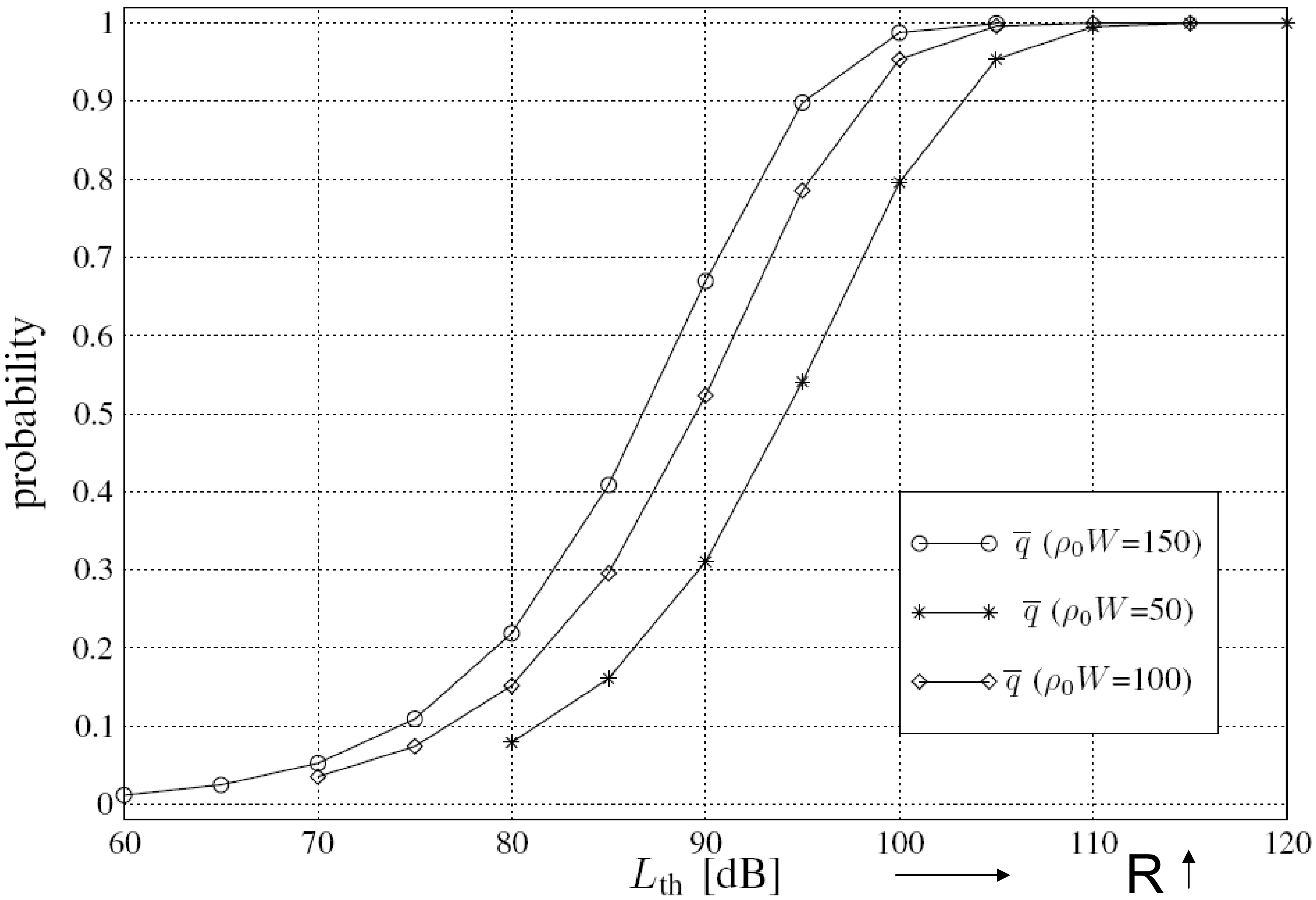
$$\overline{q} = \frac{8}{W} \int_0^{S/2} \int_0^{x} q(x,y)dydx$$

$$\text{Prob}\{F|N_s = k\} = \bar{q}^k$$

$$Z = \text{Prob}\{F\} = \sum_{k=1}^{+\infty} \text{Prob}\{F|N_s = k\}\text{Prob}\{N_s = k\}$$

$$= \sum_{k=1}^{+\infty} \bar{q}^k \cdot \frac{e^{-\rho_s W}}{k!} (\rho_s W)^k$$

Full Connectivity in Squares – Single Hop



Reachability in Squares – Single Hop

In some applications, Full Connectivity is not required

A sufficient degree or reachability is requested i.e. a given minimum number of sensors needs to reach the sinks

$$Z_{\bar{m}}(j) = \text{Prob}\left\{C_j\right\} = e^{-\bar{m}} \cdot \sum_{k=j}^{+\infty} \sum_{l=j}^{k} \binom{k}{l} \frac{\bar{m}^k \bar{q}^l (1-\bar{q})^{k-l}}{k!}$$

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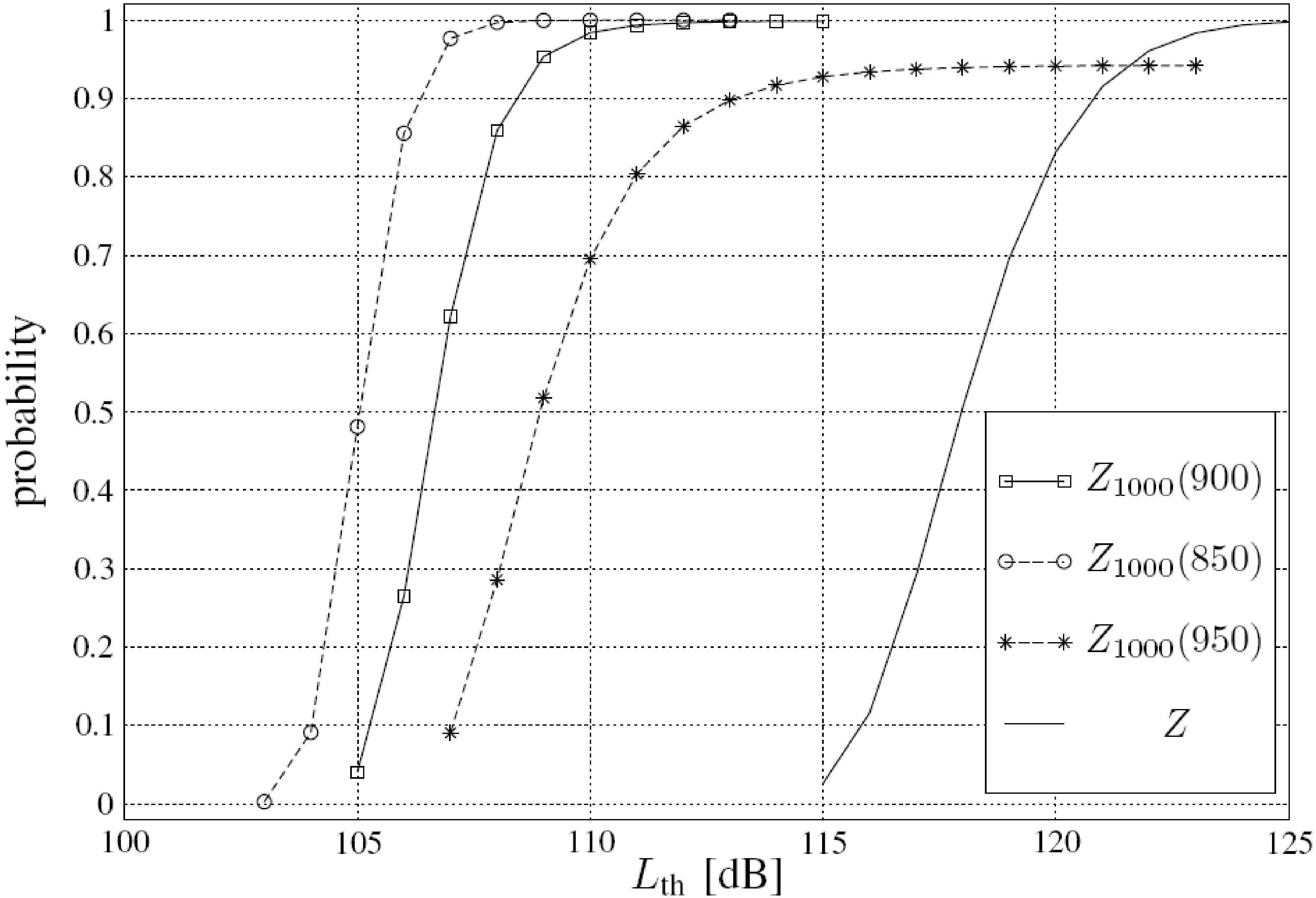
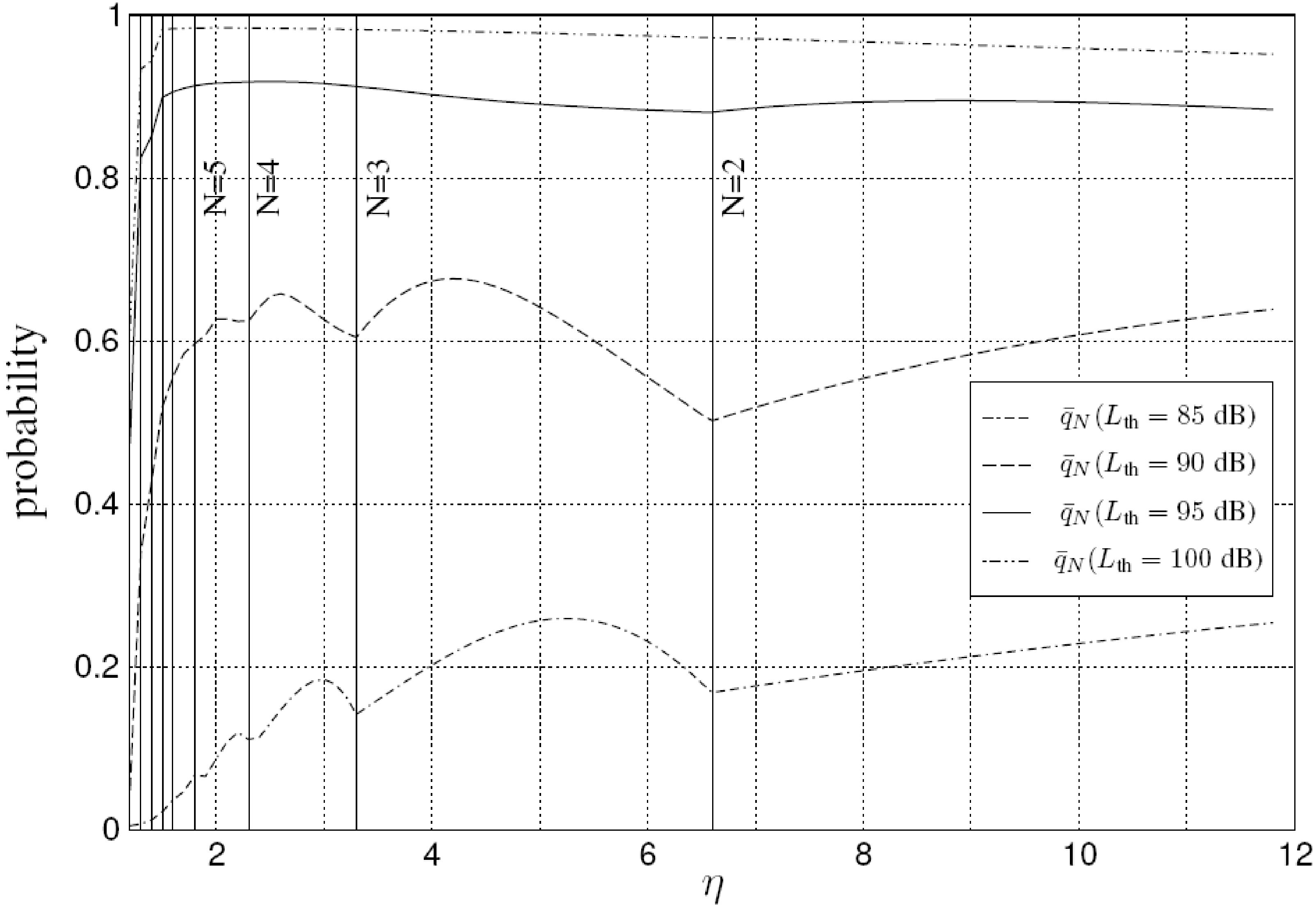
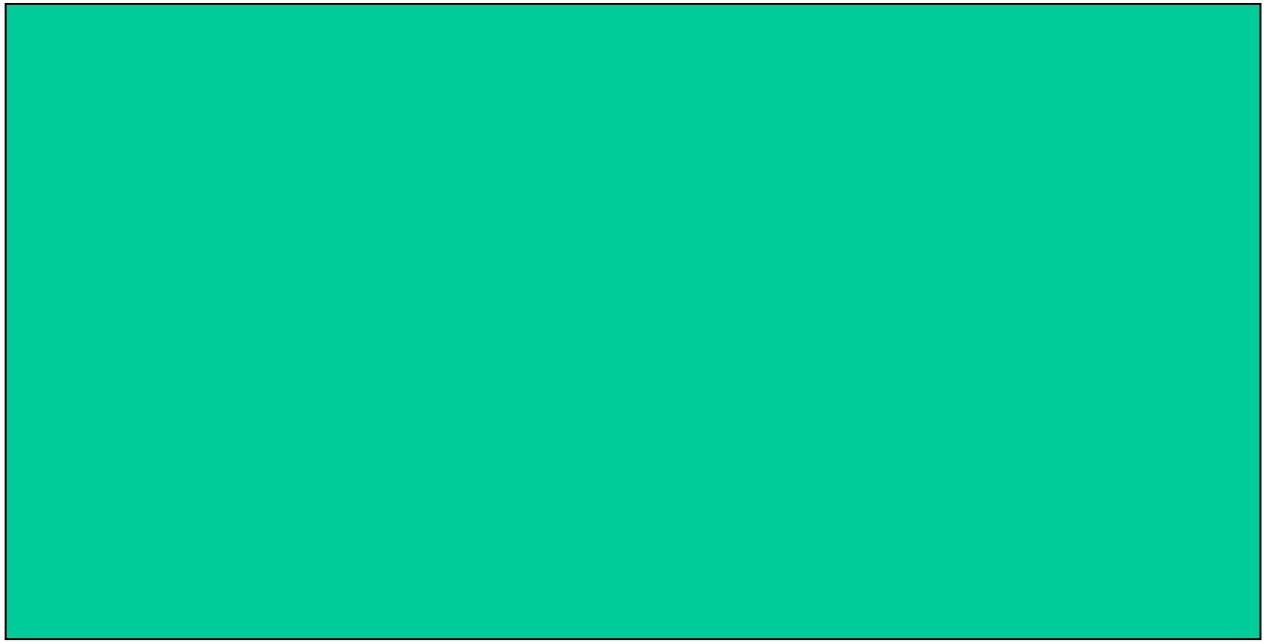
Reachability in Squares – Single Hop



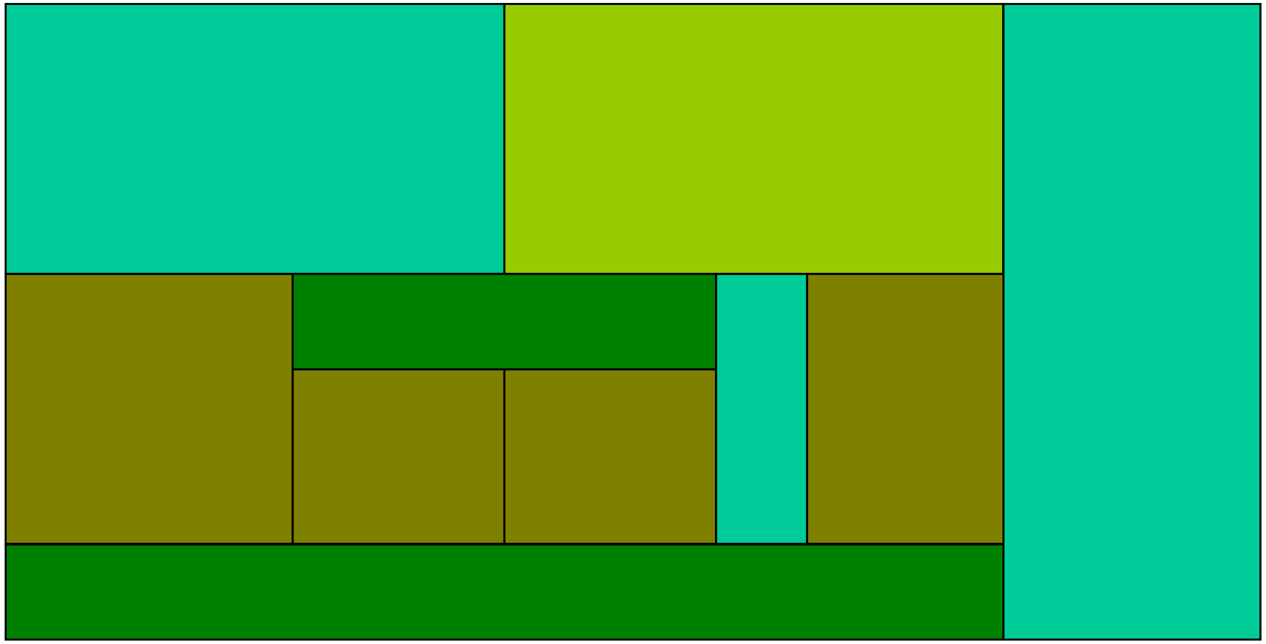
Reachability in Squares – Multiple Hops, Tree Topology



Rectangles



Tassellations



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Topology Control and Connectivity

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Thank You