Renewable Energy Sources for Distribution Generation in Smart Grids: the Role of Power Electronics

Speaker: G. Spiazzi

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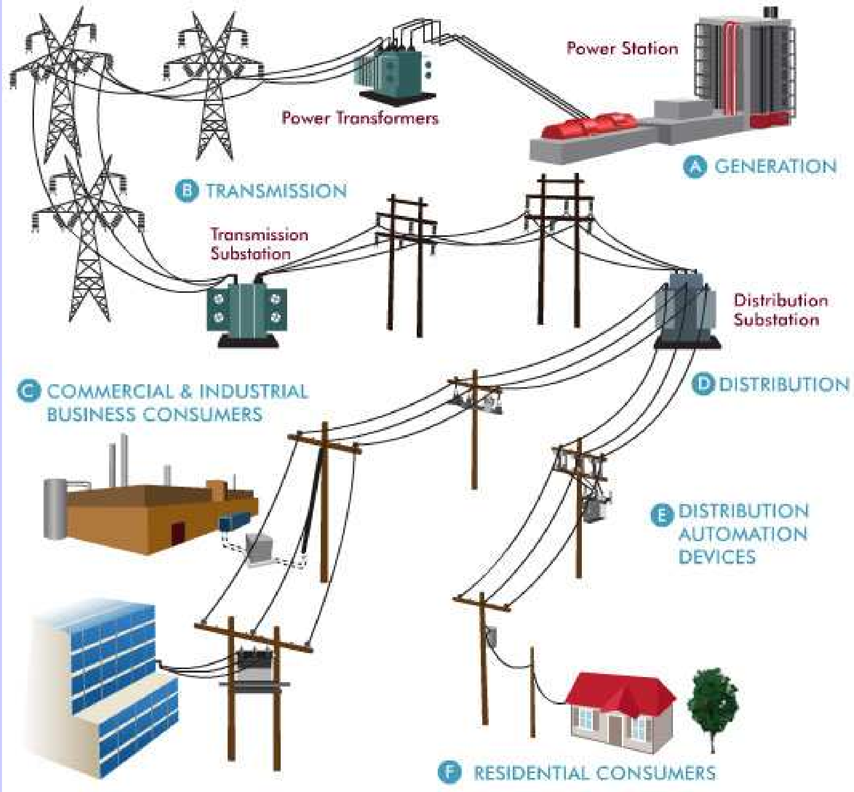
Seminar Outline

- From "Traditional" to "Smart Grids"
  - Challenges & potentialities
- Power electronics for renewable energy sources
  - Principles of photovoltaic energy generation
  - Power electronics for interfacing PV panels with the utility grid
    - Inverter operation
    - Maximum power point tracking techniques
    - Anti-islanding methods
- Traditional power theories
  - Budeanu power theory
  - Fryze power theory
  - Kusters & Moore theory (time domain)
  - Czarnecki theory (frequency domain)
- Conservative power theory
  - Mathematical and physical foundations of the theory
  - Examples

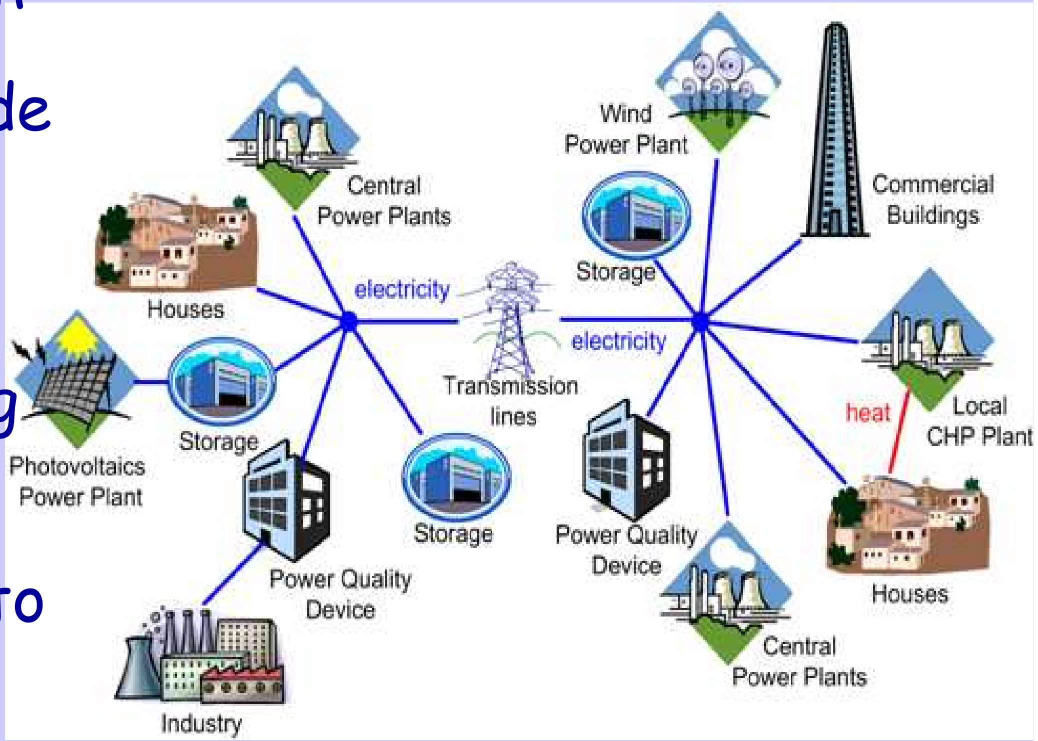
Traditional Grid

- Few large power plants feeding large number of end-users
- Power plants located in strategic sites (cost-effective generation, safety)
- Centralized control
- Unidirectional power flow
- Independent operation of each apparatus (grid performs as a nearly ideal voltage source)
- No customers' participation to power balance



Smart Grid

- Local-scale power distribution systems which can operate in grid-connected or islanded mode
- Distributed energy resources (DER)
- Bidirectional power flow
- Possibly weak grid, causing interaction of power sources and loads
- Multilateral contribution to power balance
- Intelligent electronic interfaces between energy sources and grid



Residential smart micro-grid

Energy sources at residential settlements

- Wind generators
- PV panels
- Fuel cells
- Energy storage devices (batteries)
- Micro-turbines
- Backup generators

Benefits of smart grids

- Distributed renewable resources
  - lower emission
  - energy cost reduction
- Reduction of transmission & distribution losses
  - power sources close to loads
- Improved utilization of conventional power sources
  - less active, reactive, unbalance and distortion power
- Voltage support
  - distributed injection of reactive power
- Increased power capability of network without investment in grid infrastructure

Challenges of smart grids

- Bidirectional power flow
  - new control and protection strategies
  - stabilization of voltage profiles
- Weak grid
  - compensation for voltage distortion due to distorting loads
  - compensation for voltage asymmetry due to unbalanced loads and single-phase DER units (PV, batteries, …)
- Irregular power injection by renewable energy sources
  - installation and control of energy storage devices

Vision

- Distributed generation is an increasingly important issue at planetary level: in fact, **renewable energy sources** (wind turbines/farms, photovoltaic panels/plants, hydro, tidal power etc.) and **alternative power generation** (fuel cells, micro-turbines etc.) can offer a **sustainable solution** to increasing energy demand.
- Optimal utilization of distributed energy resources and grid infrastructure requires **coordination of Power Switching Interfaces** (PSI) connecting sources to grid.
- In addition to their local duties (power flow control, voltage support), such interfaces may perform **global duties** too (distribution efficiency, power factor at PCC).

Role of Electronic Power Processors

- Optimize exploitation of energy sources
  - Power flow control
- Improve power quality at PCC
  - Reactive power compensation
  - Unbalance compensation
  - Harmonics mitigation
- Improve power quality at load terminals
  - Voltage stabilization
  - Limitation of voltage distortion
- Improve system efficiency
  - Minimization of distribution losses associated to useless current terms flowing in distribution grid

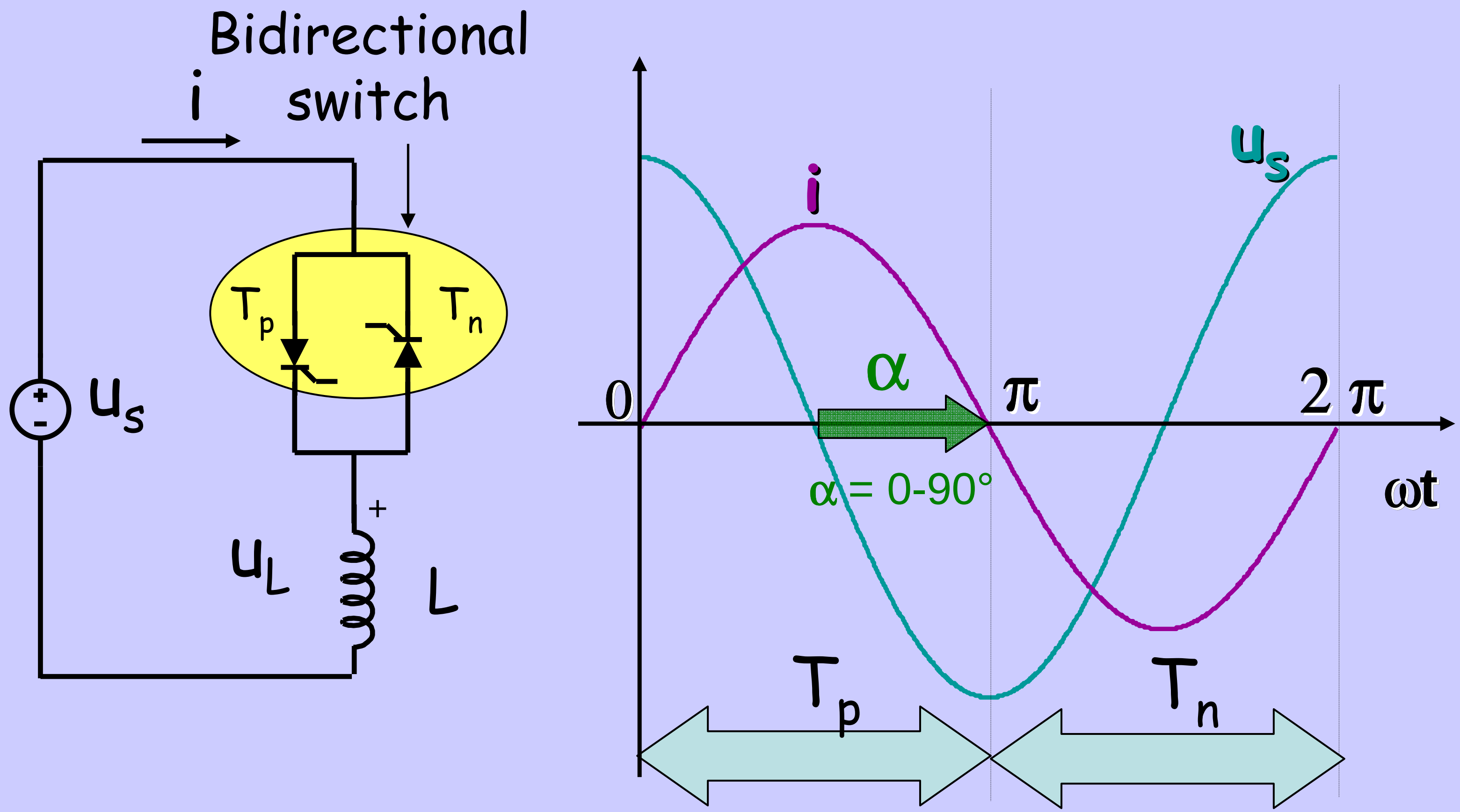
EPP Types (1)

- Smart grids may include a variety of Electronic Power Processors (EPP)
- Quasi-stationary EPP
  - Static VAR Compensators (SVC), which provide controllable amount of reactive power
  - TCR (thyristor-controlled reactors)
  - TSC (thyristor-switched capacitors)
  - STATCOM (low-frequency switching converters)

- **Quite cheap**
- **Unbalance and reactive power compensation**

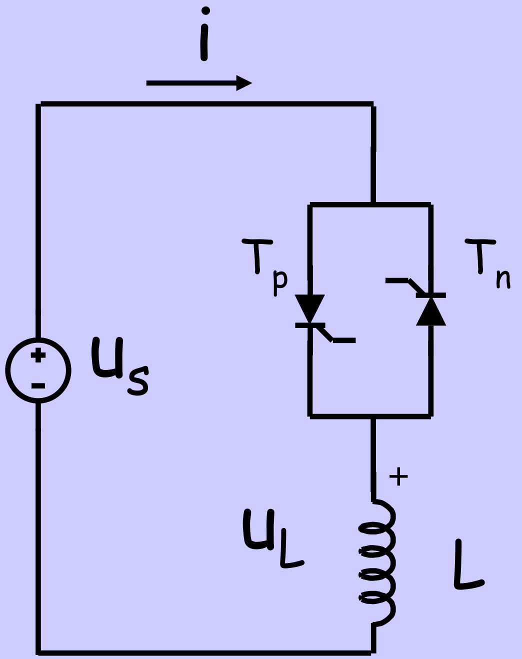
- **Harmonic injection at connection point**
- **Slow regulation**

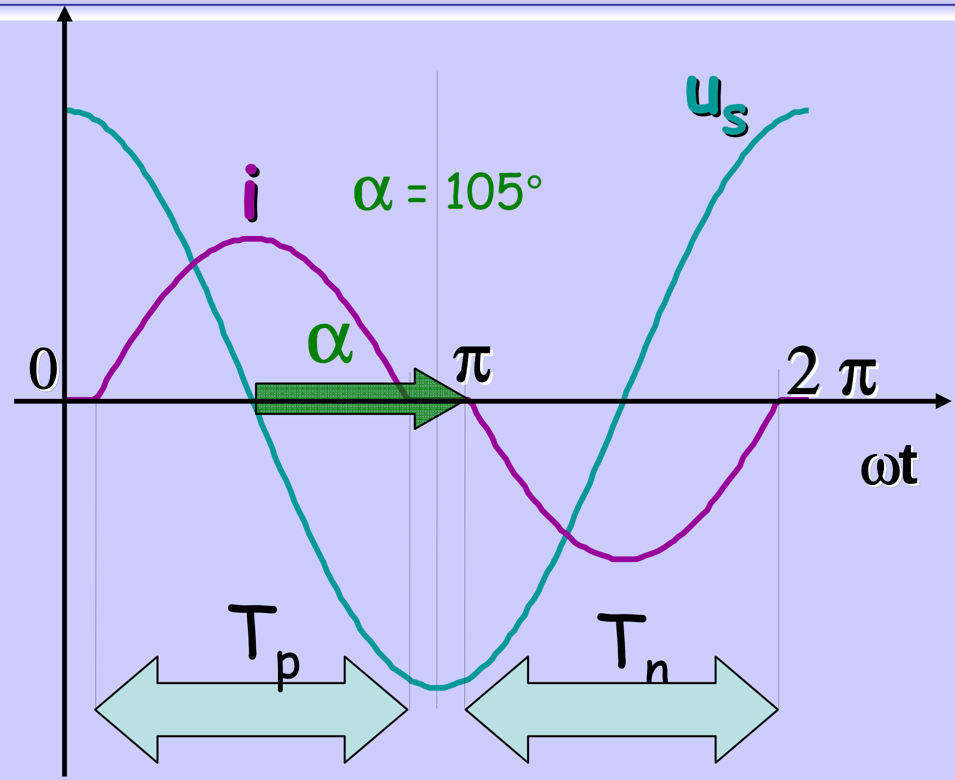
Example of TCR



When α = 0-90° a sinusoidal current is obtained

Example of TCR



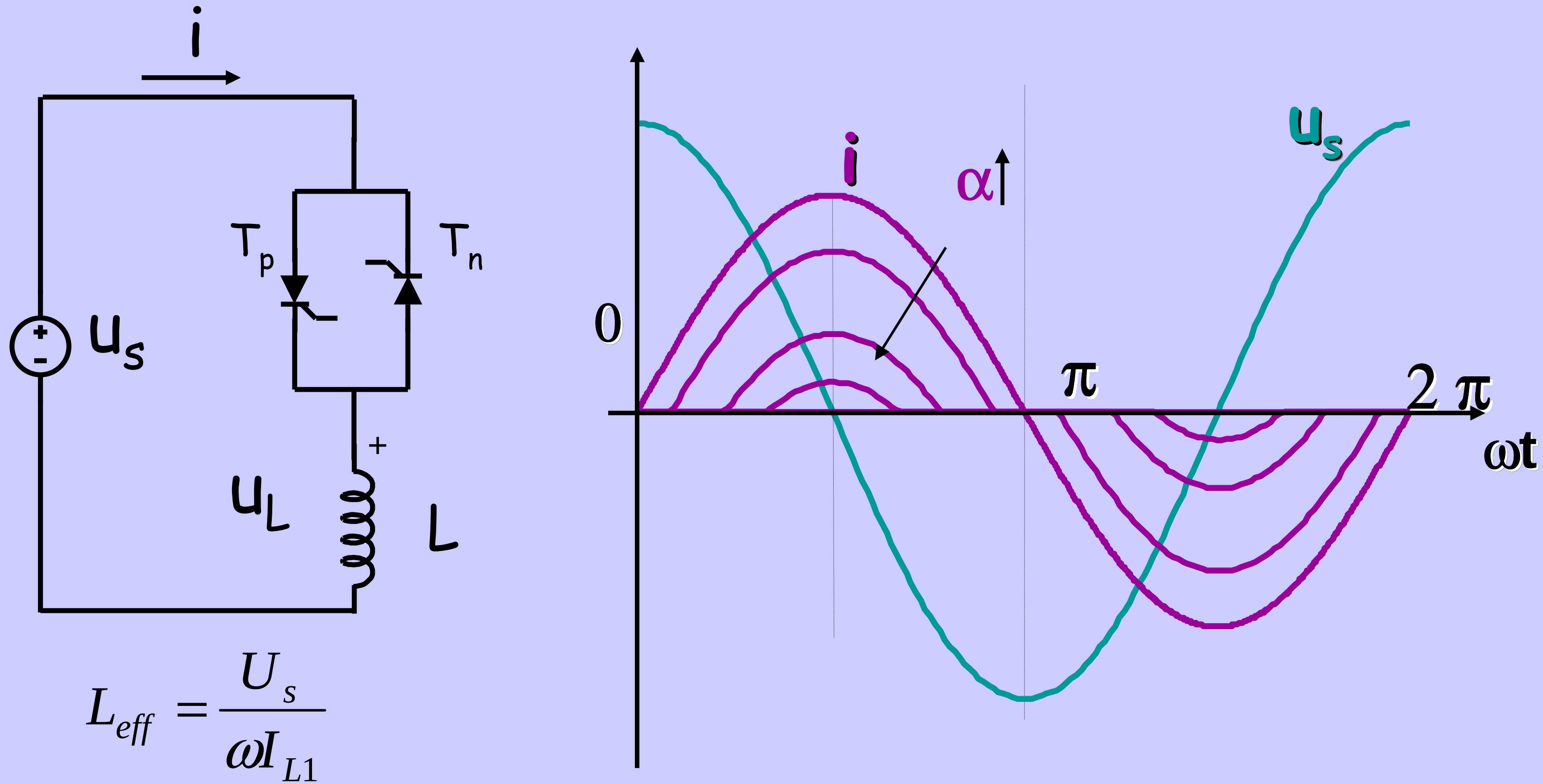


$$I_{L1} = \frac{U_s}{\pi\omega L}(2\pi - 2\alpha + \sin 2\alpha), \quad \alpha \in [\frac{\pi}{2}, \pi]$$

$$L_{eff} = \frac{U_s}{\omega I_{L1}}$$

When $\alpha > 90°$ odd current harmonics are generated

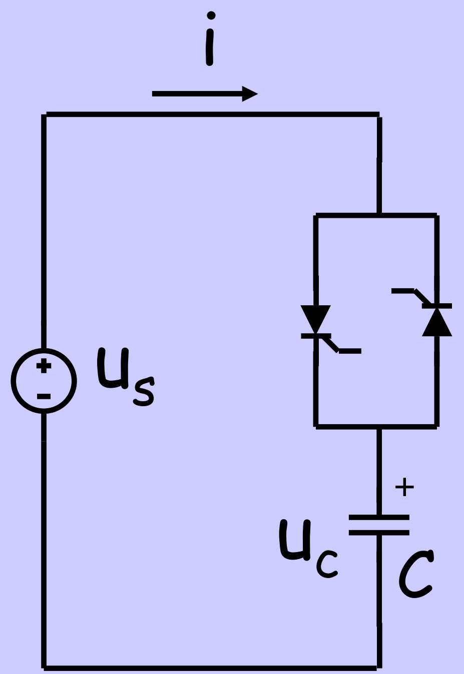
Example of TCR



$$L_{eff} = \frac{U_s}{\omega I_{L1}}$$

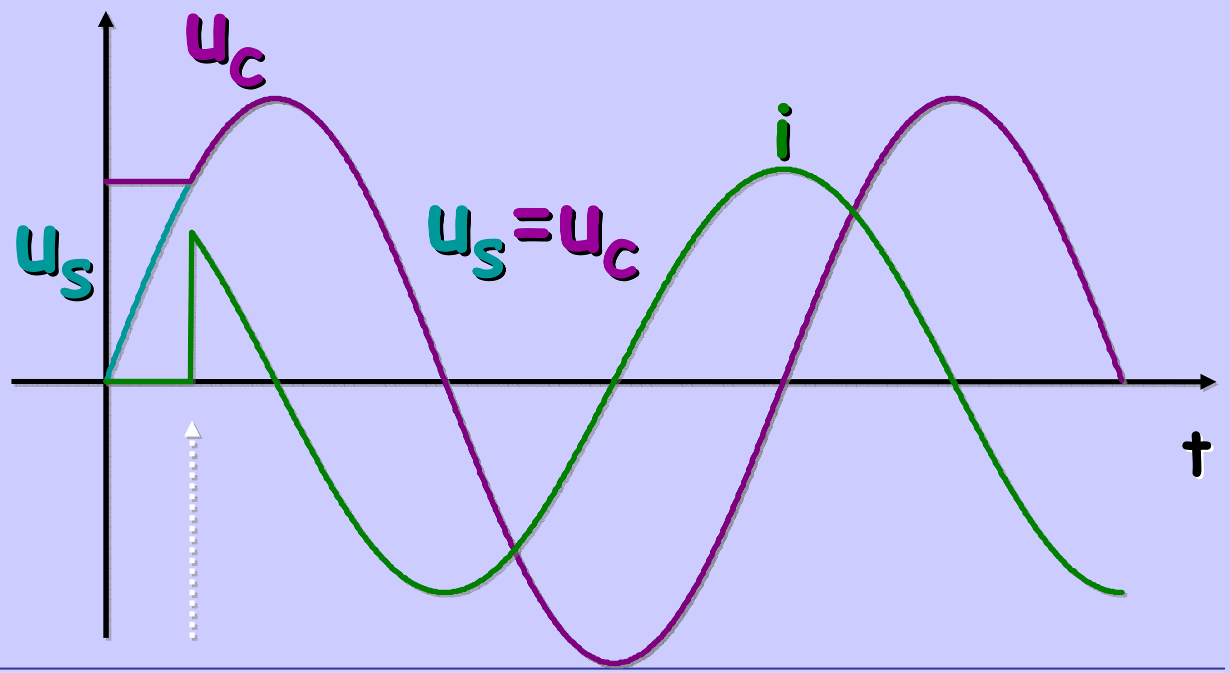
Firing angle α controls the current fundamental component

Example of TSC

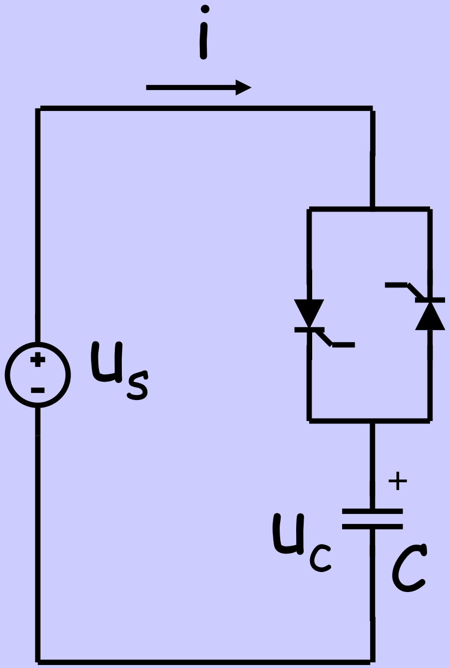


$$i = C\frac{du_C}{dt}$$

SCR must be fired only when $u_s = u_c$, so as to avoid dangerous current pulses

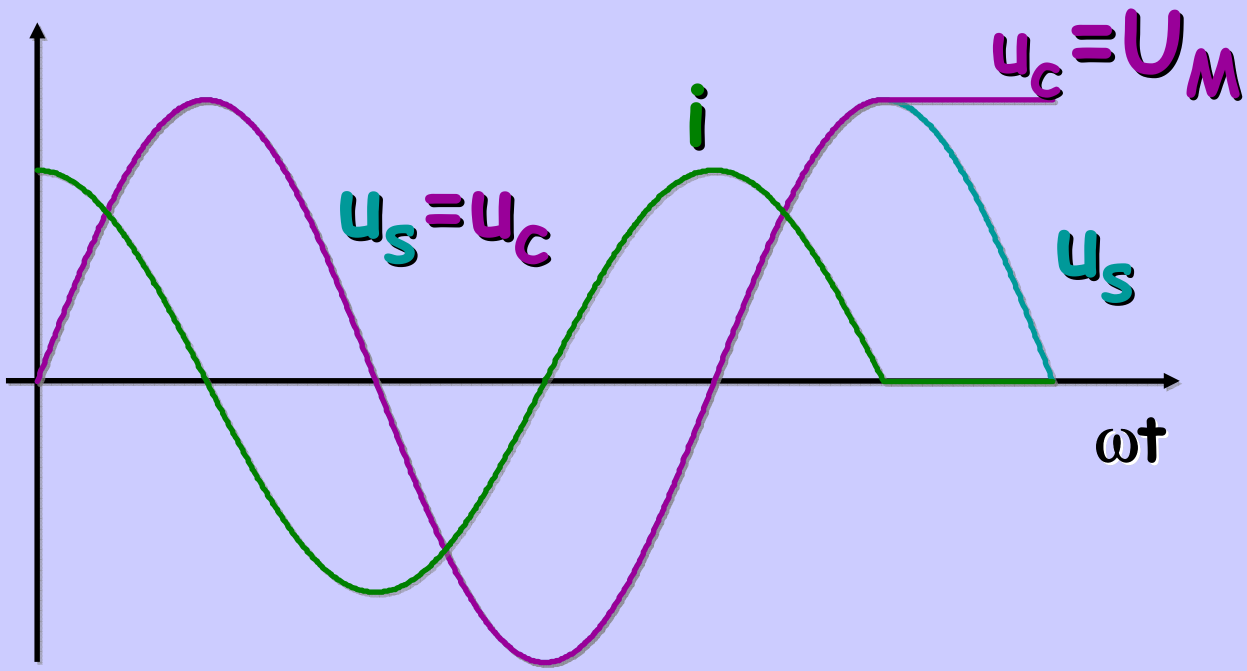


Example of TSC

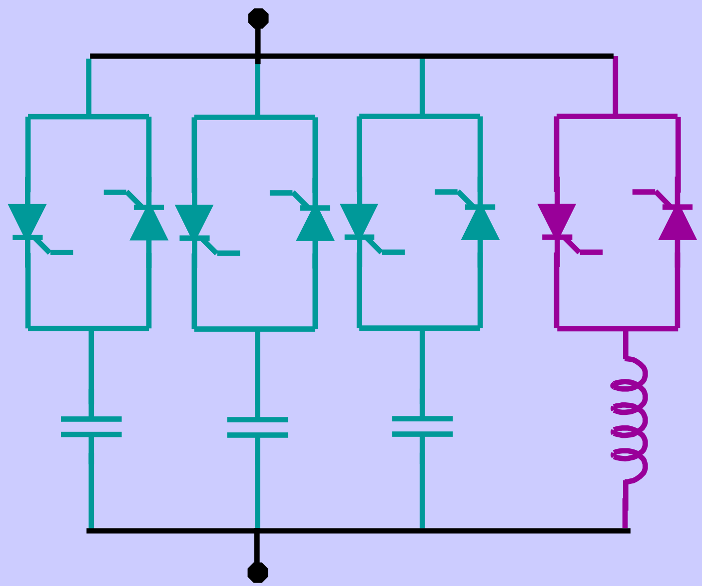


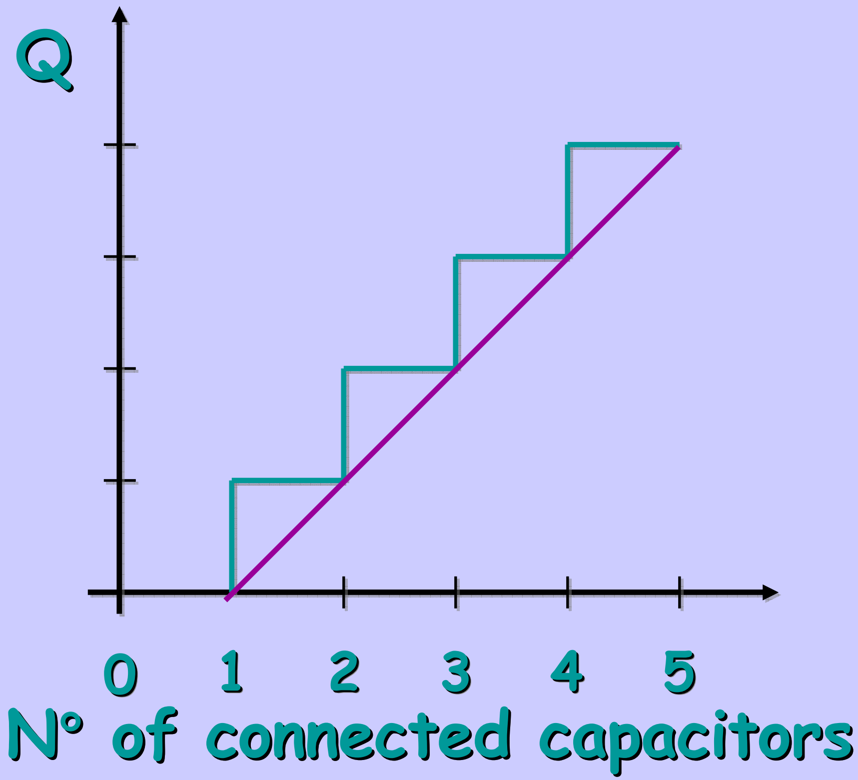
$$i = 0$$

The turn off occurs at the current zero crossing



Ibrid compensators





Combining TCR and TSC, a continuous regulation of reactive power is achieved

EPP Types (2)

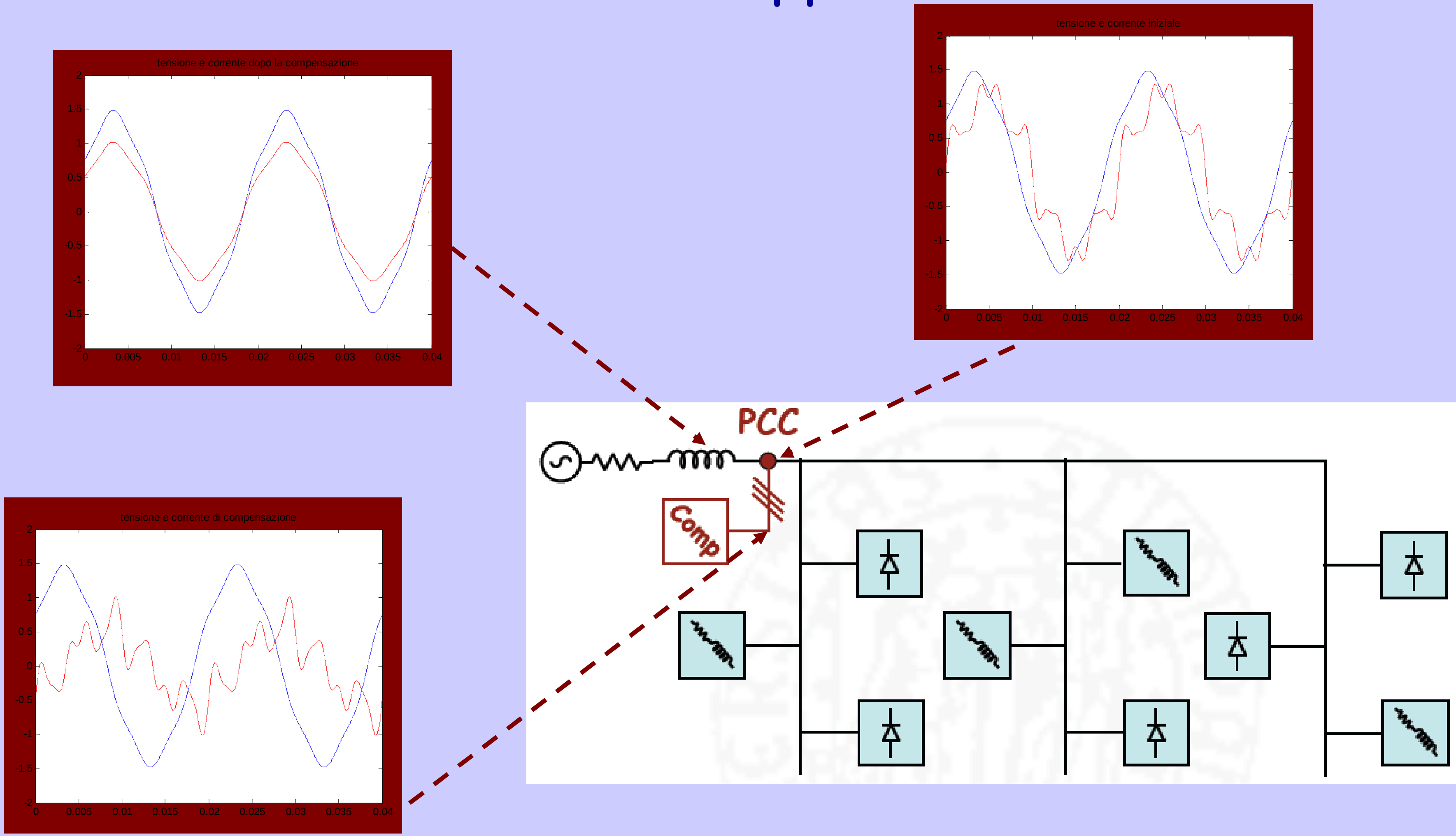
- Smart grids may include a variety of Electronic Power Processors (EPP)
- Dynamic EPP
  - Switching Power Compensators (SPC), which can compensate for transient and/or high-frequency current components absorbed by time-varying and distorting loads
  - APF (Active Power Filters)

- Fast control
- Compensation of unbalanced & reactive loads and harmonic distortion.

- Expensive (APF)

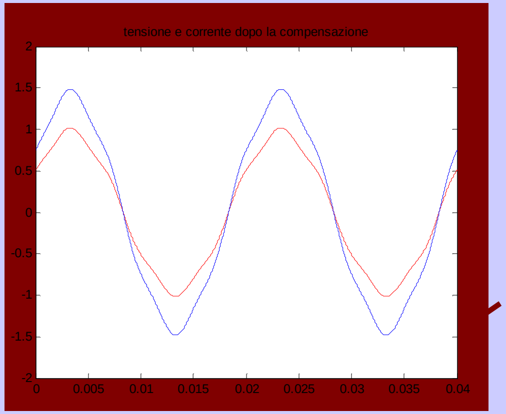
Example: Improving PQ at PCC

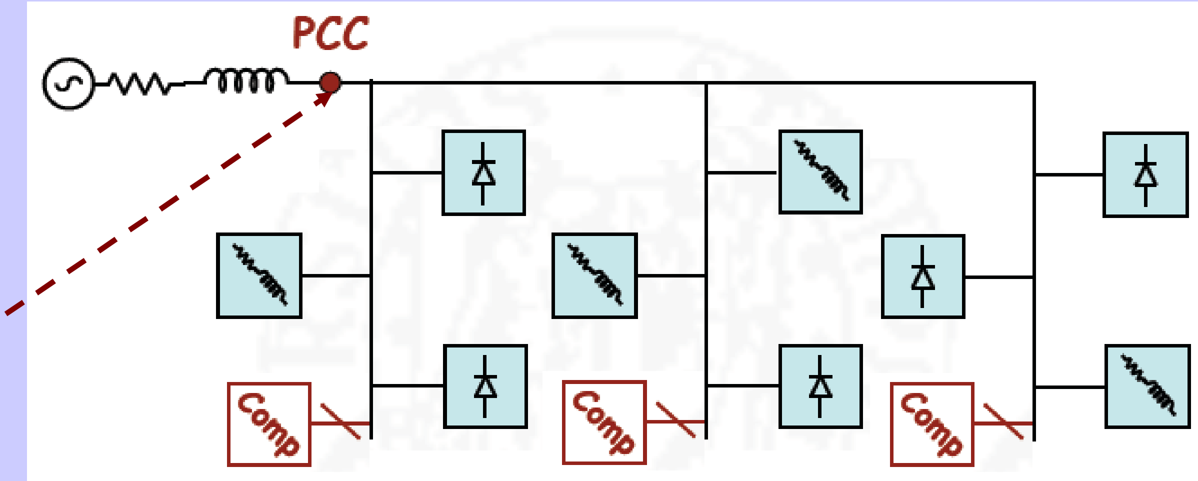
An ideal approach



Example: Improving PQ at PCC

Distributed control approach





Problem

Need to control cooperatively EPPs of any kind, connected at various network terminals, fed by different voltages (amplitude and phase shift induced by transformers) and affected by voltage and current distortion

Cooperative control of distributed EPP

Advantages:

- Full exploitation of distributed energy resources
- Improved energy efficiency and power quality
- Sharing of control duties among EPPs according to global and local (glocal) optimization strategies
- Full exploitation of EPPs, avoiding redundancy and over-rating
- Control of network dynamics
- Improvement of network stability
- Elimination of unwanted interactions

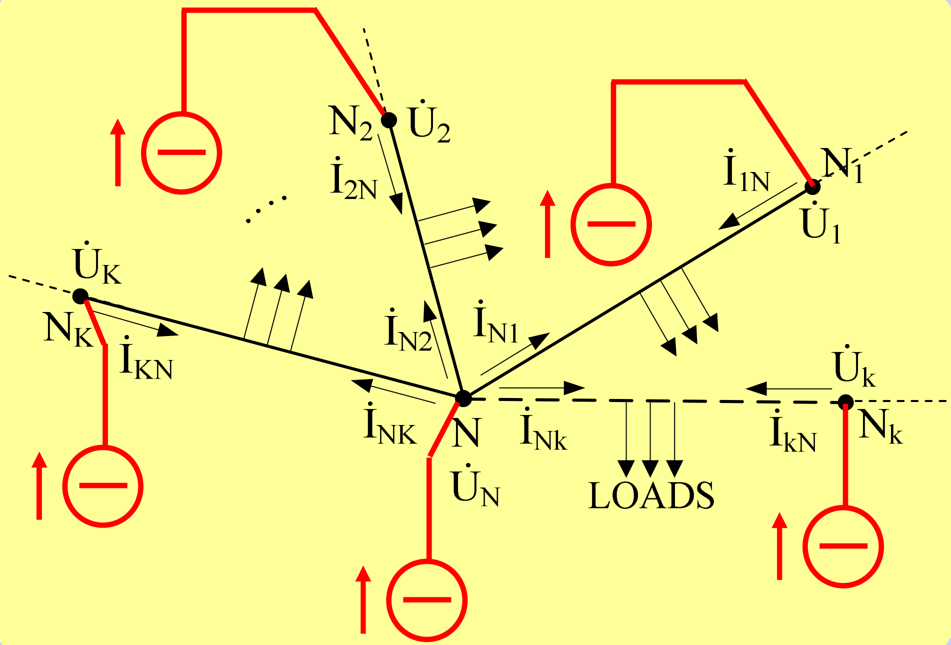
Example of control of DPP in micro-grids

(DPP: Distributed Power Processors)

Assumptions:

- **Grid nodes coincide with the power meters**, equipped by suitable measurement, synchronization and communication capability
  - Active grid nodes correspond to prosumers, i.e., buildings or residential settlements equipped with energy sources and power processors (inverters) capable to control the active and reactive power fed to the grid
  - Passive grid nodes correspond to traditional consumers
- A **distributed plug & play control** is envisaged, to ensure flexibility and scalability of the micro-grid
- Communication occurs only among neighbor nodes
- Though not needed, a central controller can also be implemented to manage the utility interface at the point of common coupling

Example: tokeng ring control



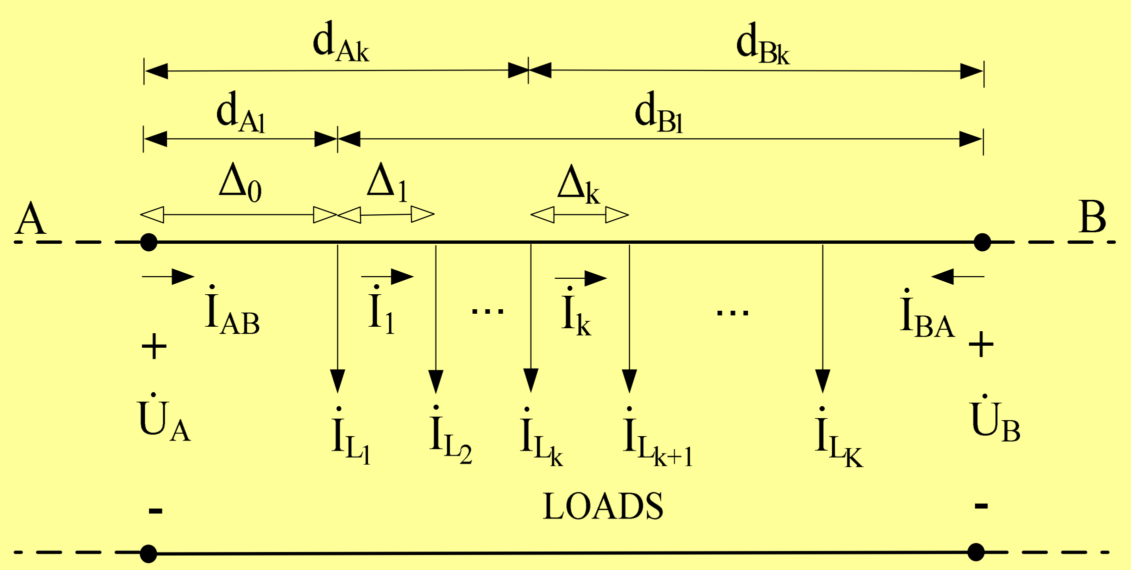
DPPs operate as current sources
(to stabilize grid impedances)

- DPPs cyclically update their current references (**control phase**).
- Outside the control phase, DPPs keep constant current references (**hold phase**).
- When a DPP is in the control phase, the neighbors keep the hold phase. This prevents detrimental control interactions.

Minimization of distribution losses

Optimization goal: find the values of I_{AB} and I_{BA} that minimize conduction losses in path A-B

$$\left(\frac{\partial P_{LOSS}}{\partial \dot{I}_{AB}}, \frac{\partial P_{LOSS}}{\partial \dot{I}_{BA}}\right) = (0,0)$$



$$\begin{cases} \dot{I}_{AB}^{opt} = \dfrac{1}{d_{AB}} \displaystyle\sum_{k=1}^{K} \dot{I}_{Lk} d_{Bk} \\ \dot{I}_{BA}^{opt} = \dfrac{1}{d_{AB}} \displaystyle\sum_{k=1}^{K} \dot{I}_{Lk} d_{Ak} \end{cases}$$

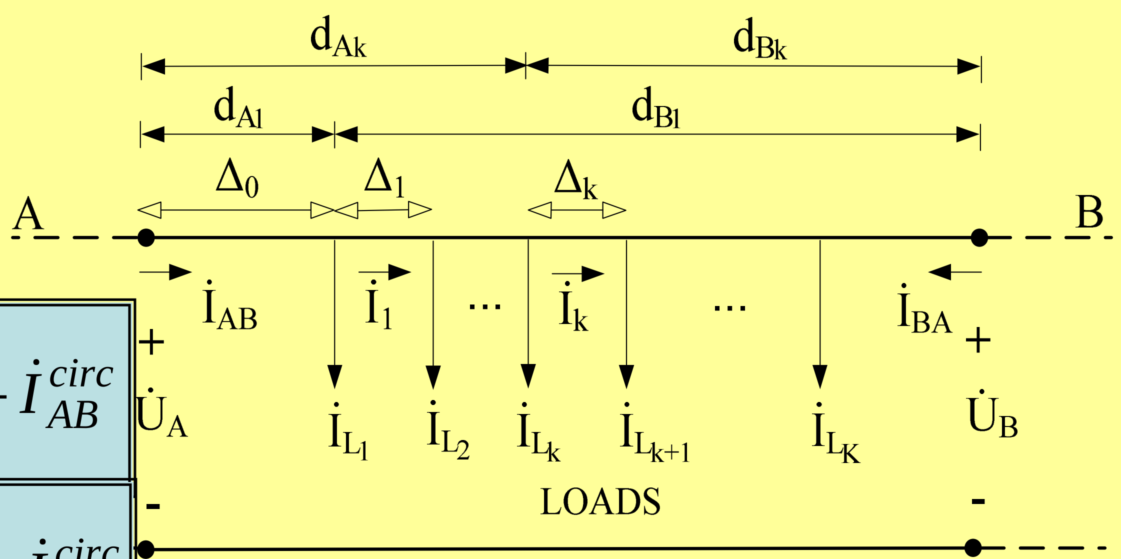
The optimum node currents depend only on the loads and their distribution along path A-B

Moreover:

$$\begin{cases} \dot{I}_{AB} = \dot{I}_{AB}^{opt} \\ \dot{I}_{BA} = \dot{I}_{BA}^{opt} \end{cases} \Leftrightarrow \quad \dot{U}_A = \dot{U}_B$$

Minimization of distribution losses

In general, nodes A and B are not equipotential, thus:



$$\dot{I}_{AB} = \dot{I}_{AB}^{opt} + \frac{\dot{U}_A - \dot{U}_B}{\dot{z}\, d_{AB}} = \dot{I}_{AB}^{opt} + \dot{I}_{AB}^{circ}$$

$$\dot{I}_{BA} = \dot{I}_{BA}^{opt} + \frac{\dot{U}_B - \dot{U}_A}{\dot{z}\, d_{AB}} = \dot{I}_{BA}^{opt} + \dot{I}_{BA}^{circ}$$

Impedance per unit of length of distribution line

Optimum current

Circulation current

Minimization of distribution losses

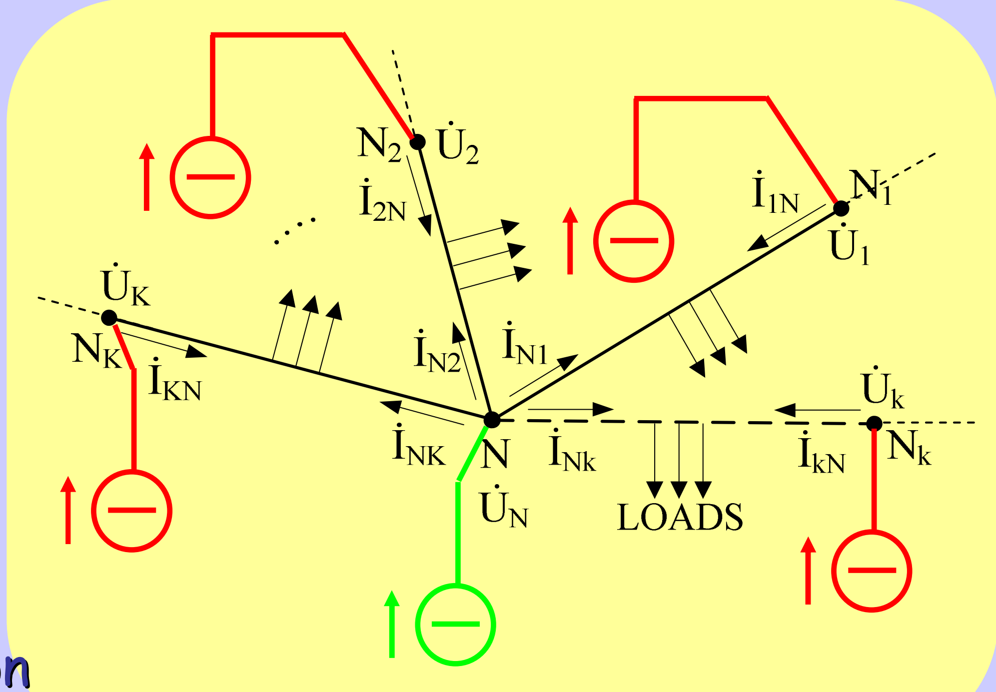
Node current and voltage optimization

Current at node N:

$$\dot{I}_N = \sum_{k=1}^{K} \dot{I}_{Nk} = \underbrace{\sum_{k=1}^{K} \dot{I}_{Nk}^{opt}}_{\dot{I}_N^{opt}} + \underbrace{\sum_{k=1}^{K} \frac{\dot{U}_N - \dot{U}_k}{\dot{Z}_k}}_{\dot{I}_N^{circ}}$$

Depends on loads connected to paths L_1 - L_K

Depends on voltage differences



Micro-grid cut set

Minimum distribution loss condition

$$\dot{I}_N^{circ} = 0 \quad \Rightarrow \quad \begin{cases} \dot{I}_N = \dot{I}_N^{opt} \\ \dot{U}_N = \dot{U}_N^{opt} = \sum_{k=1}^{K} \dfrac{\dot{U}_k}{\dot{Z}_k} \Big/ \sum_{k=1}^{K} \dfrac{1}{\dot{Z}_k} \end{cases}$$

DPP in control phase

DPPs in hold phase

Plug & Play surround control

$$\dot{I}_N = \dot{I}_N^{opt} = \sum_{k=1}^{K} \dot{I}_{Nk}^{opt} = \sum_{k=1}^{K} \frac{1}{d_{Nk}} \sum_{m=1}^{M_{Nk}} \dot{I}_{L_{Nk}} d_{k_{Nk}}$$

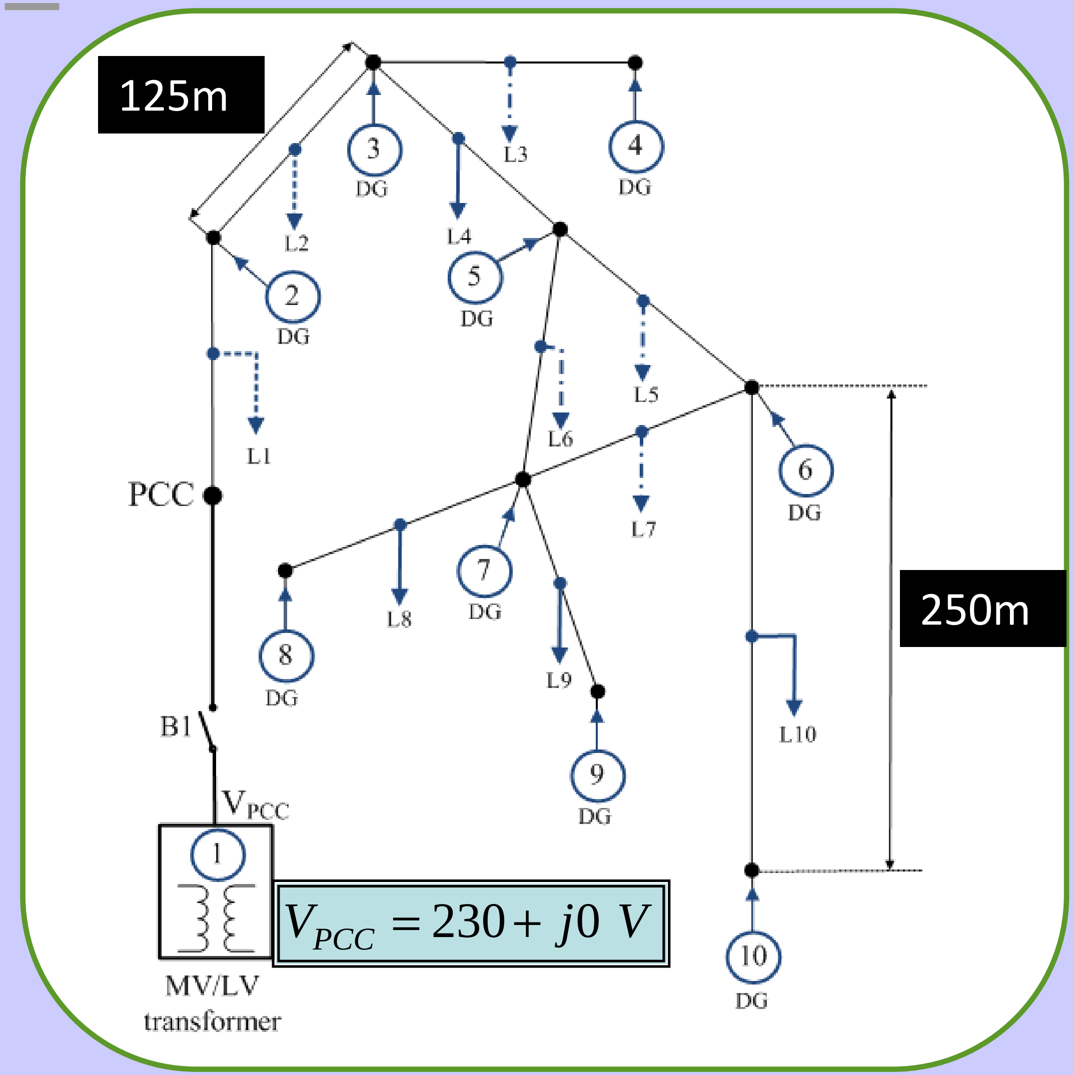
This equation holds separately for active and reactive terms, thus optimization can be done by acting on active currents, reactive currents, or both

Computation of **optimum current reference** requires *distance estimation* (ranging), *local grid mapping*, and measurement of currents at surrounding passive nodes)

$$\dot{U}_N = \dot{U}_N^{opt} = \frac{\sum_{k=1}^{K} \frac{\dot{U}_k}{\dot{Z}_k}}{\sum_{k=1}^{K} \frac{1}{\dot{Z}_k}} \approx \frac{\sum_{k=1}^{K} \frac{\dot{U}_k}{d_k}}{\sum_{k=1}^{K} \frac{1}{d_k}}$$

Computation of **optimum voltage reference** requires *local grid mapping*, knowledge of *path impedances* (or node distances), and measurement of voltages at surrounding active nodes

Application example



LOADS

- P_{L1}=5kW at $\cos\varphi_{L1}$=0.91
- P_{L2}=10kW at $\cos\varphi_{L2}$=0.8
- P_{L3}=20kW at $\cos\varphi_{L3}$=0.75

P_{TOT}=100kW at $\cos\varphi_{TOT}$=0.8

DISTRIBUTED GENERATORS

9 distributed generators rated for **3kW** each with DPPs (Distributed Power Processors) rated for **6kVA**

DISTRIBUTION LINE

r=0.0348 Ω/km

l=275 μH/km

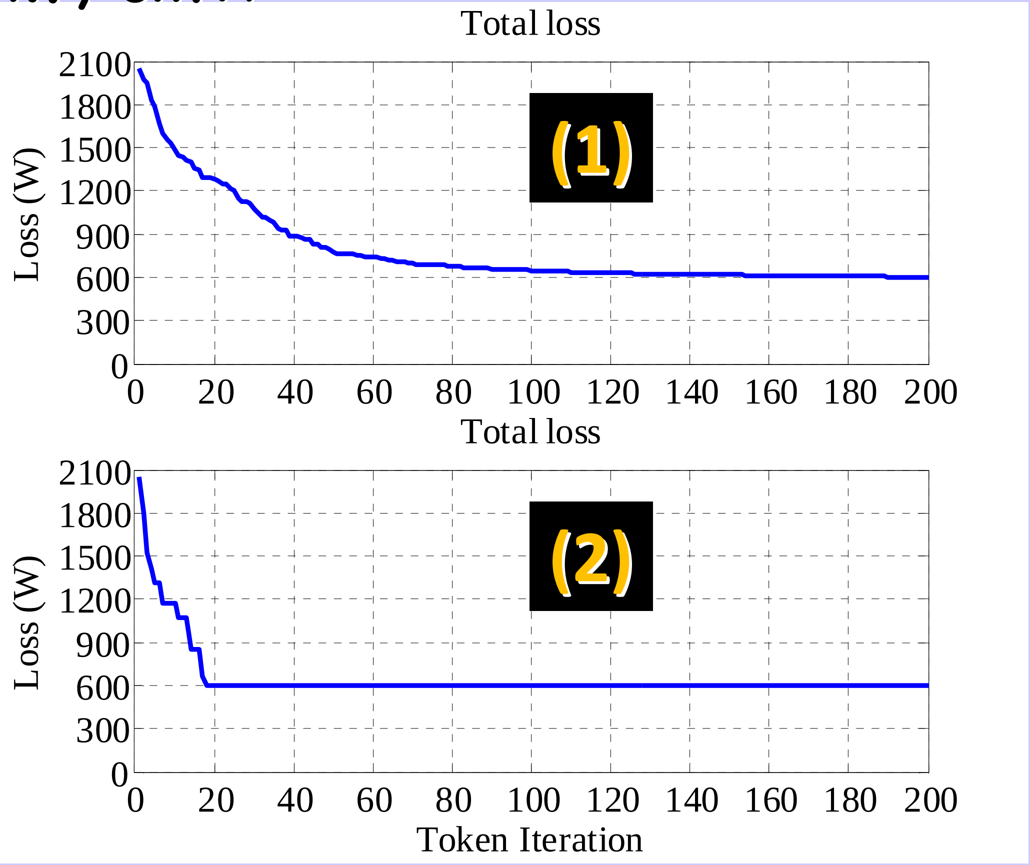
Application example

Comparison between Optimum current-mode control (2) and Optimum voltage-mode control (1)

Active and reactive compensation with DGs current capability limited to rated power - 3kW, 6kVA

Loss reduced from 2100 W to 600 W

70% total loss reduction



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Photovoltaic Effect

- It is based on the generation of electron-holes pairs in a semiconductor material illuminated by solar light.
- A typical silicon photovoltaic cell produces an open circuit voltage around **0.6-0.7 V** with a short-circuit current density in the order of **0.5-0.6 mA/mm^2**.
- A photovoltaic module is composed by series connection of many photovoltaic cells (e.g. 36 or 72)

Mathematical Model of a Photovoltaic Cell

- I-V relation:

$$I_{cell} = I_{ph} - I_o\left(e^{\frac{qV_{cell}}{nkT}} - 1\right)$$

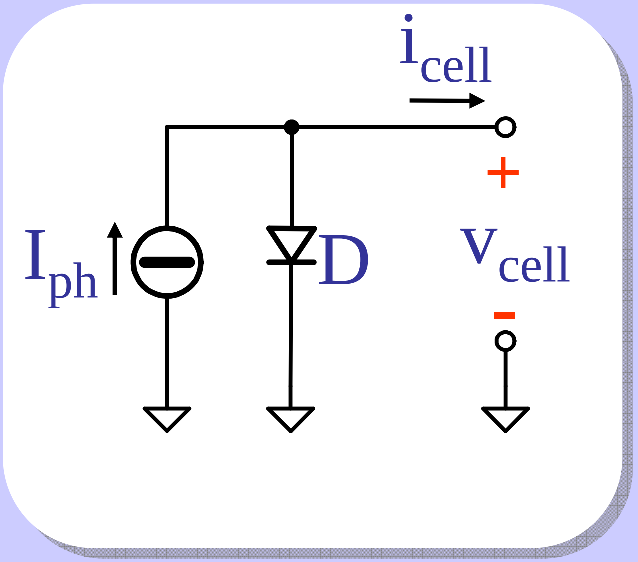
where:

$$I_{ph} = I_{phD} + I_{phn} + I_{php}$$

is the sum of the photo-generated currents in three different semiconductor regions (p- and n-doped regions as well as depletion region), and *n* is the ideality factor (value between **1** and **2**).

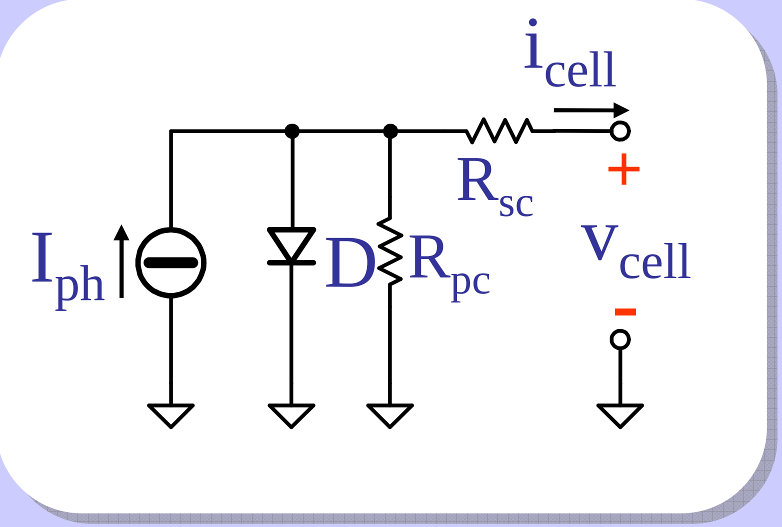
Equivalent Electric Model

$$I_{cell} = I_{ph} - I_o\left(e^{\frac{qV_{cell}}{nkT}} - 1\right)$$

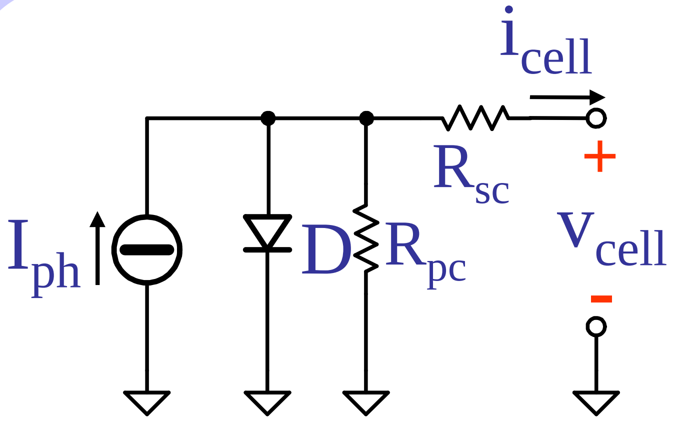


Equivalent Electric Model

- Dissipative phenomena:
  - Non negligible resistivity of quasi-neutral regions as well as of metal contacts (modeled with a series resistance in the range **2-20 mΩ**);
  - Internal recombination of photo-generated hole-electron pairs (modeled with a shunt resistance in the range **0.5-5 Ω**). This effect is often neglected!



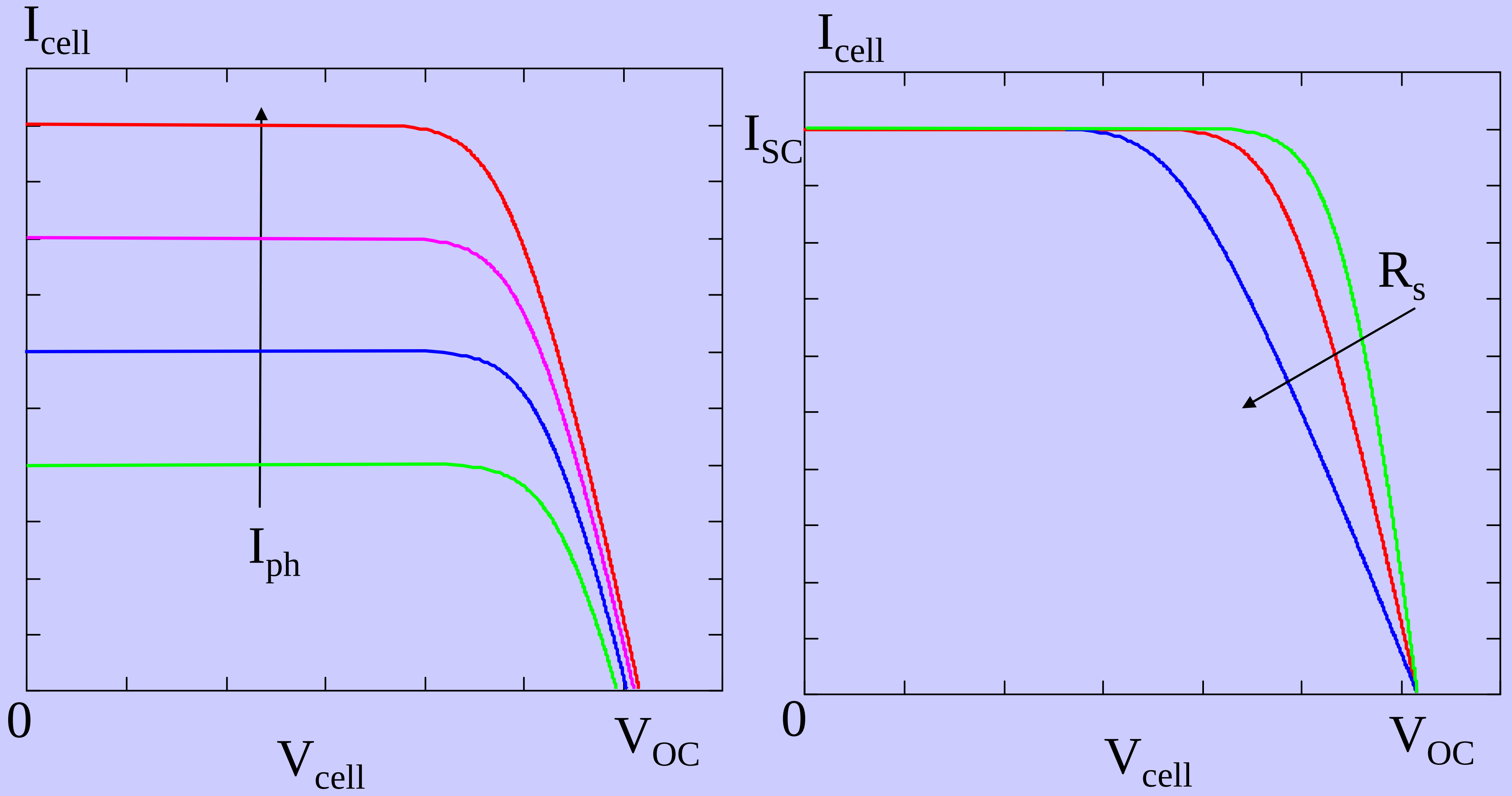
Equivalent Electric Model



- The I-V relation becomes:

$$I_{cell} = I_{ph} - I_o\left(e^{\frac{q(V_{cell}+R_{sc}I_{cell})}{nkT}} - 1 \right) - \frac{V_{cell} + R_{sc}I_{cell}}{R_{pc}}$$

I-V Characteristic



I-V Characteristic

- Fundamental parameters:
  - Short circuit current I_{SC} (function of the luminous flux);
  - Open circuit voltage V_{OC} (depends mainly on temperature);

$$I_{SC} = I_{cell}\Big/_{V_{cell}=0} = I_{ph} - I_o\left(e^{\frac{qR_{sc}I_{SC}}{nkT}} - 1\right) - \frac{R_{sp}I_{SC}}{R_{pc}} \approx I_{ph}$$

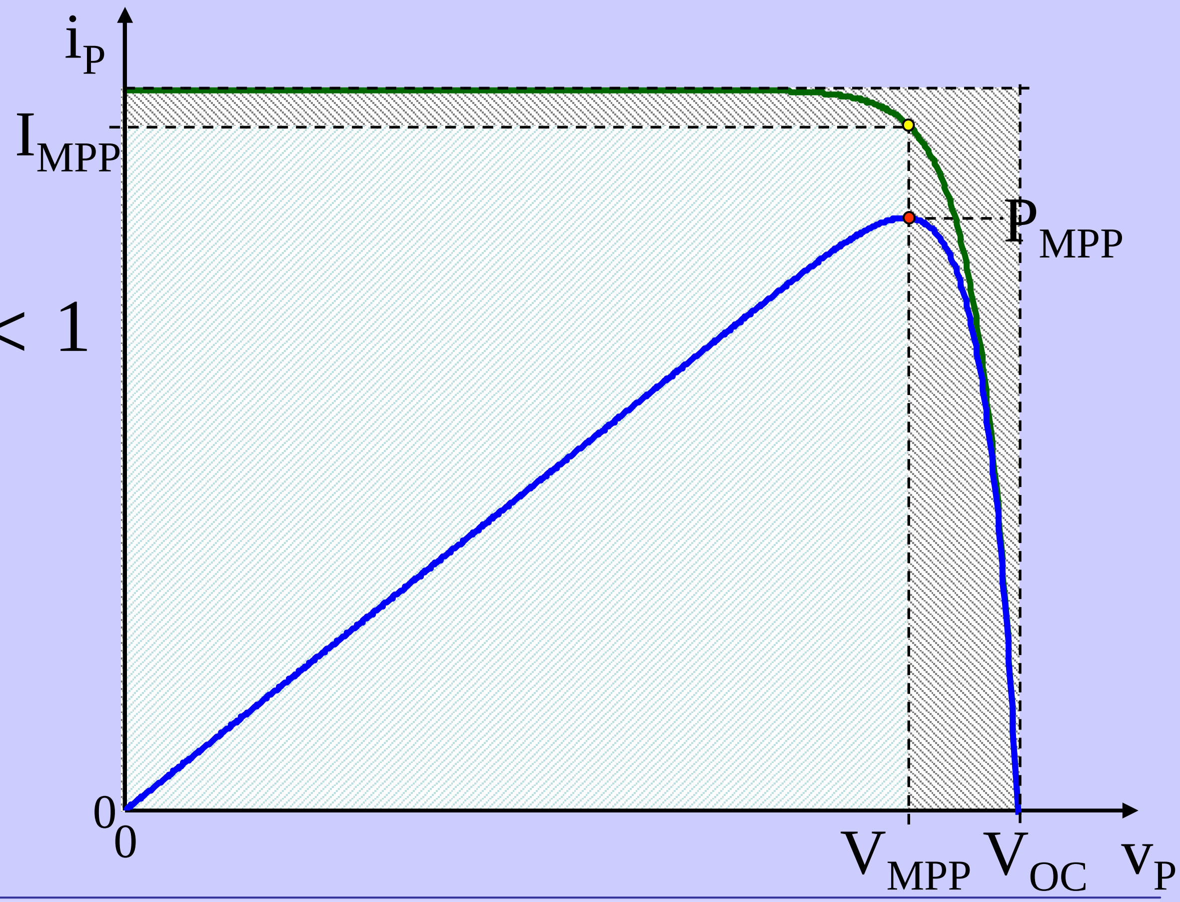
$$V_{OC} = V_{cell}\Big/_{I_{cell}=0} \approx \frac{kT}{q}\ln\left(\frac{I_{ph}+I_o}{I_o}\right)$$

I-V Characteristic

- Fundamental parameters:
  - Fill factor;
  - Conversion *efficiency* (R = radiation, W/m^2, A = area, m^2);

$$FF = \frac{V_{MPP}\ I_{MPP}}{V_{OC}\ I_{SC}} < 1$$

$$\eta = \frac{P_{MPP}}{R \cdot A}$$

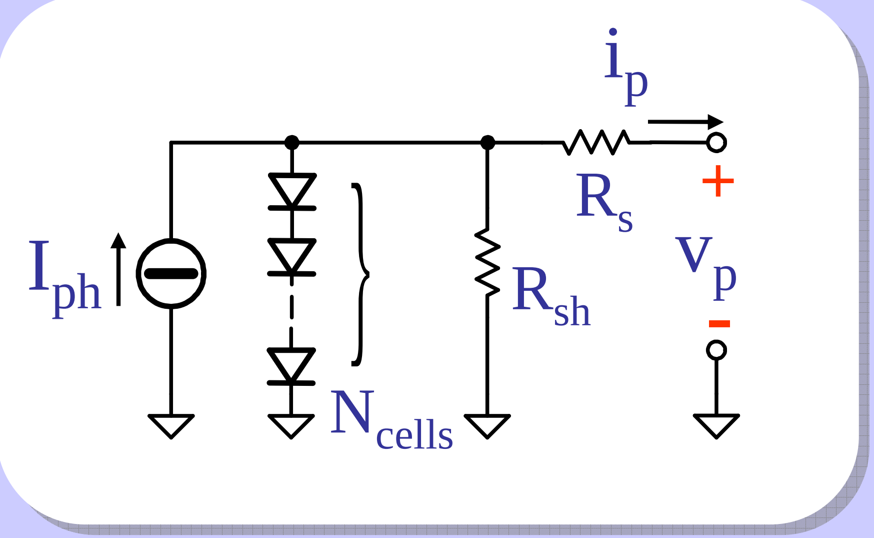


Photovoltaic Module

- A photovoltaic module is, in general, composed by the series connection of N cells ($R_s = N_{cells} \cdot R_{sc}$, and $R_{sh} = N_{cells} \cdot R_{pc}$):

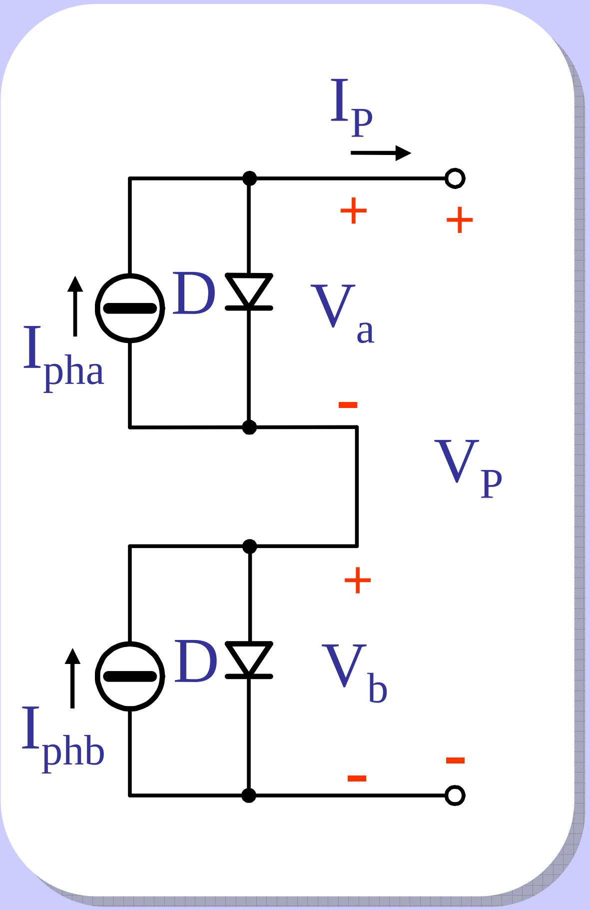
$$V_P = \sum_{i=1}^{N_{cells}} V_{cell}$$

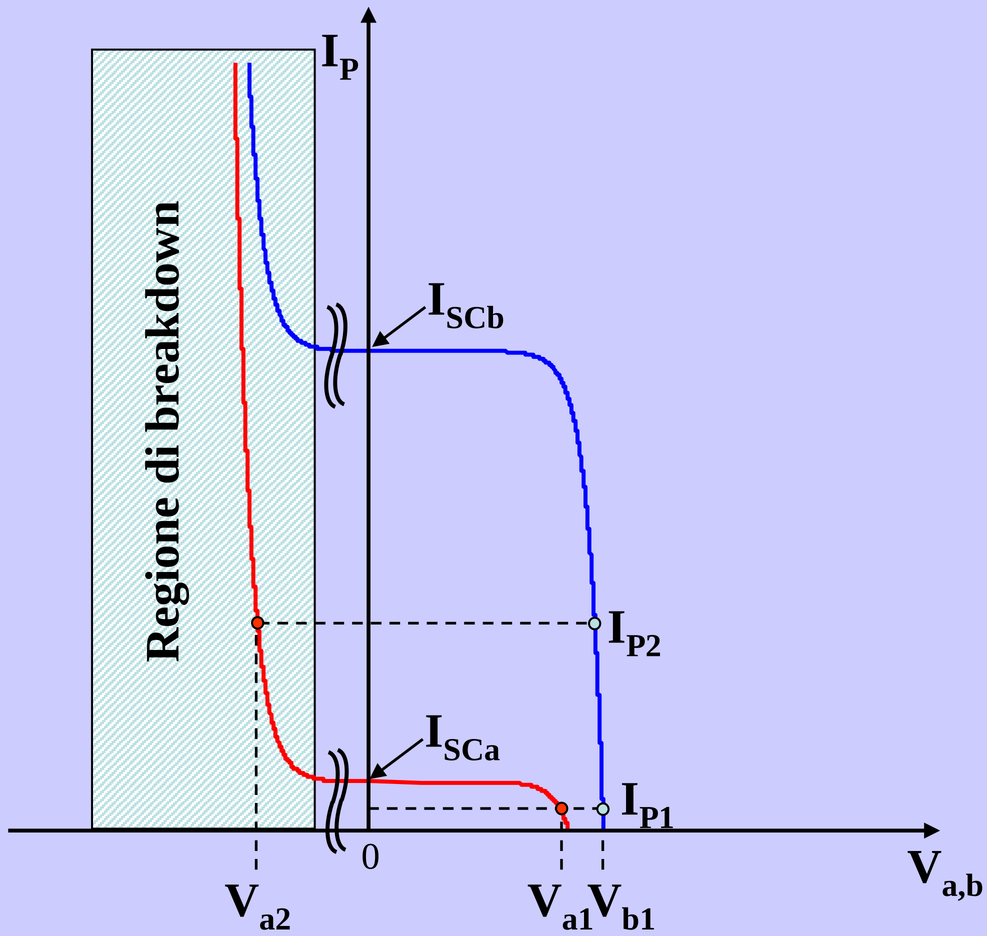
$$I_P = I_{cell}$$



Shadowing

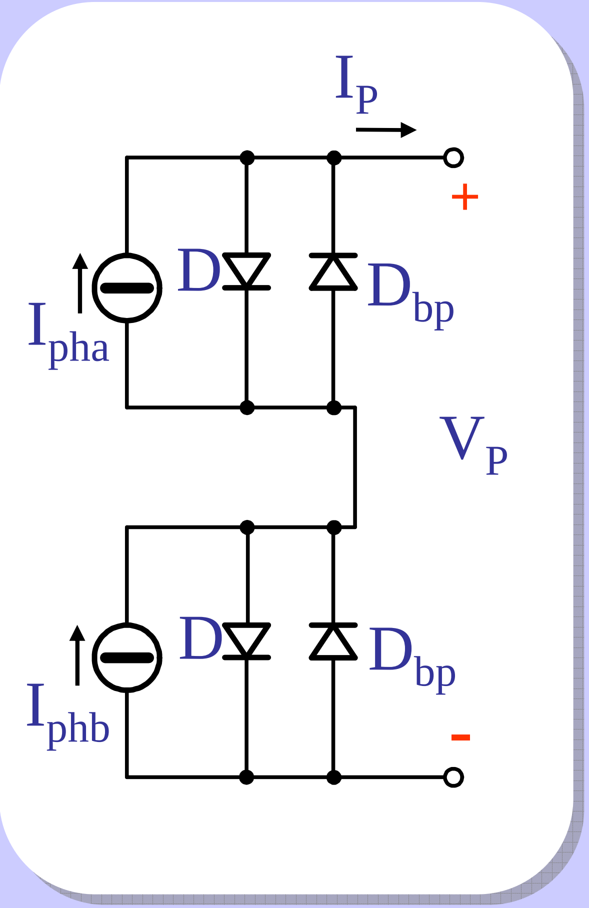
- $I_{pha} \neq I_{phb}$
- At operating point P_2, cell *a* behaves as a dissipative load

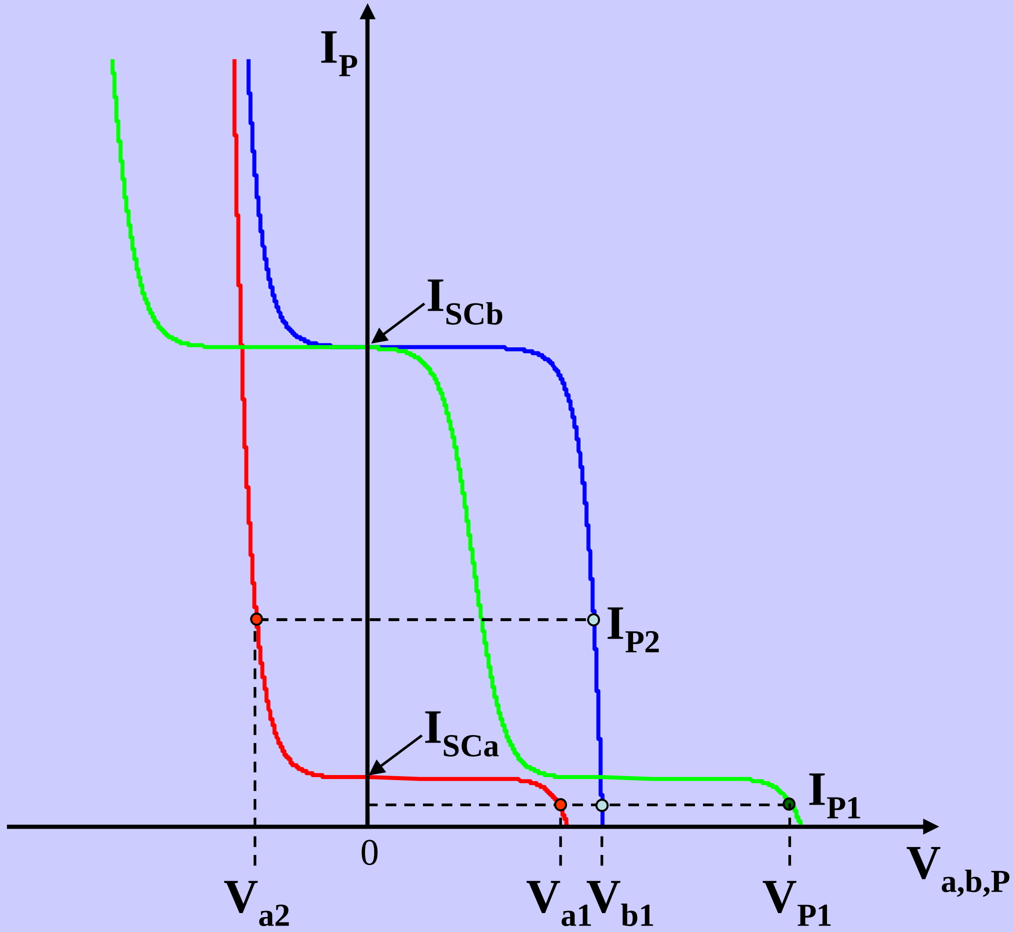




Shadowing: Using Bypass Diodes

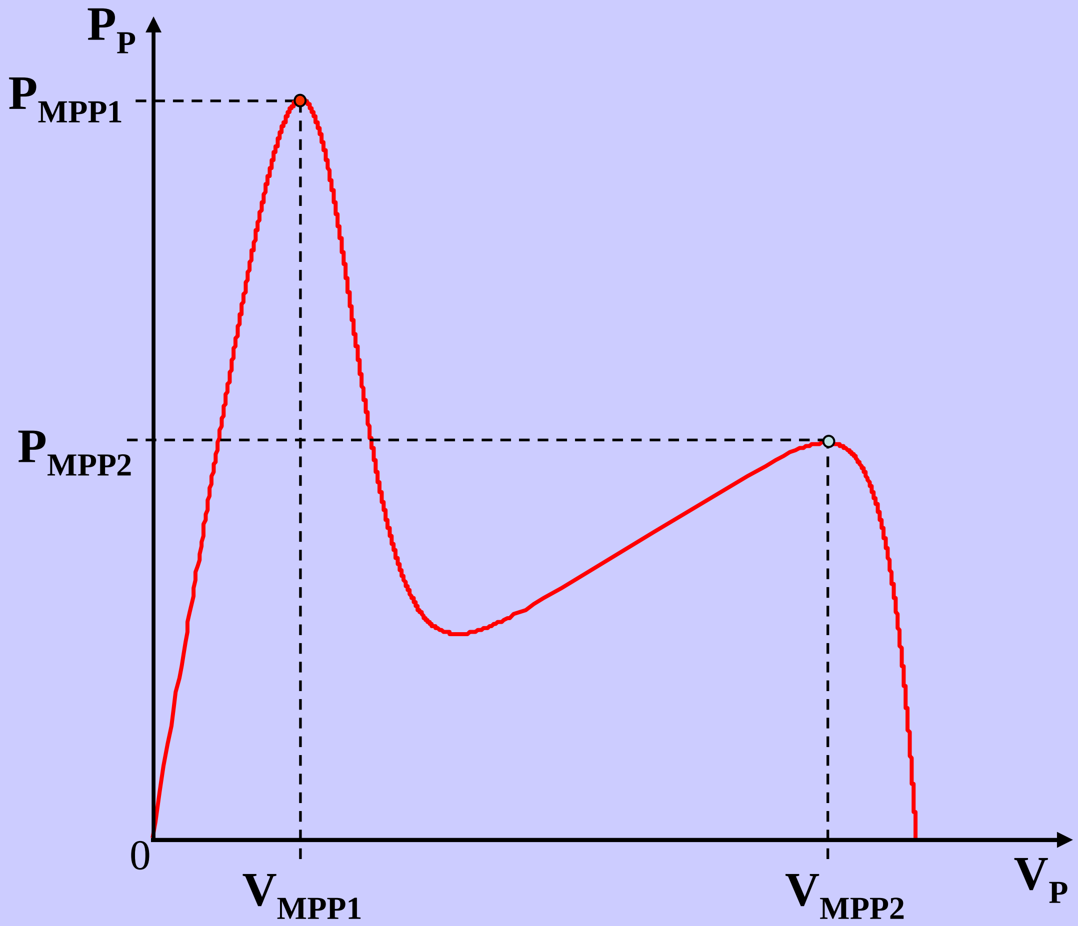
- $I_{pha} \neq I_{phb}$





Multiple Maxima

- Power to voltage characteristic: multiple maxima



Commercial Photovoltaic Modules

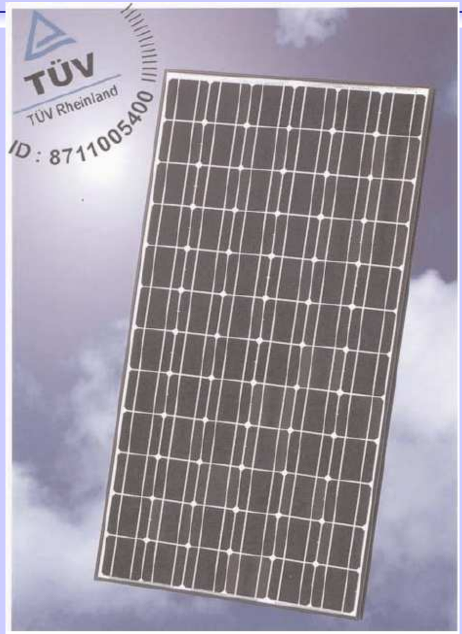
Sanyo **HIP210 module**

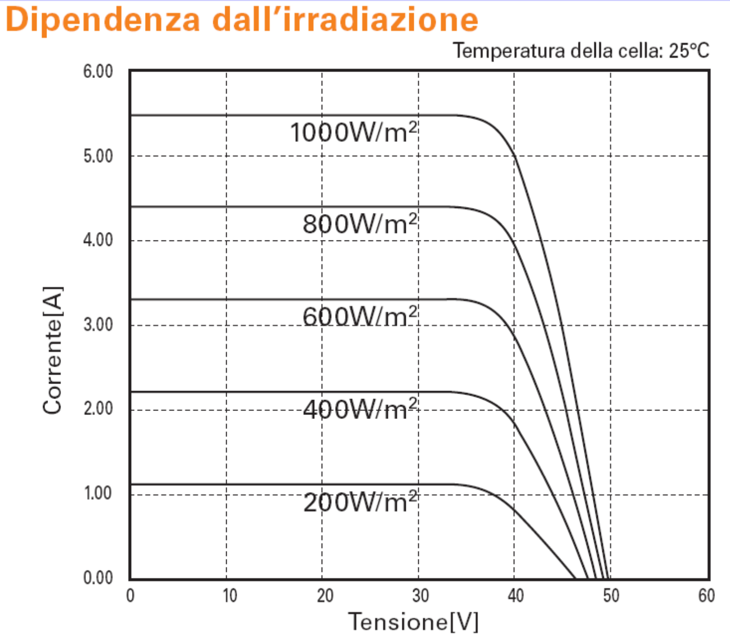
72 cells series connected

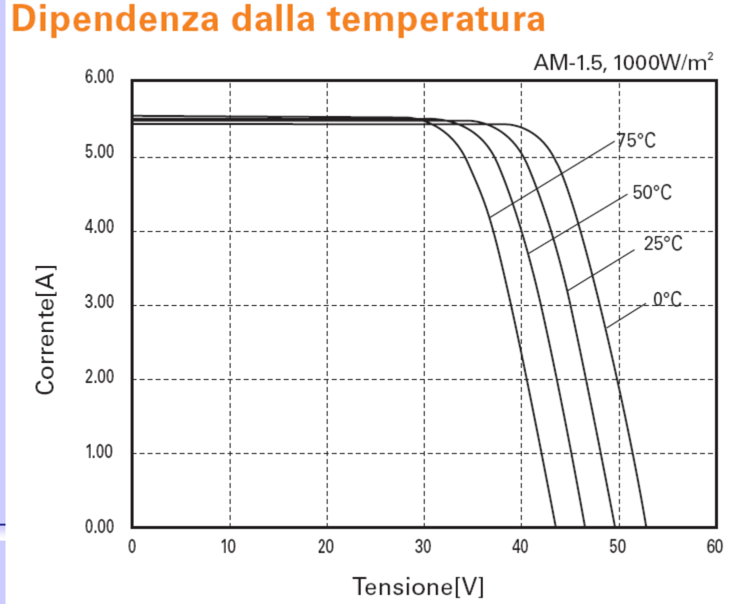
V_{oc} = 50.9 V

I_{sc} = 5.57 A

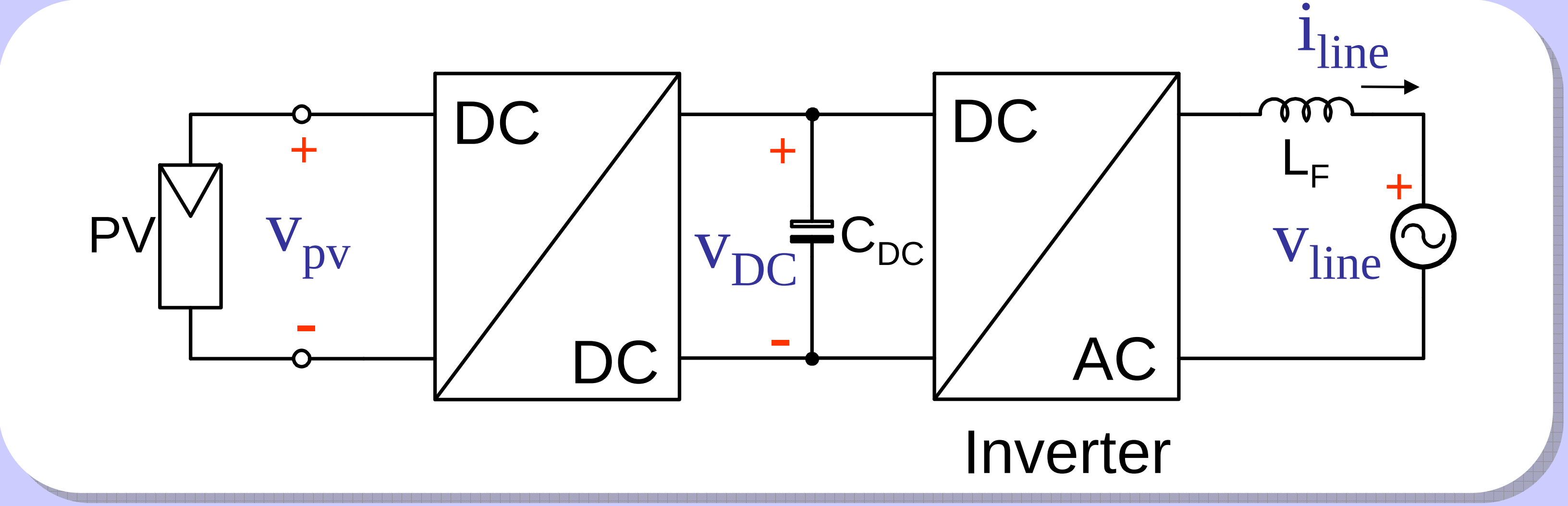
Dimensions: 1570 mm x 798 mm







Example of Grid-Connected System



- European Conversion Efficiency:

$$\eta_{EU} = 0.03\eta_{5\%} + 0.06\eta_{10\%} + 0.13\eta_{20\%} + 0.1\eta_{30\%} + 0.48\eta_{50\%} + 0.2\eta_{100\%}$$

$\eta_{X\%}$ = conversion efficiency measured at

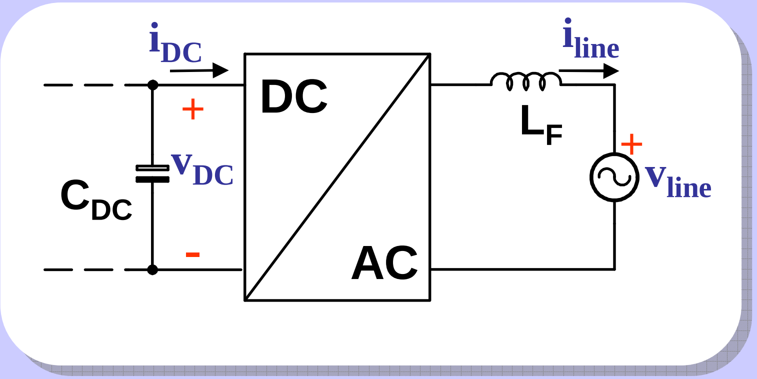
$$P = X\% \cdot P_{nom}$$

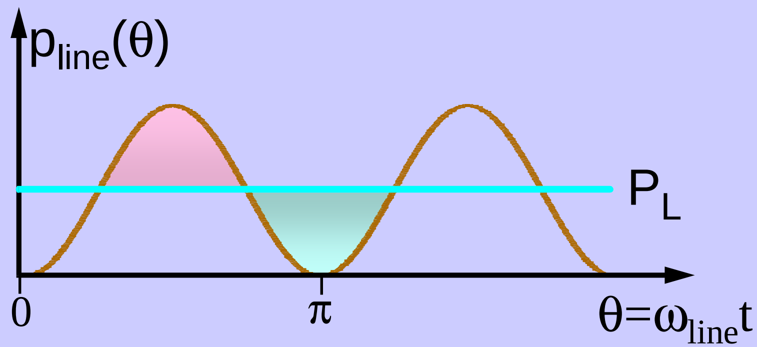
Single-Phase Grid Connection

Line voltage and current (unity power factor):

$$v_{line}(\theta) = \sqrt{2}V_L \sin(\theta), \qquad \theta = \omega_{line} t \qquad i_{line}(\theta) = \sqrt{2}I_L \sin(\theta)$$

$$p_{line}(\theta) = v_{line}(\theta) i_{line}(\theta) = 2V_L I_L \sin^2(\theta) = V_L I_L (1 - \cos(2\theta))$$





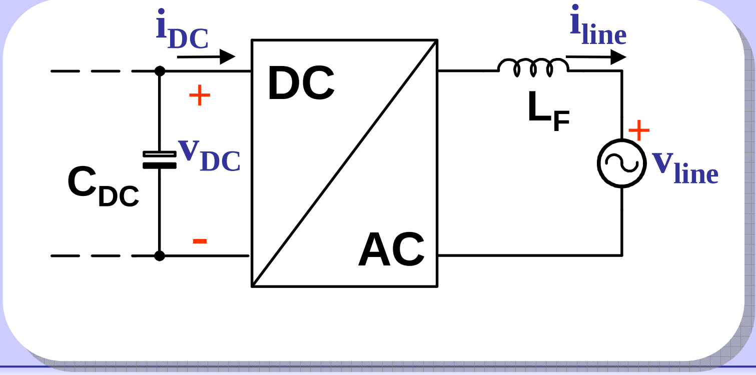
- The power delivered to the grid has a dc value plus a sinusoidal term at twice the line frequency

Single-Phase Grid Connection

Hp: negligible energy processing at line frequency

$$p_{DC}(\theta) = p_{line}(\theta) \quad \Rightarrow \quad i_{DC}(\theta) = \frac{P_L}{V_{DC}}\left(1 - \cos(2\theta)\right) = I_{DC} - I_{DC}\cos(2\theta)$$

$$\Delta V_{DC} = \frac{1}{2\omega_{line} C_{DC}} \Delta I_{DC} = \frac{P_L}{2\omega_{line} C_{DC} V_{DC}}$$

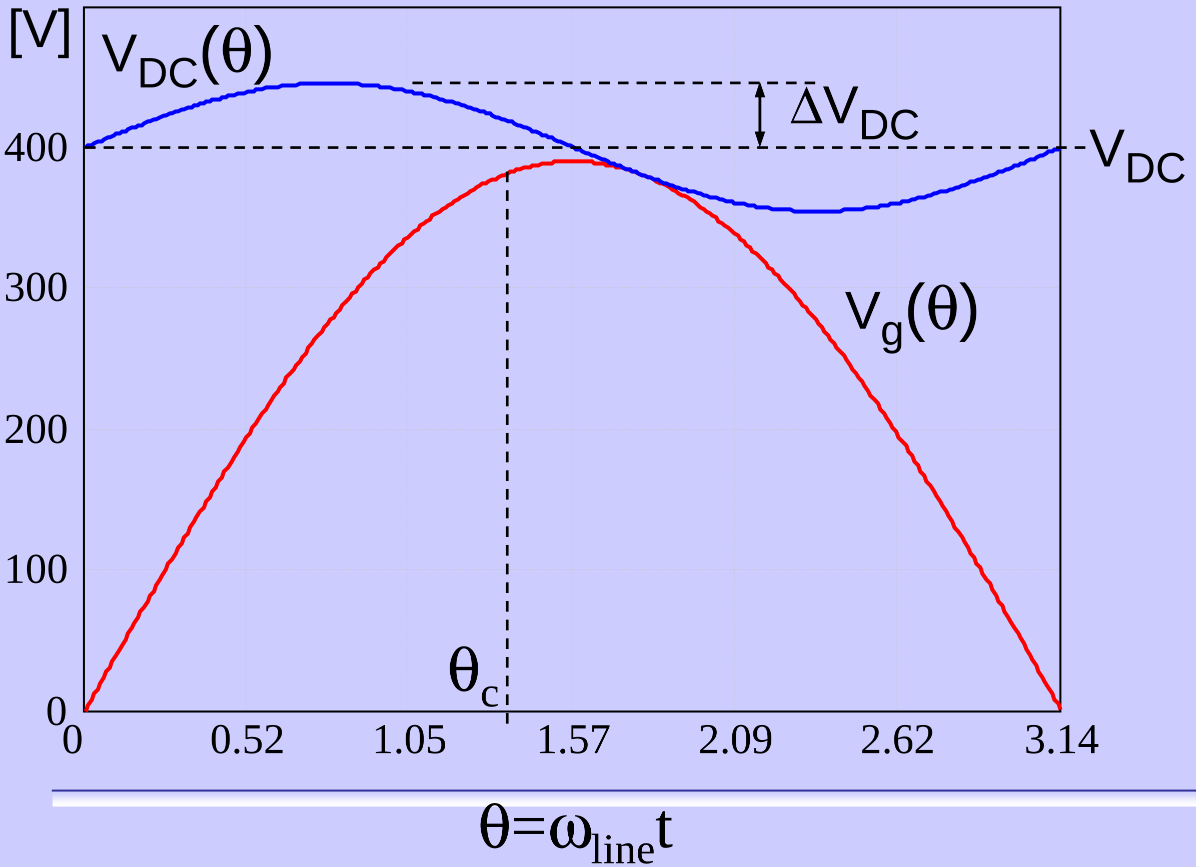


$$C_{DC} = \frac{P_L}{2\omega_{line} \dfrac{\Delta V_{DC}}{V_{DC}} V_{DC}^2} = \frac{P_L}{2\omega_{line} r_{V_{DC}} V_{DC}^2}$$

Single-Phase Grid Connection

Maximum allowed voltage ripple across DC link capacitor (step-down inverter)

$$v_{DC}(\theta) = V_{DC} + \Delta V_{DC} \sin(2\theta)$$



Example:

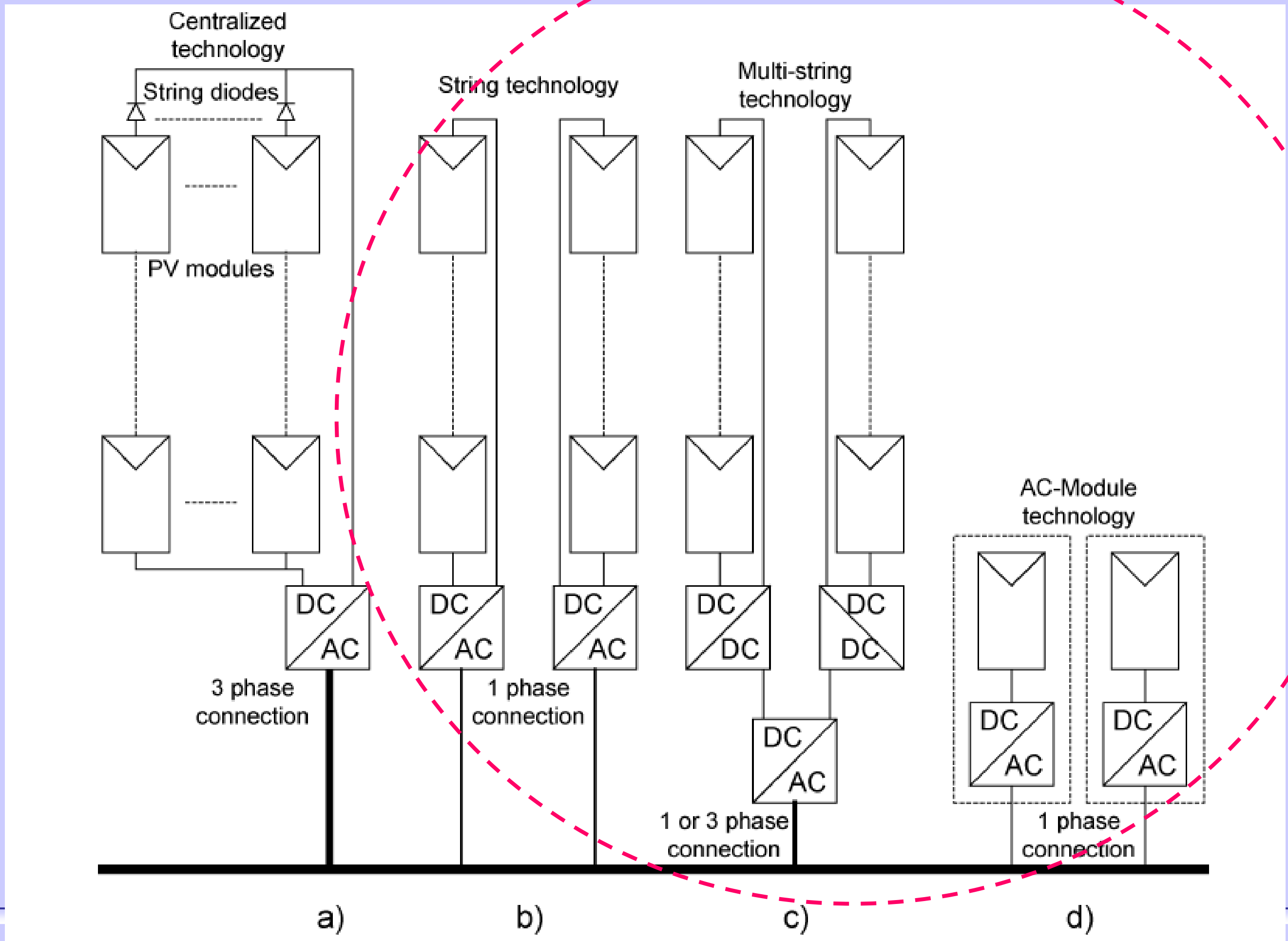
V_{gpk} = 390 V

V_{DC} = 400 V

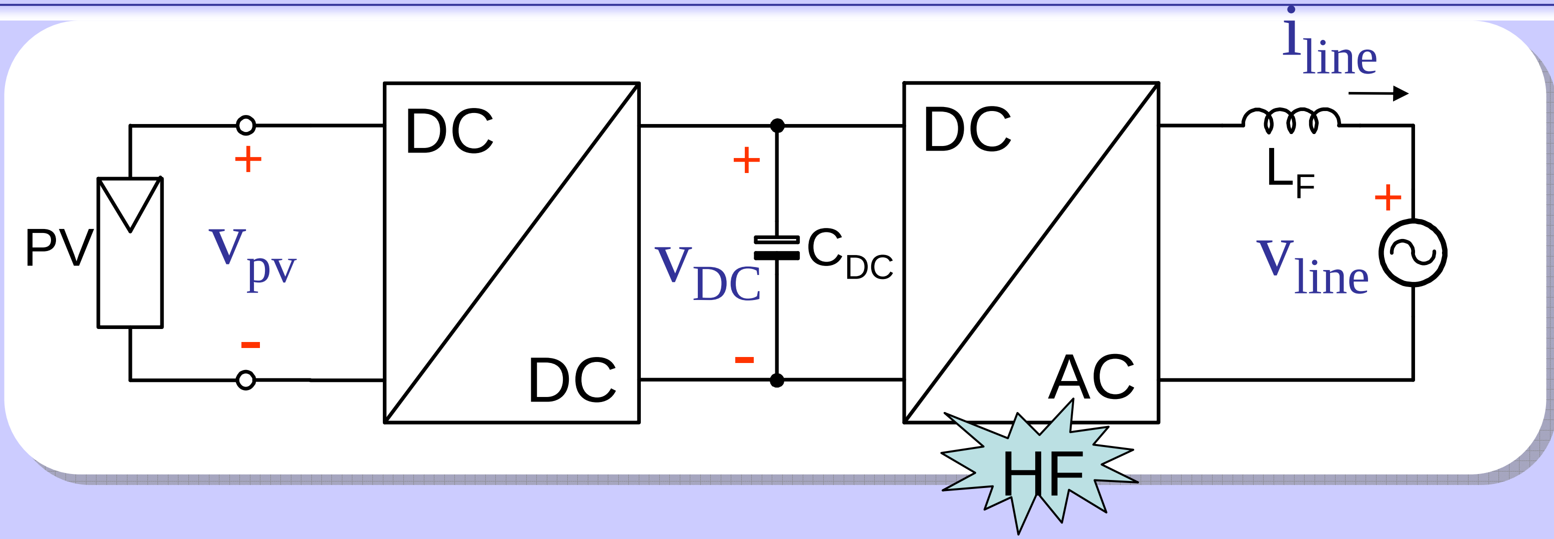
ΔV_{DC} = 45.5 V

System Configuration

For domestic applications:

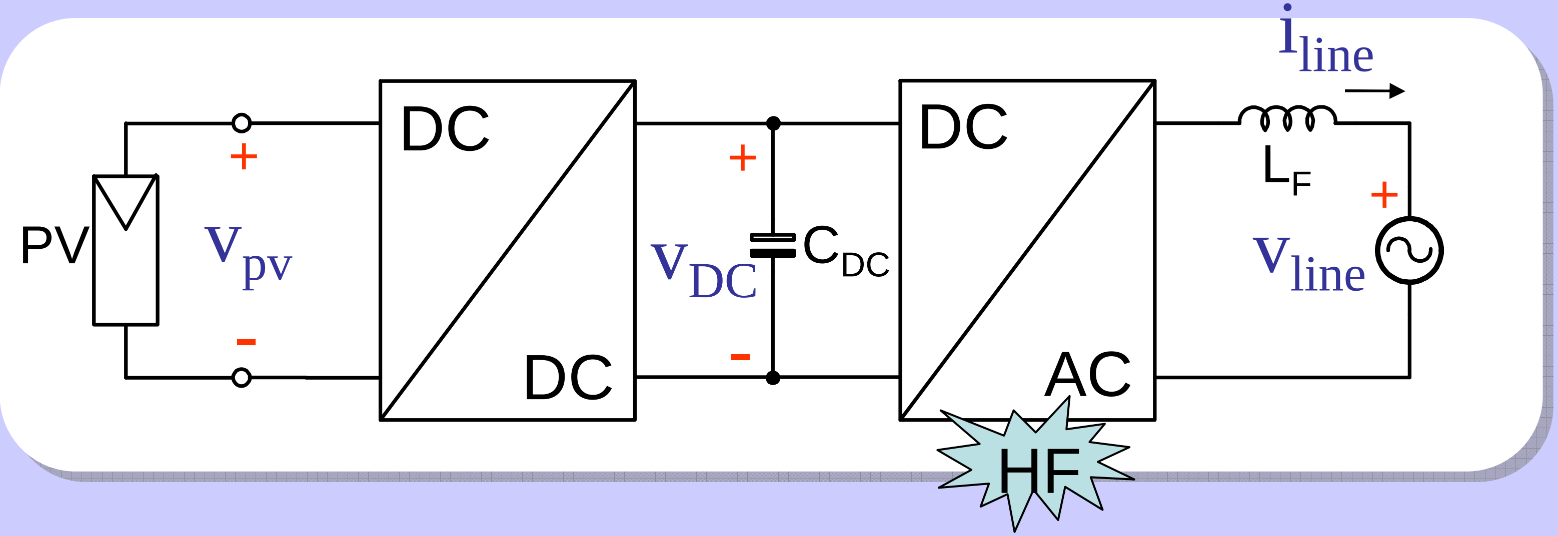


Dual-Stage Configuration



- The DC-DC stage controls the PV string so as to operate at the MPP and works under a constant output voltage V_{DC}
- The DC-AC inverter injects a sinusoidal current into the grid at a unity power factor and controls the DC link voltage V_{DC}
- The DC link capacitor is located between the two converters (high voltage means lower capacitor value)

Dual-Stage Configuration

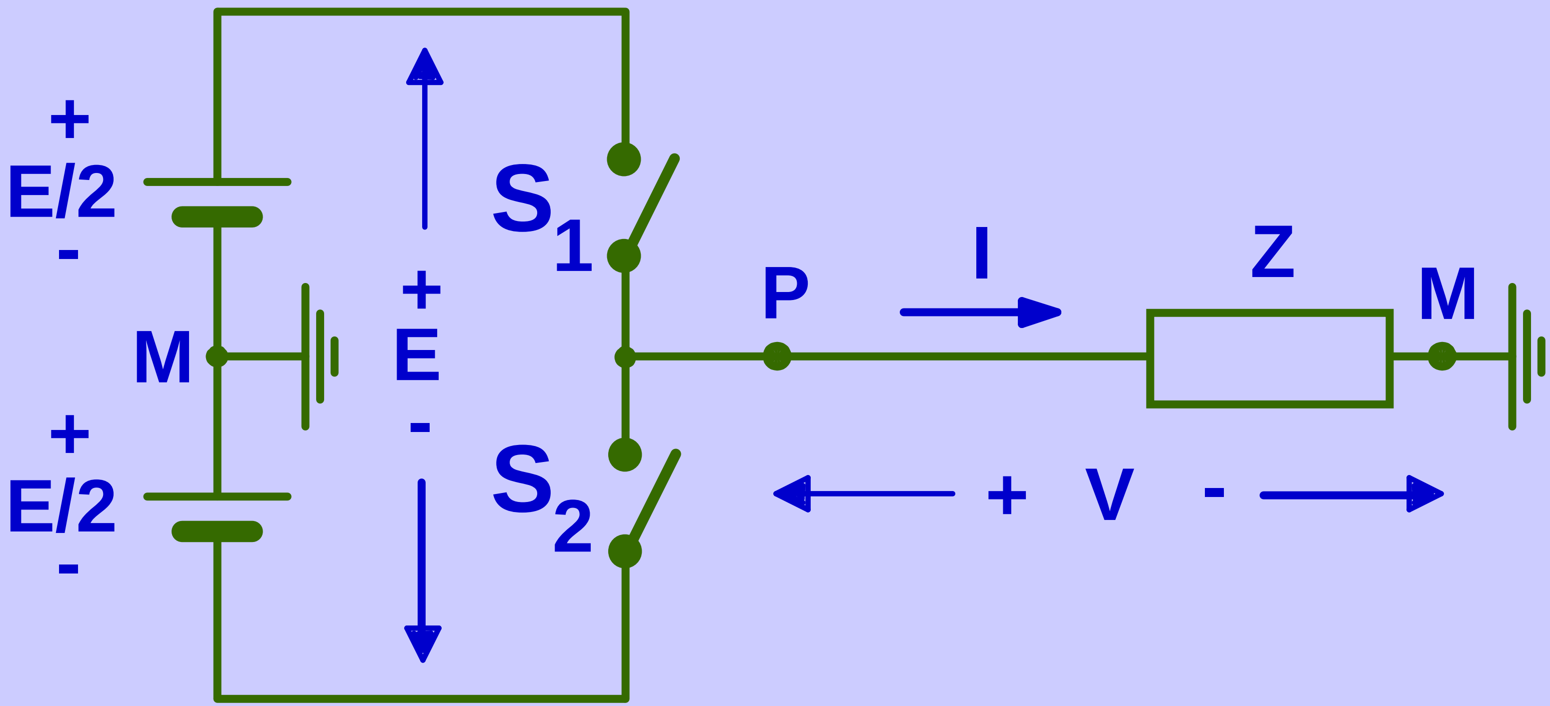


- The DC-AC inverter is usually a step-down topology

How it works?

Ideal half-bridge inverter

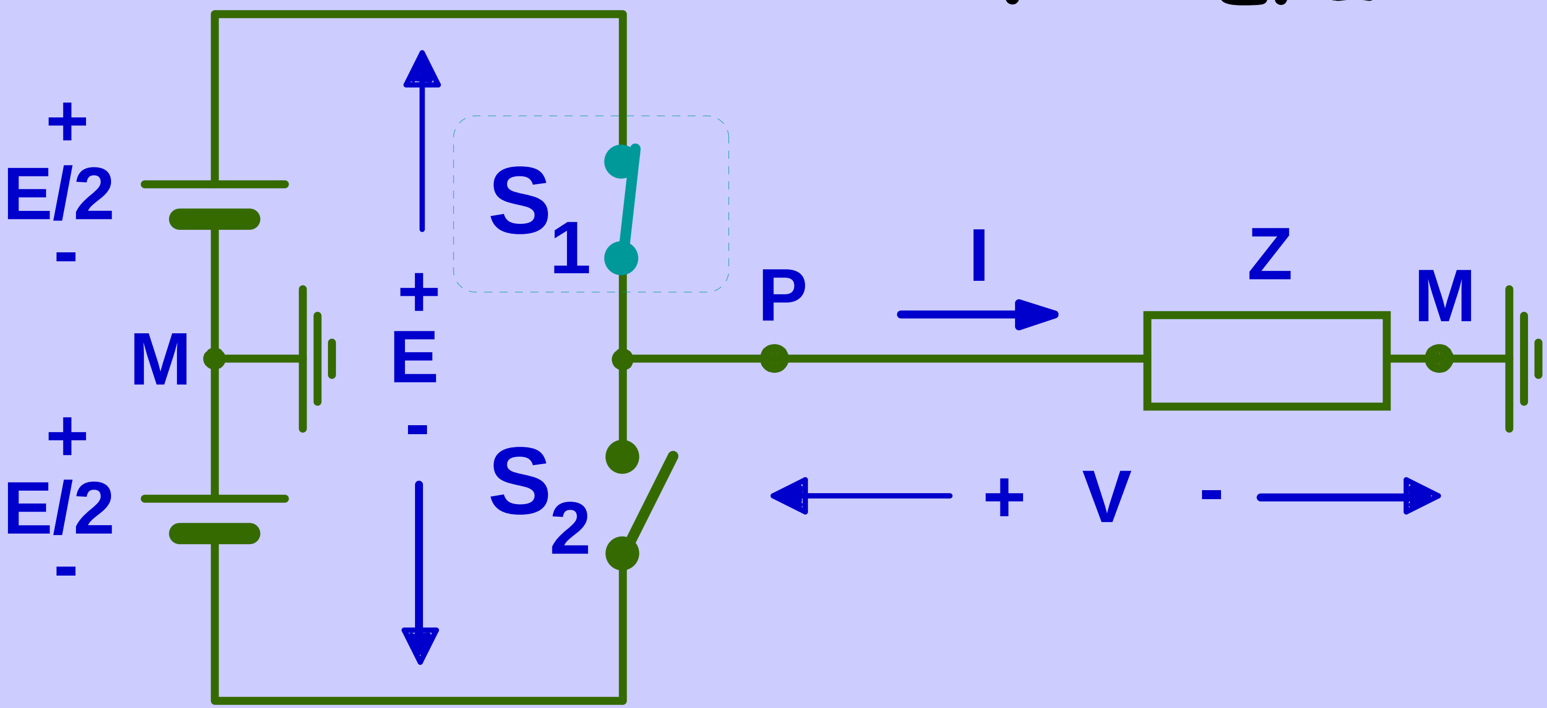
- Z = generic load



Ideal half-bridge inverter

- 1° topological state: S_1 = "On", S_2 = "Off"

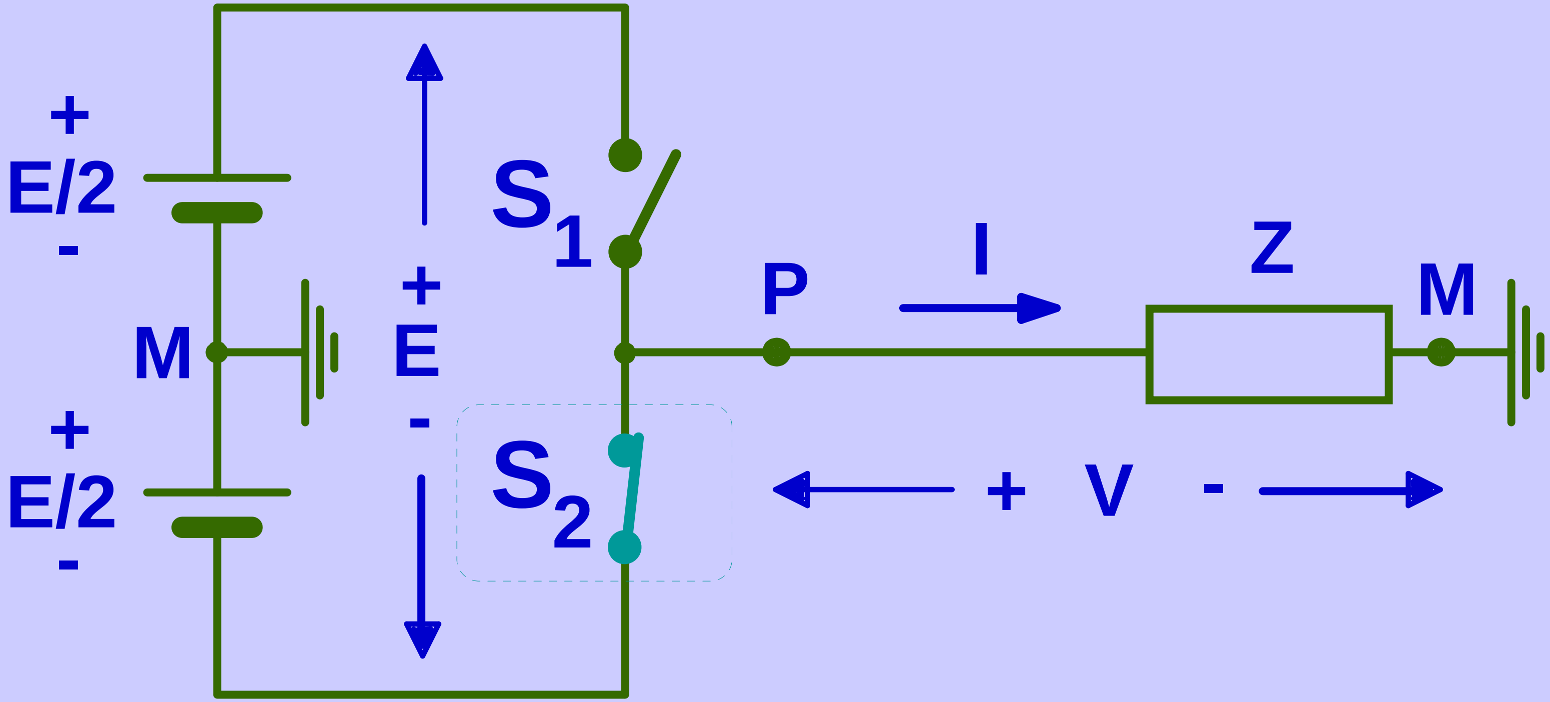
V = +E/2



Ideal half-bridge inverter

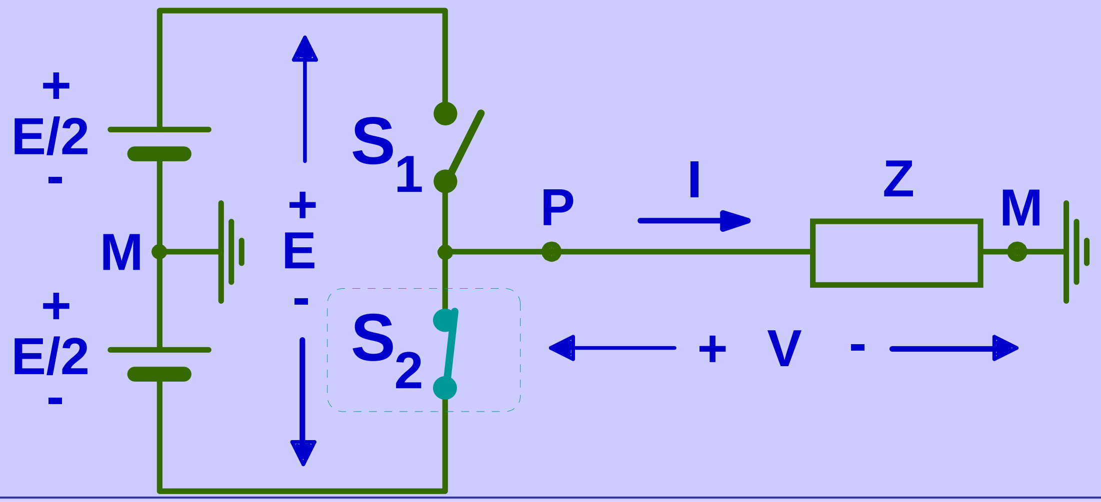
- 2° topological state: S_1 = "Off", S_2 = "On"

$$V = -E/2$$



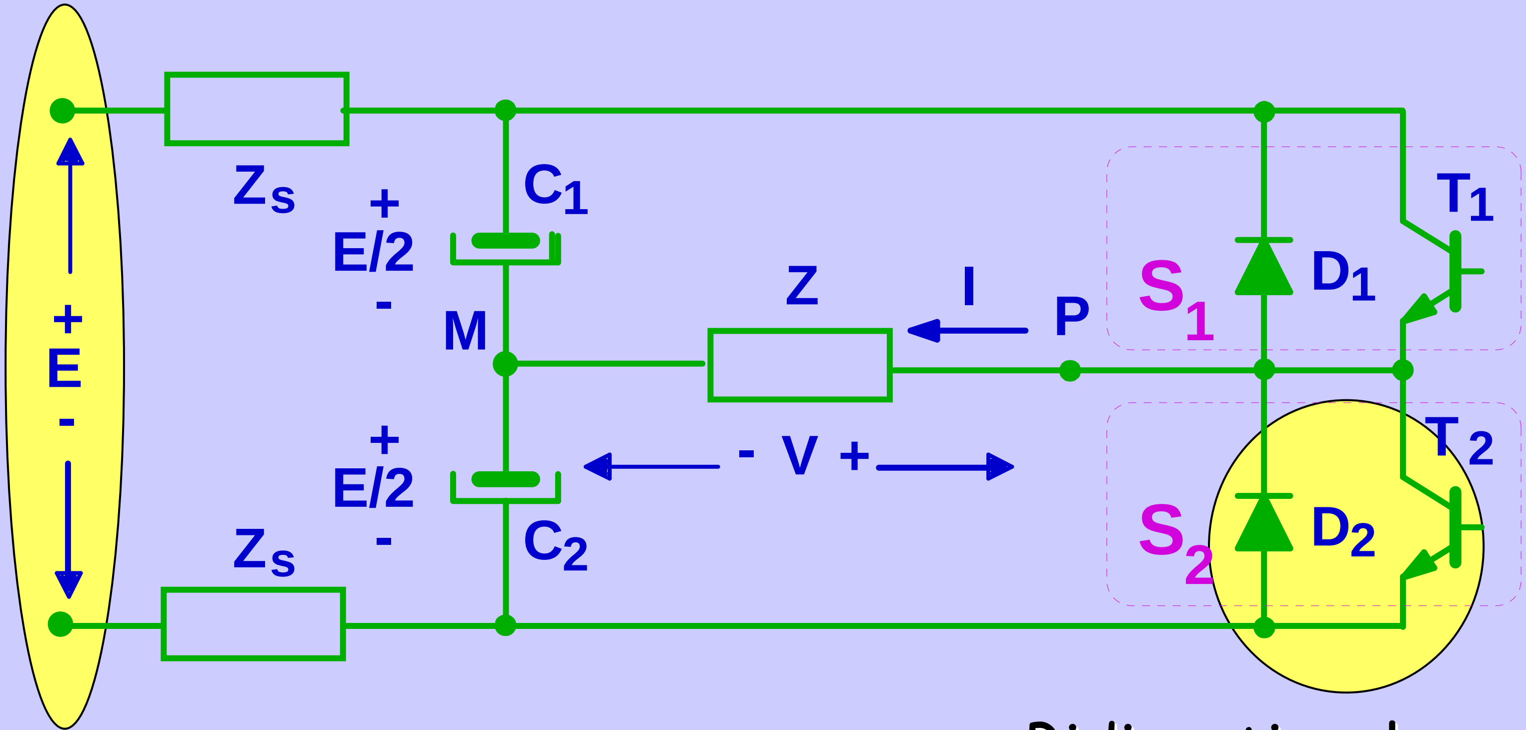
Ideal half-bridge inverter

- Switches S_1 and S_2 cannot be "On" at the same time
  - A suitable dead time has to be inserted
- During the dead time the load current must be allowed to flow (e.g. inductive load)
  - Bidirectional current switches must be used



Real half-bridge PWM inverter

- Z = generic load

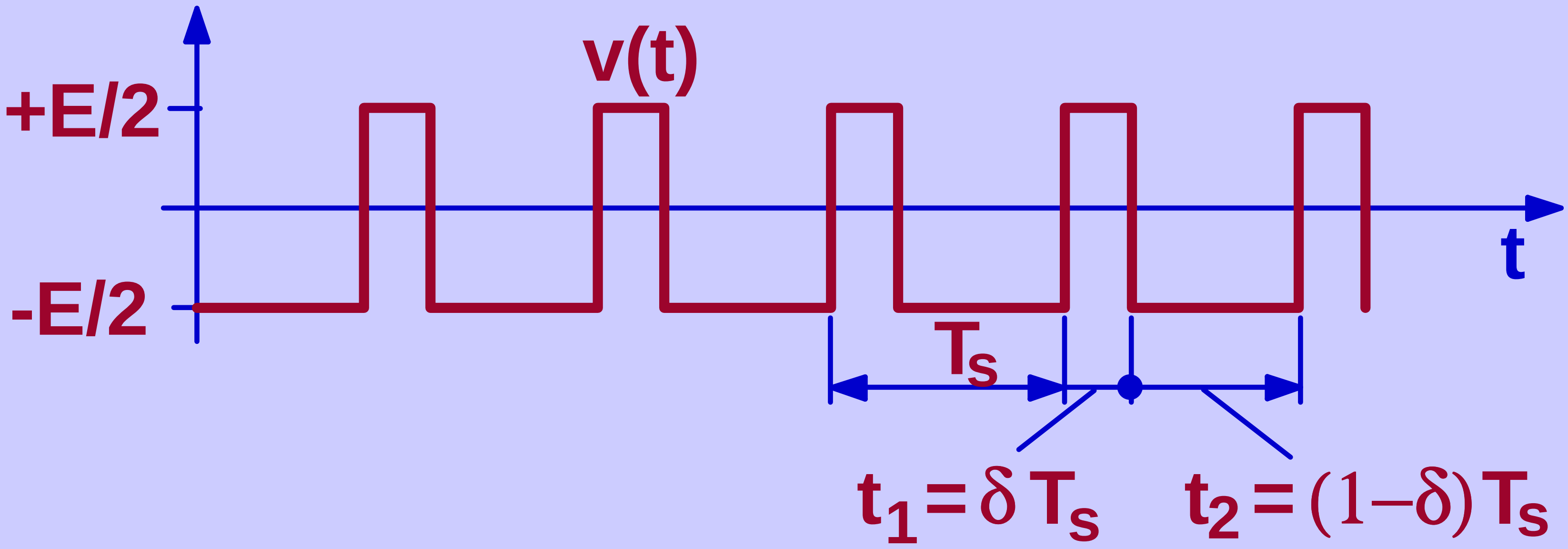


Single supply voltage

Bidirectional current switches

Pulse Width Modulation - PWM

- Control of each topological state duration

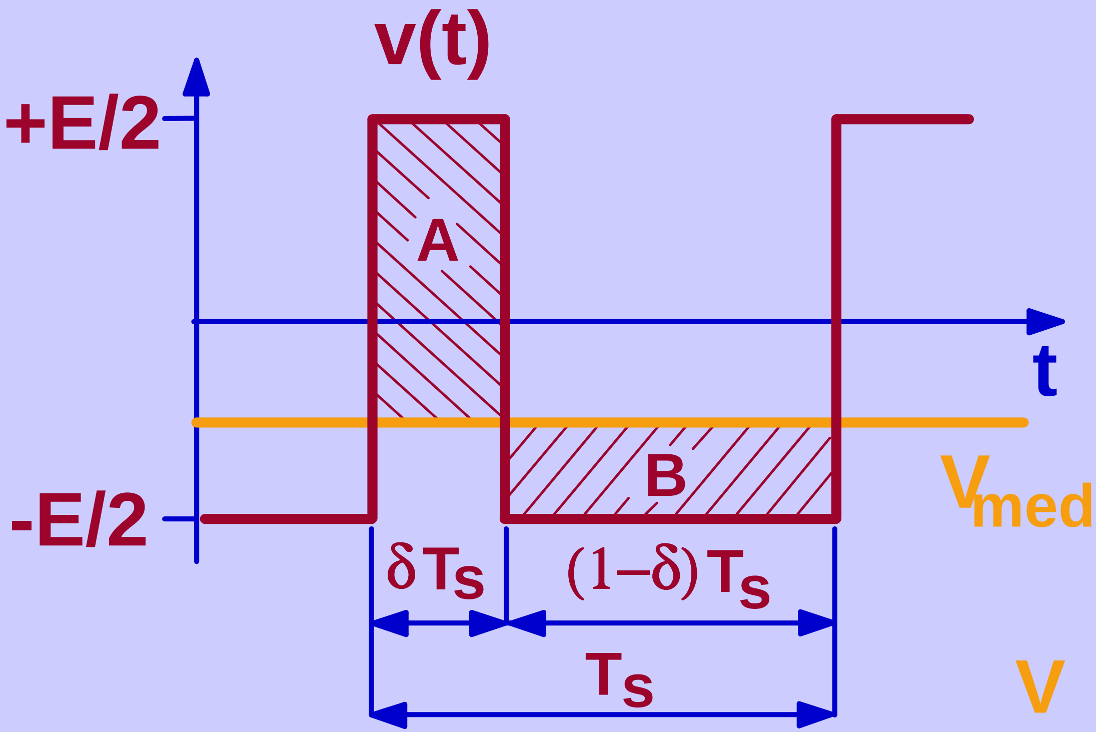


$f_s = 1/T_s$ = switching frequency

$\delta = t_1/T_s$ = duty-cycle

Pulse Width Modulation - PWM

Average output voltage

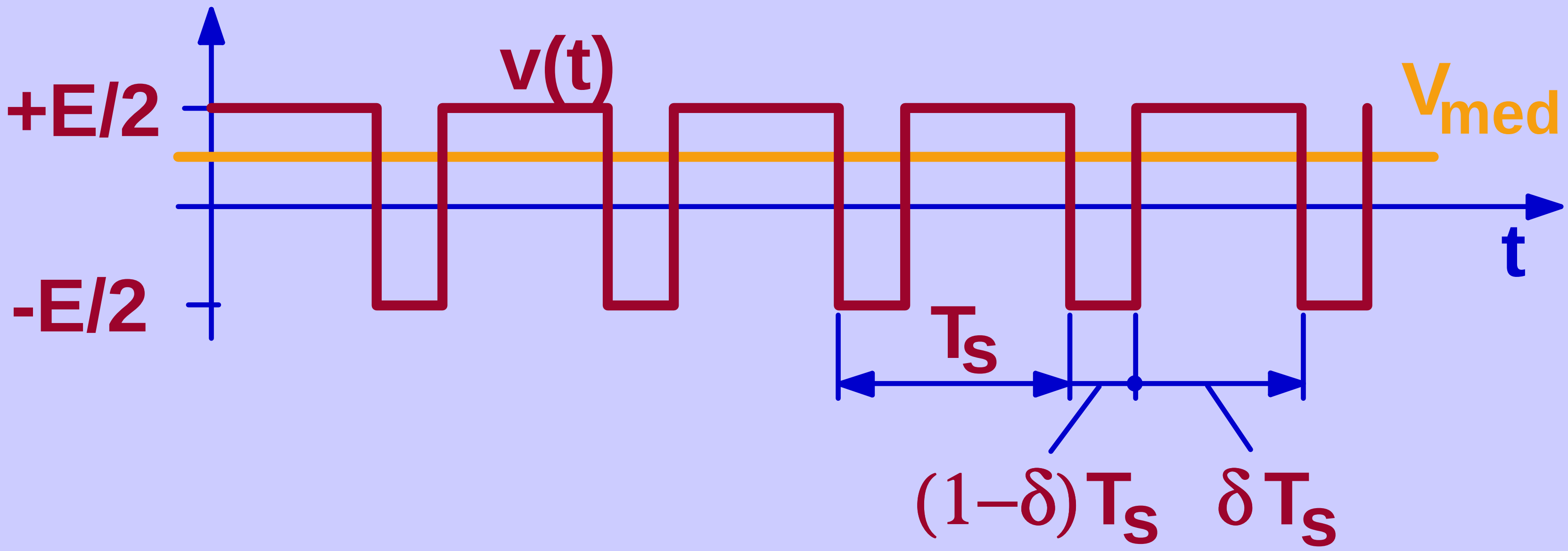


$$V_{med} = (\delta - 1/2)\, E$$

Pulse Width Modulation - PWM

Average output voltage

$\delta > 0.5$ $\quad$ $V_{med} > 0$

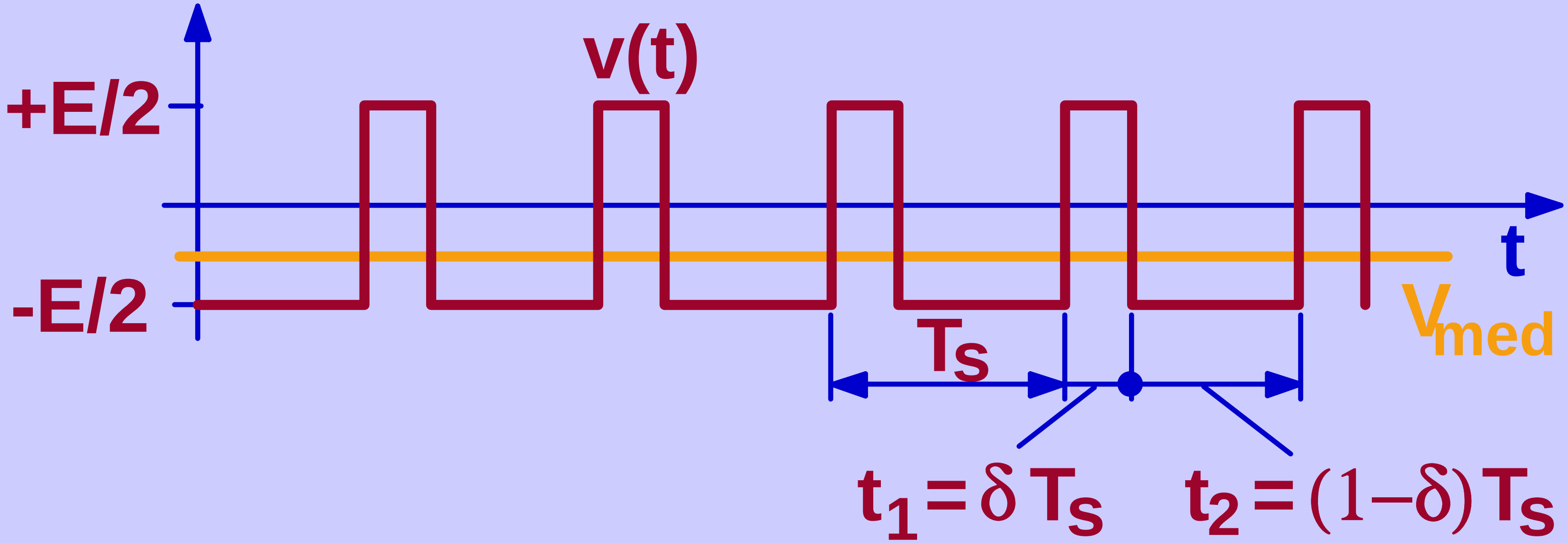


$$V_{med} = (\delta - 1/2)\,E$$

Pulse Width Modulation - PWM

Average output voltage

$\delta < 0.5 \longrightarrow V_{med} < 0$



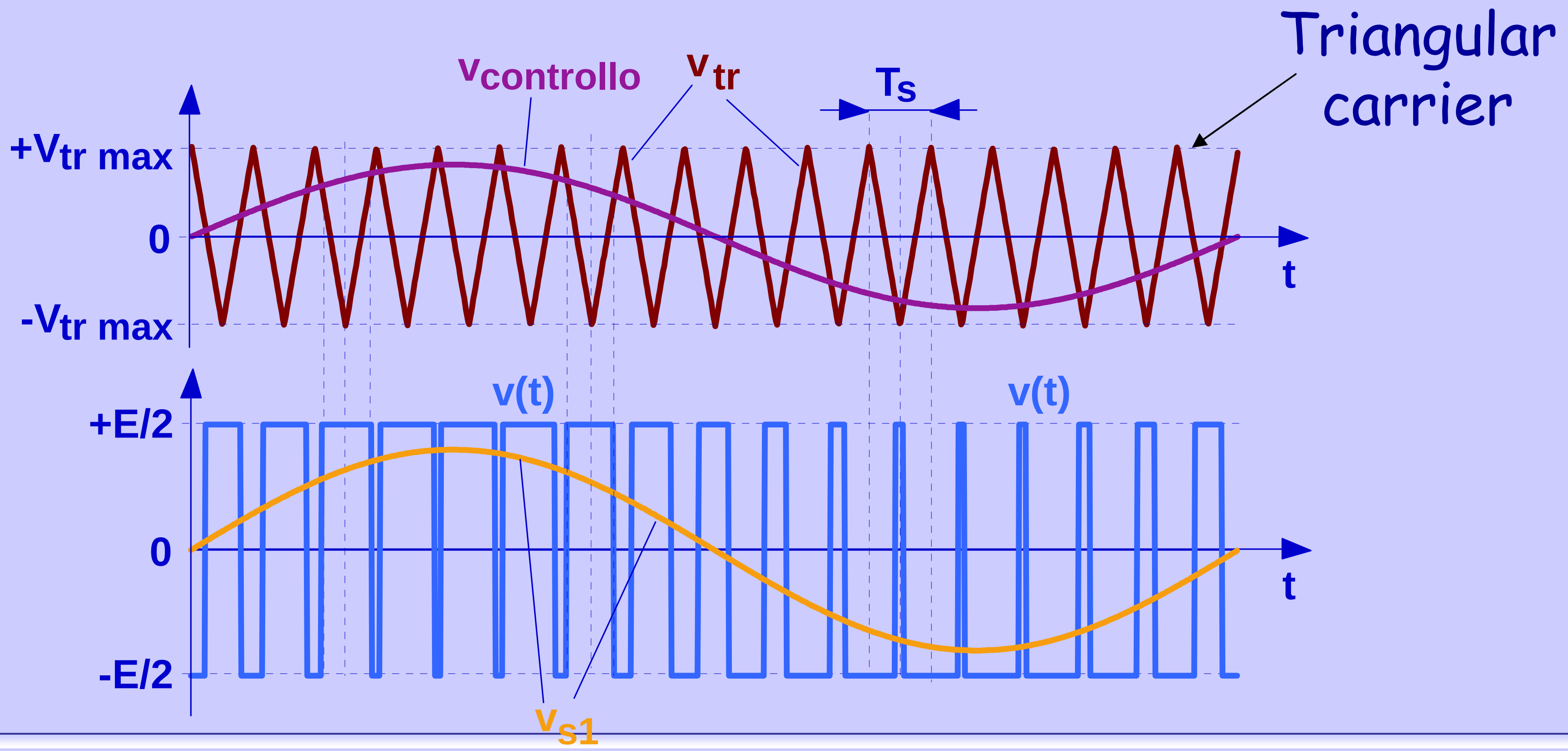
$$V_{med} = (\delta - 1/2)\,E$$

Generation of a sinusoidal voltage

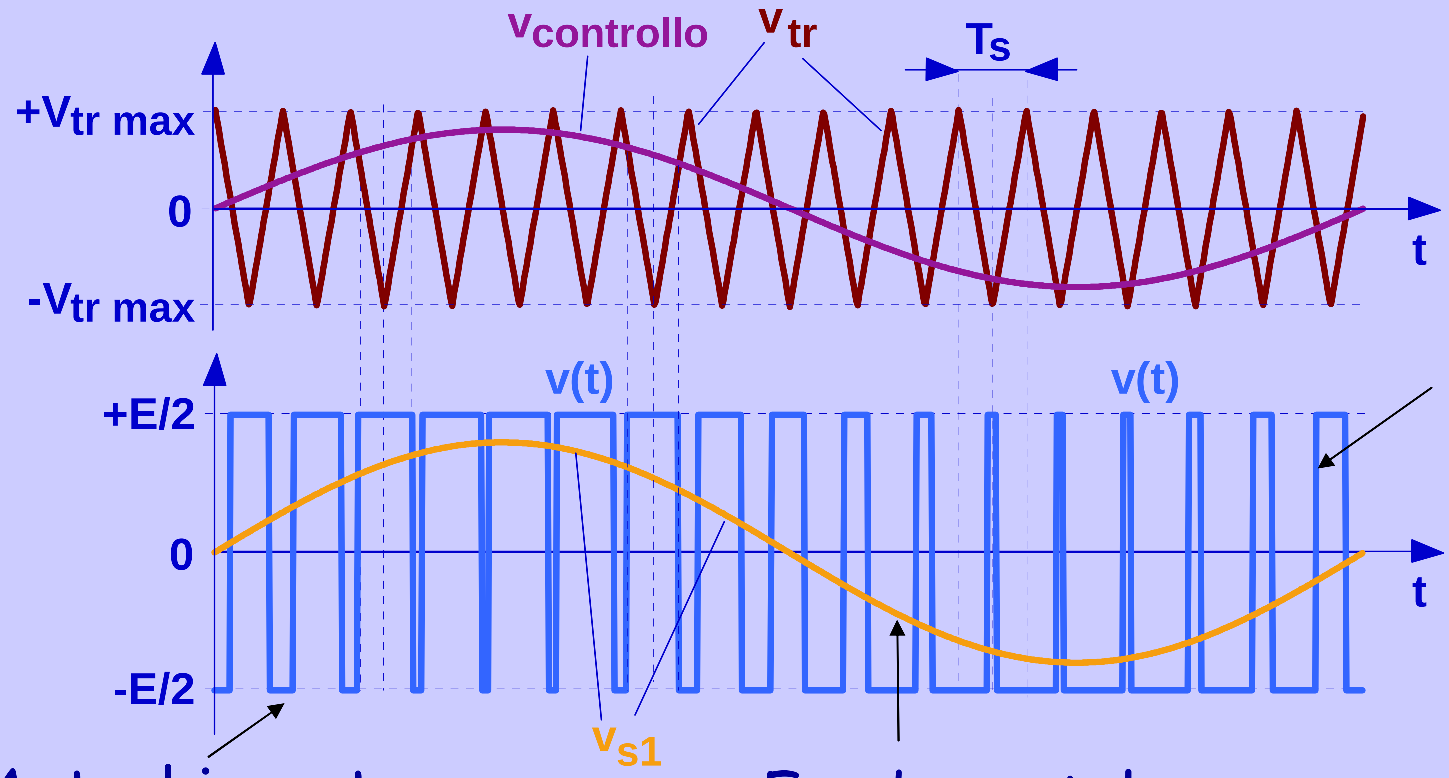
Control law:

$V_c > V_{tr}$ ⇨ S_1 = "On", S_2 = "Off"

$V_c < V_{tr}$ ⇨ S_1 = "Off", S_2 = "On"



Generation of a sinusoidal voltage



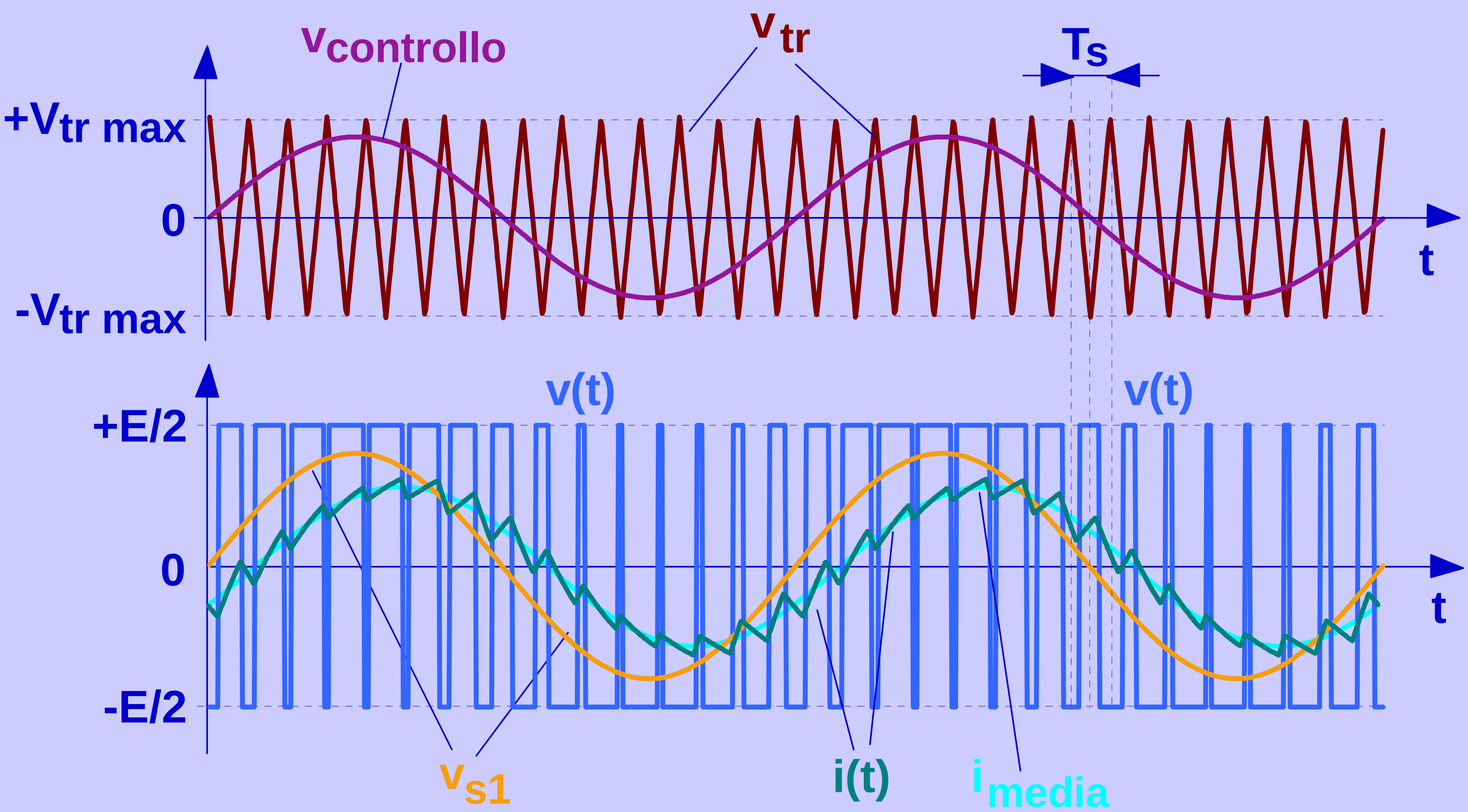
Actual inverter output voltage

Fundamental component of the inverter output voltage

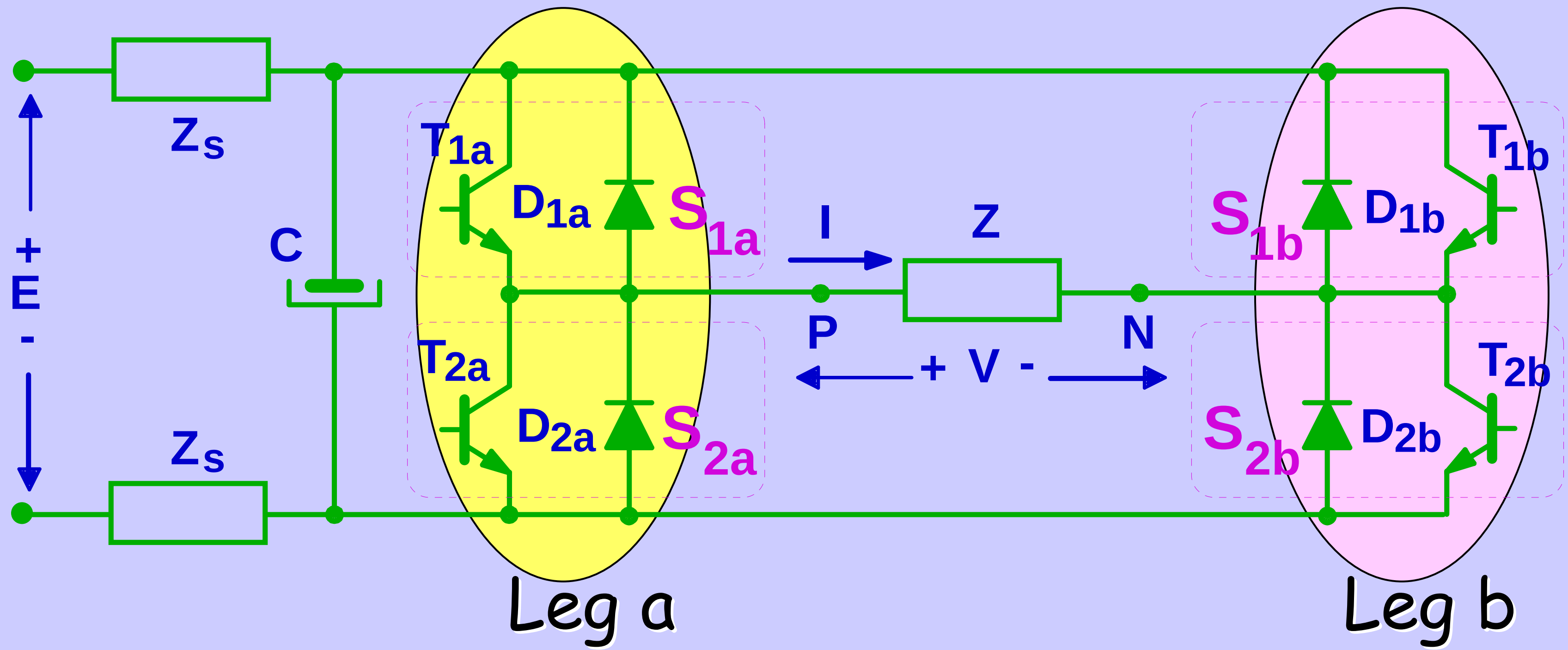
Need of a suitable filtering of the inverter output voltage to remove switching frequency related harmonics

Pulse Width Modulation - PWM

Sinusoidal generation with R-L-C load



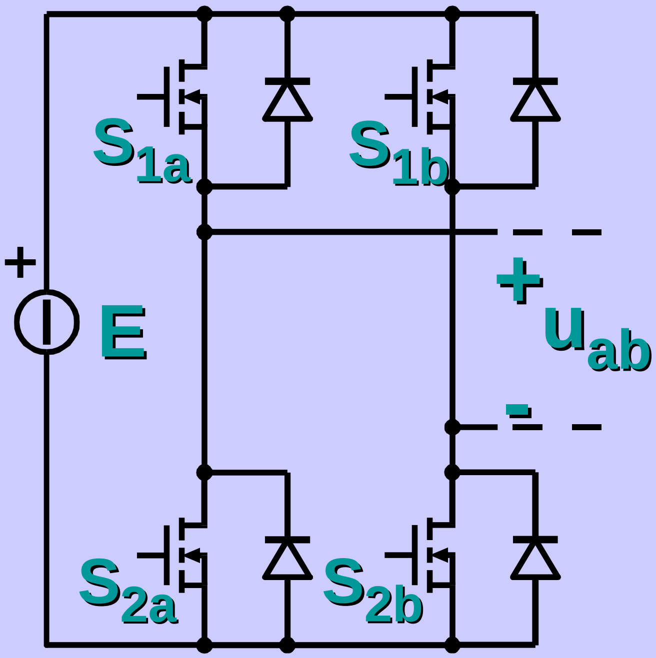
Full-bridge single-phase inverter

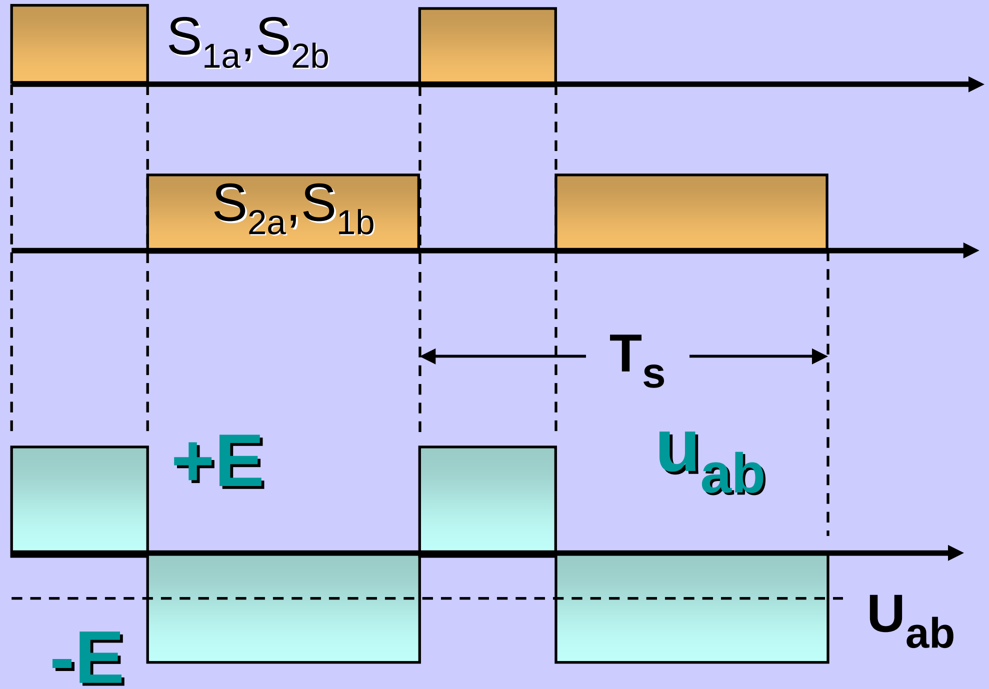


$$V = V_P - V_N$$

Full-bridge single-phase inverter

Two-level modulation

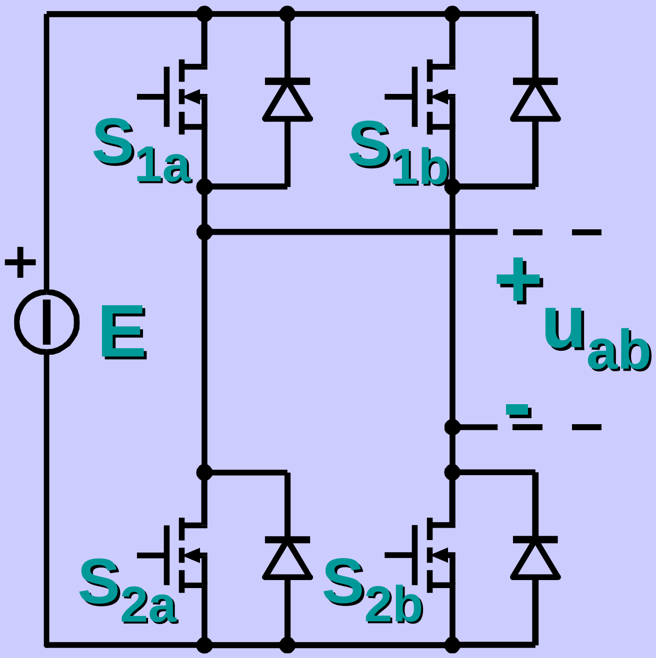


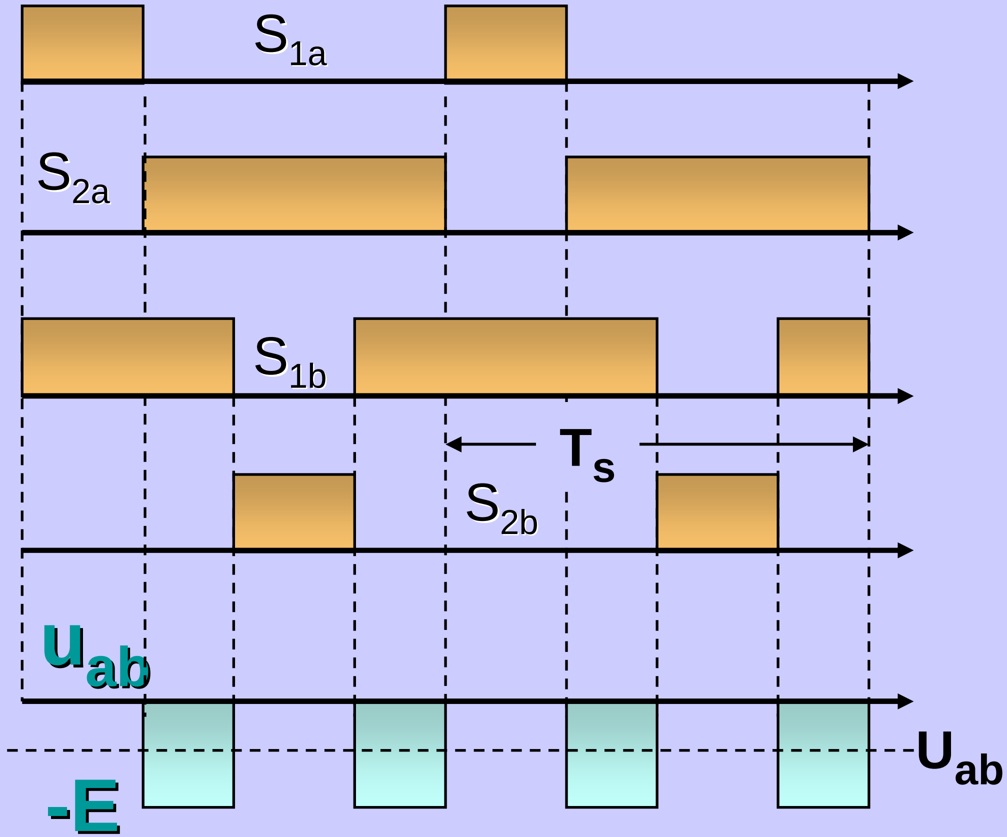


$$U_{ab} = \delta E - (1 - \delta)E = (2\delta - 1)E$$

Full-bridge single-phase inverter

Three-level modulation

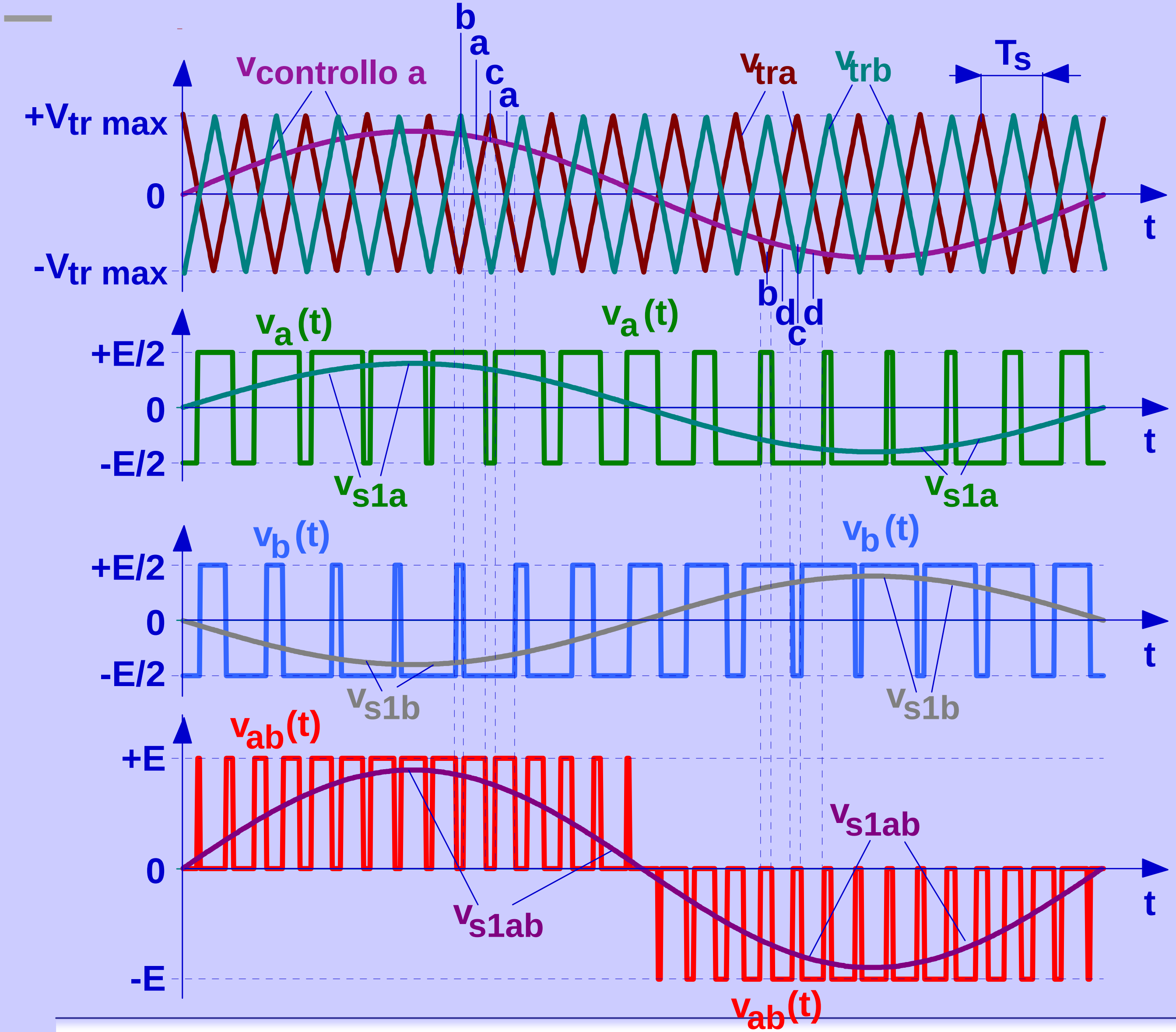




$$U_{ab} = -E[(1-\delta)-\delta] = (2\delta - 1)E$$

Much less harmonic content

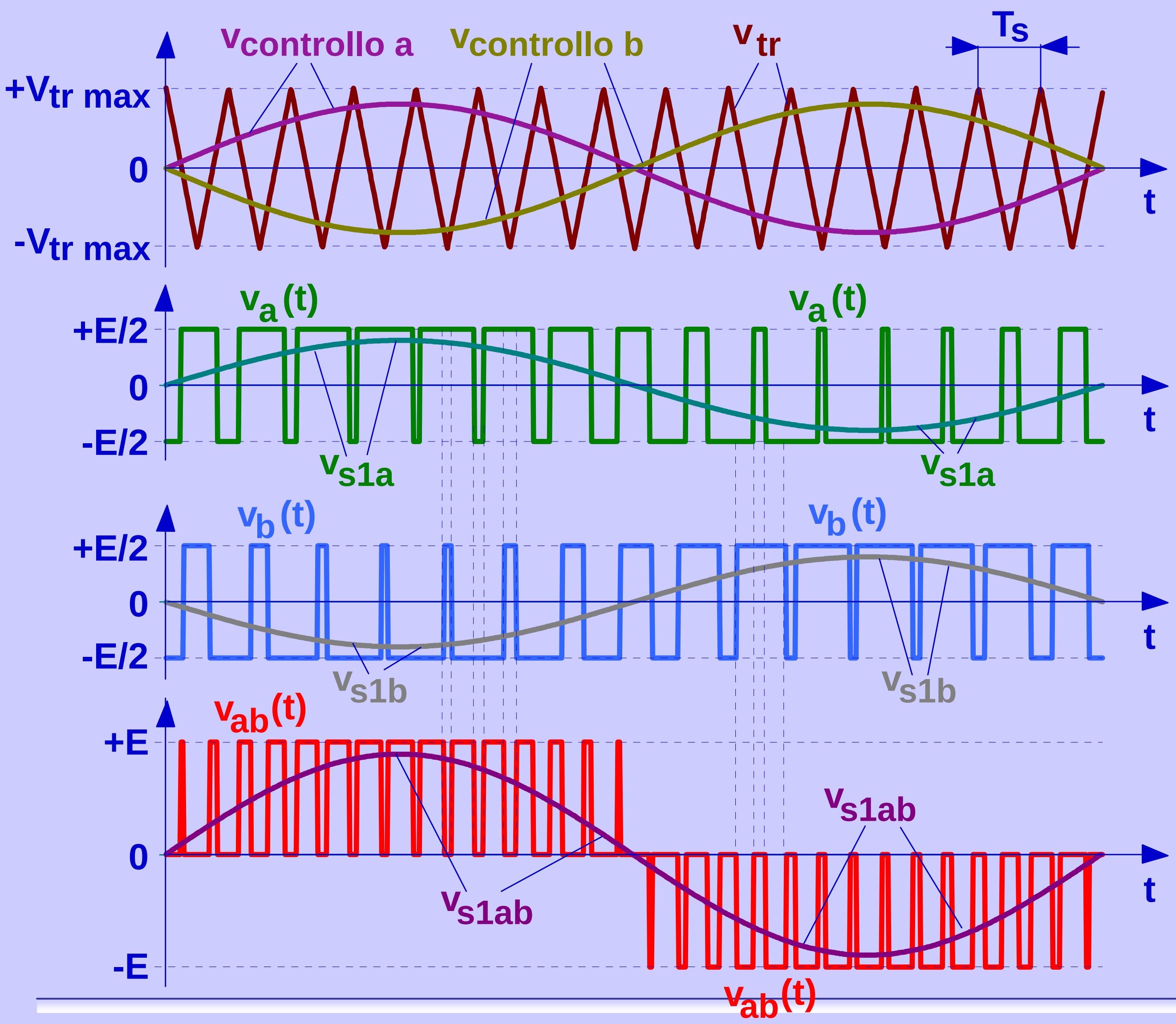
Full-bridge single-phase inverter



Three-level modulation

Same control signal but **triangular carrier** waveforms phase shifted by 180°

Full-bridge single-phase inverter

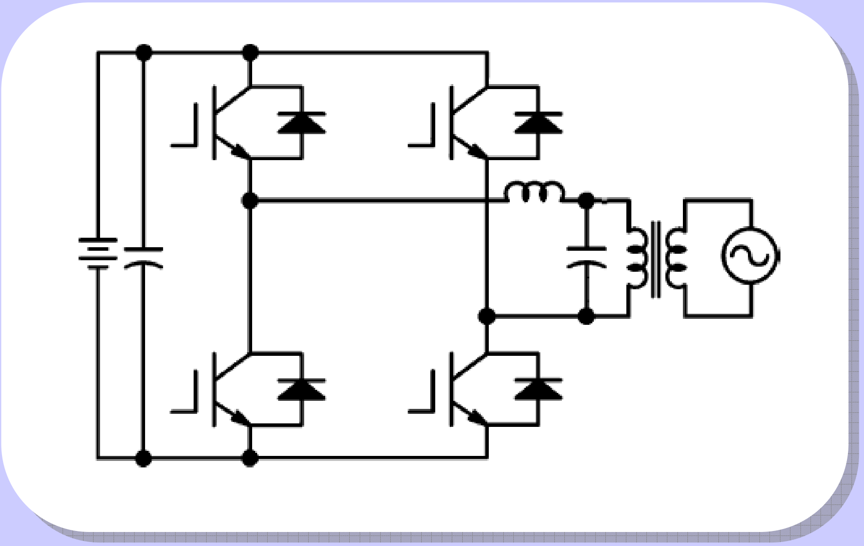


Three-level modulation

Same triangular carrier waveform but control signals phase shifted by 180°

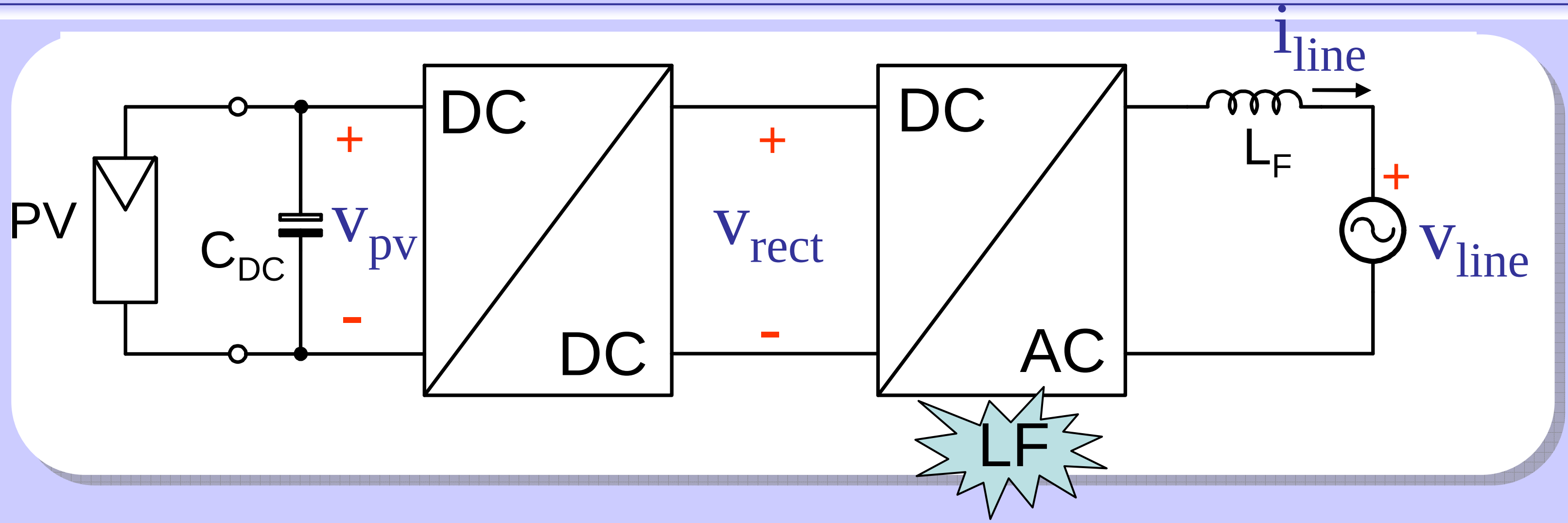
Example of Single-Stage Solutions

- Step-down inverter with low-frequency transformer



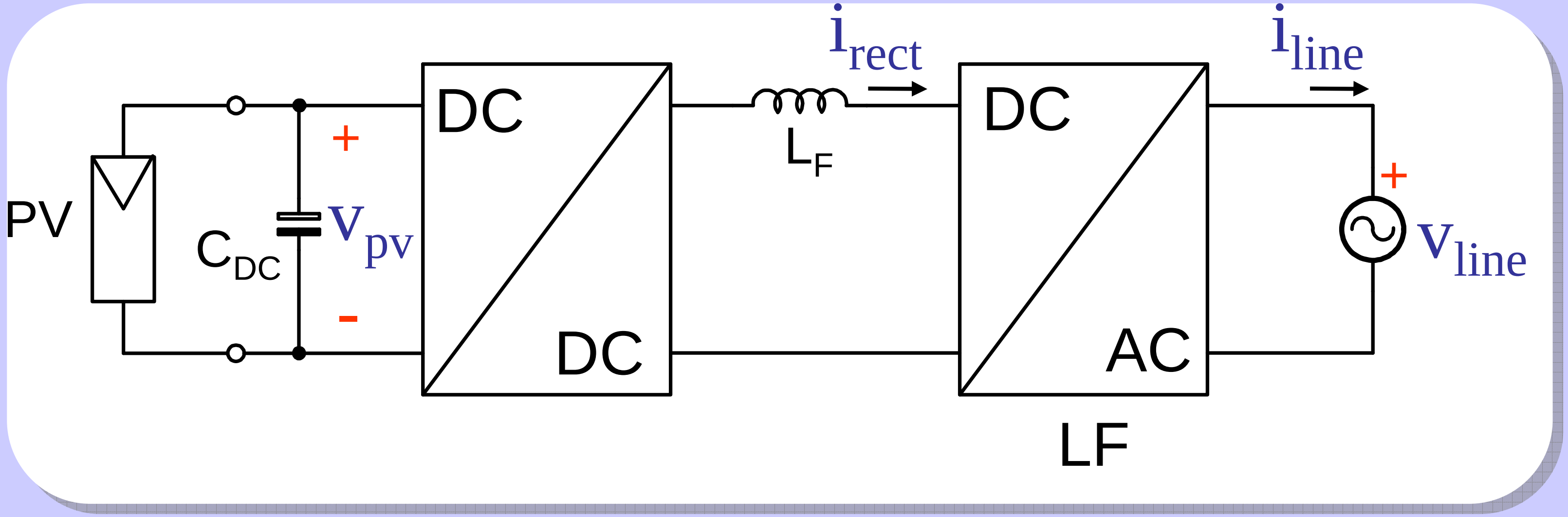
- No DC current injection into the grid

Different dual-Stage Configuration



- The DC-DC stage controls the PV string so as to operate at the MPP and generates a rectified sinusoidal voltage at its output
- The *maximum* instantaneous power delivered by the DC-DC stage is twice the average value
- The DC-AC inverter operates at line frequency and unfolds the rectified sinusoidal voltage into a sine wave

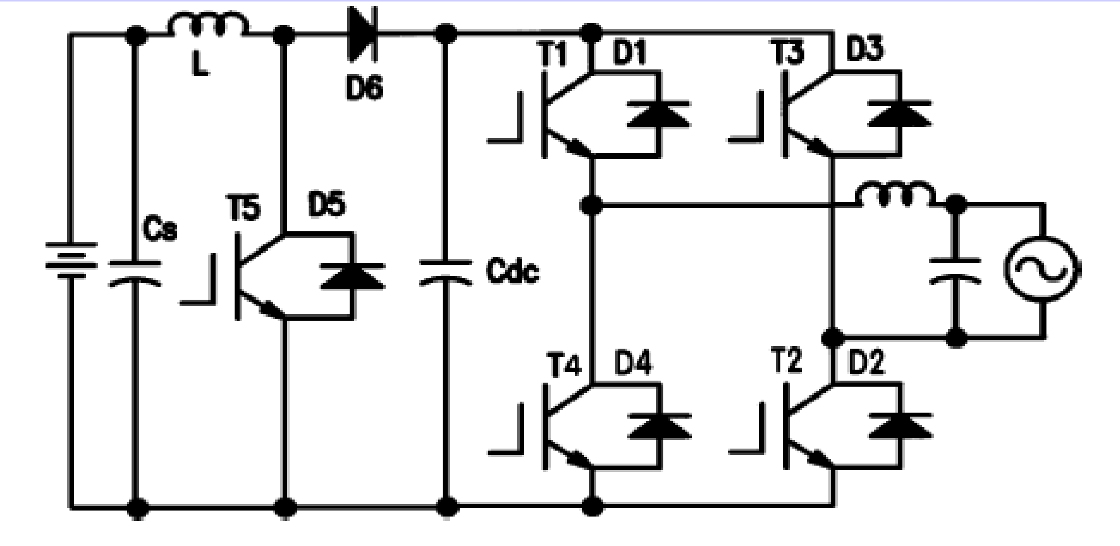
Different dual-Stage Configurations

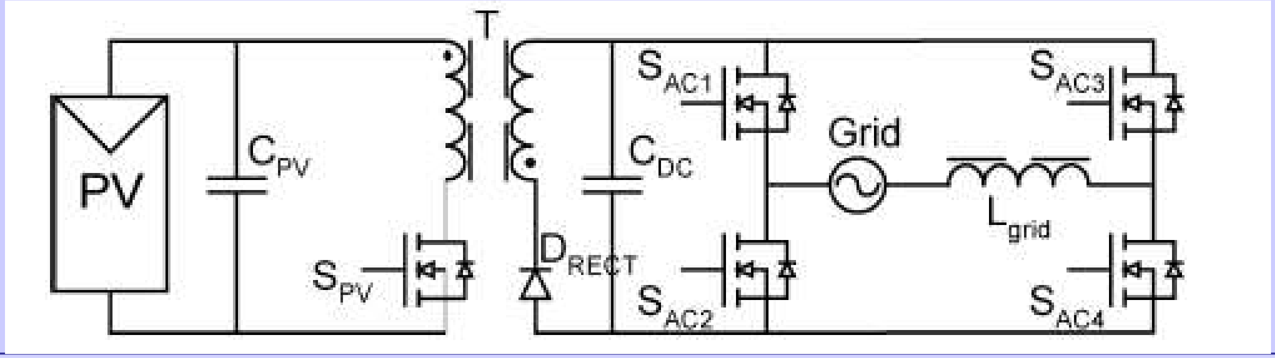


- The DC-DC stage controls the PV string so as to operate at the MPP and generates a rectified sinusoidal current at its output
- The *maximum* instantaneous power delivered by the DC-DC stage is twice the average value
- The DC-AC inverter operates at line frequency and unfolds the rectified sinusoidal current into a sine wave

Example of Dual-Stage Solutions

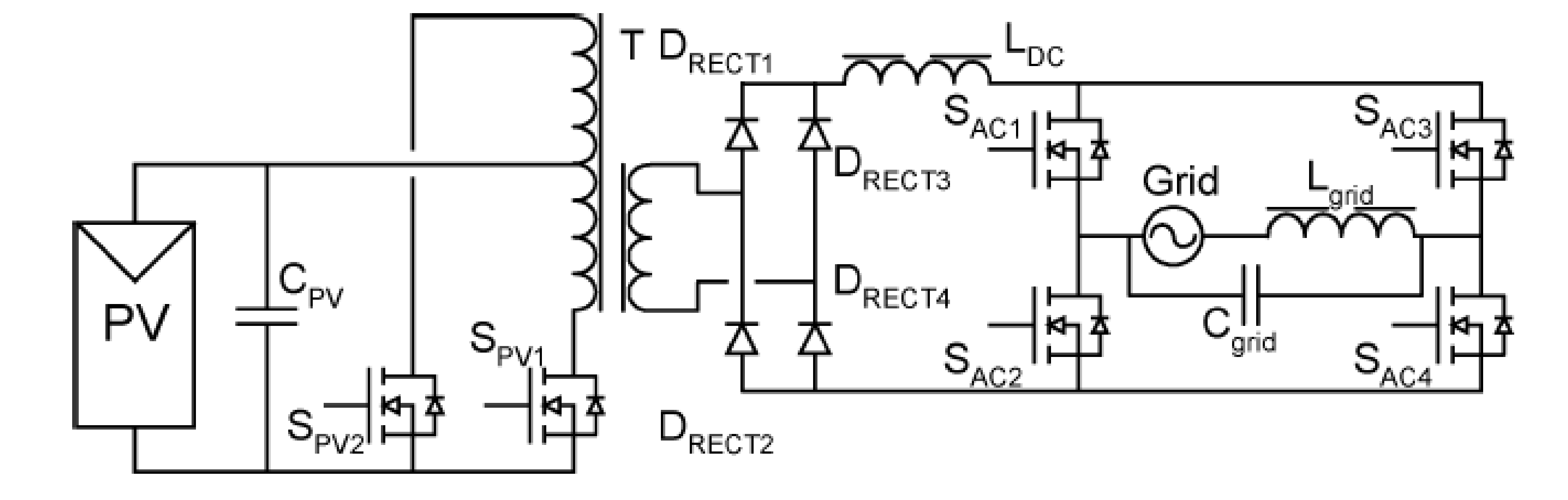
- Boost or flyback converter plus a PWM modulated inverter





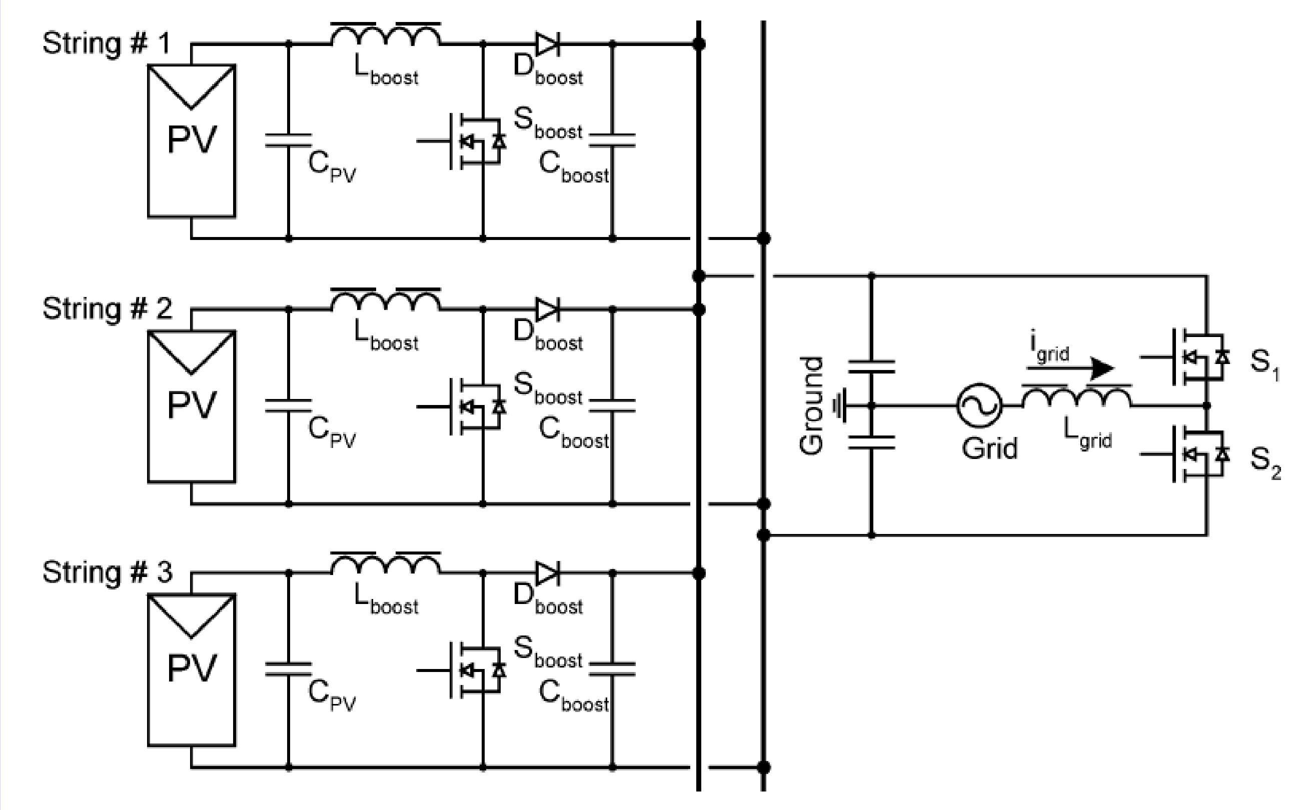
Example of Dual-Stage Solutions

- Push-pull converter plus a line-frequency commutated inverter (Mastervolt Soladin 120)



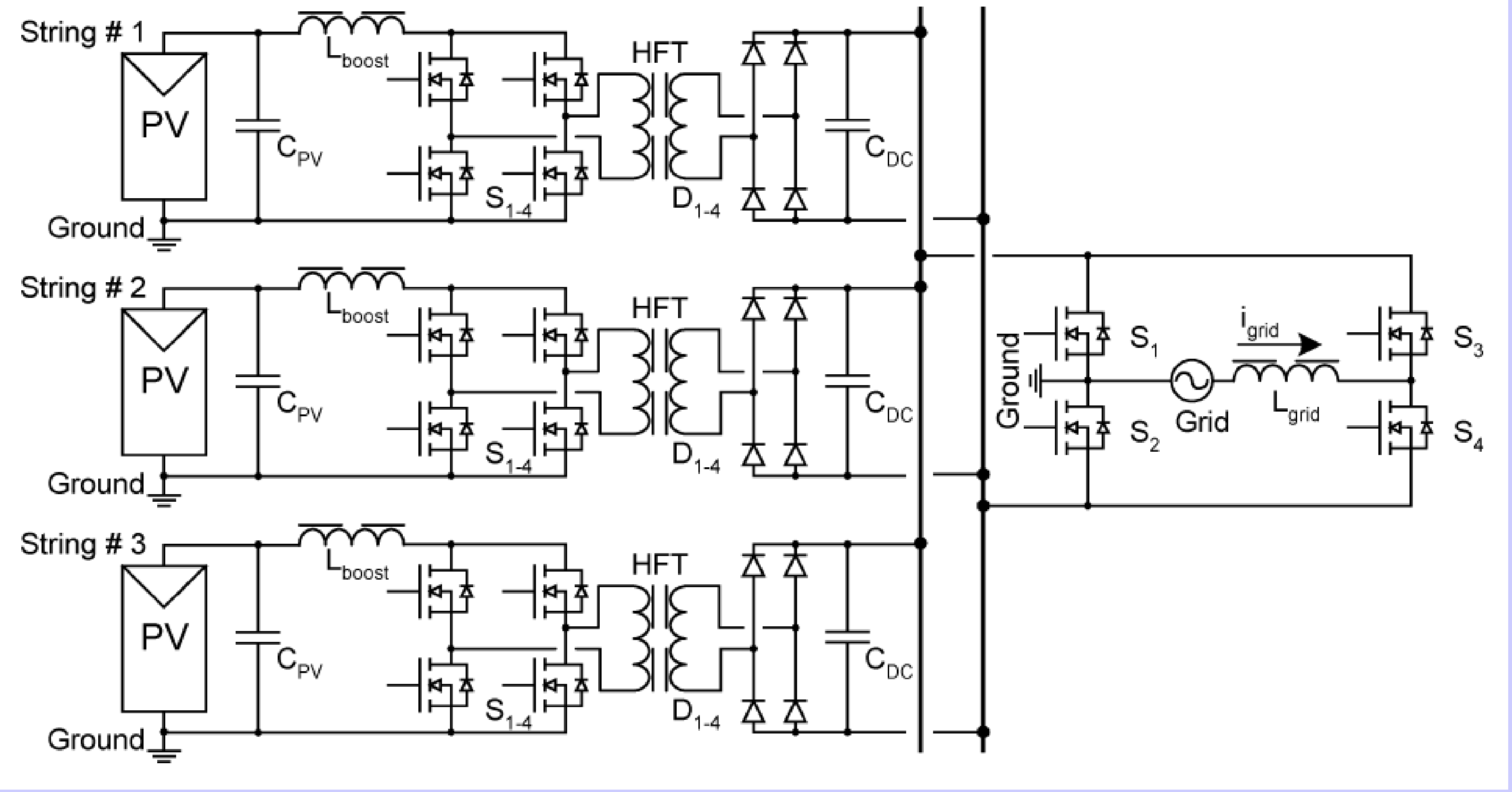
Example of Dual-Stage Solutions

- Boost converters plus a PWM modulated half-bridge inverter (SMA SunnyBoy 5000TL)



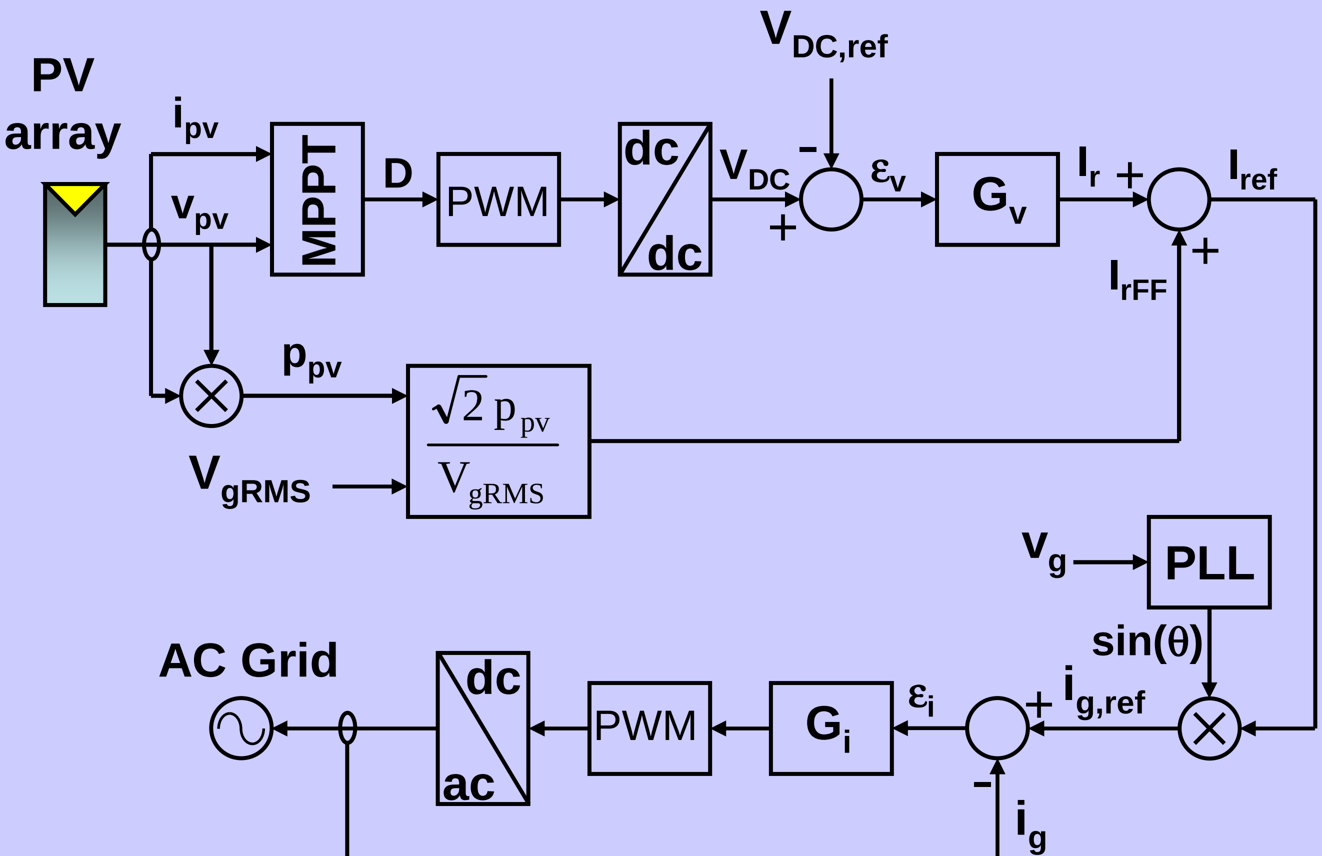
Example of Dual-Stage Solutions

- Isolated boost converters plus a PWM modulated full-bridge inverter (Powerlink PV 4.5kW)



Control Implementation

- Dual stage system configuration



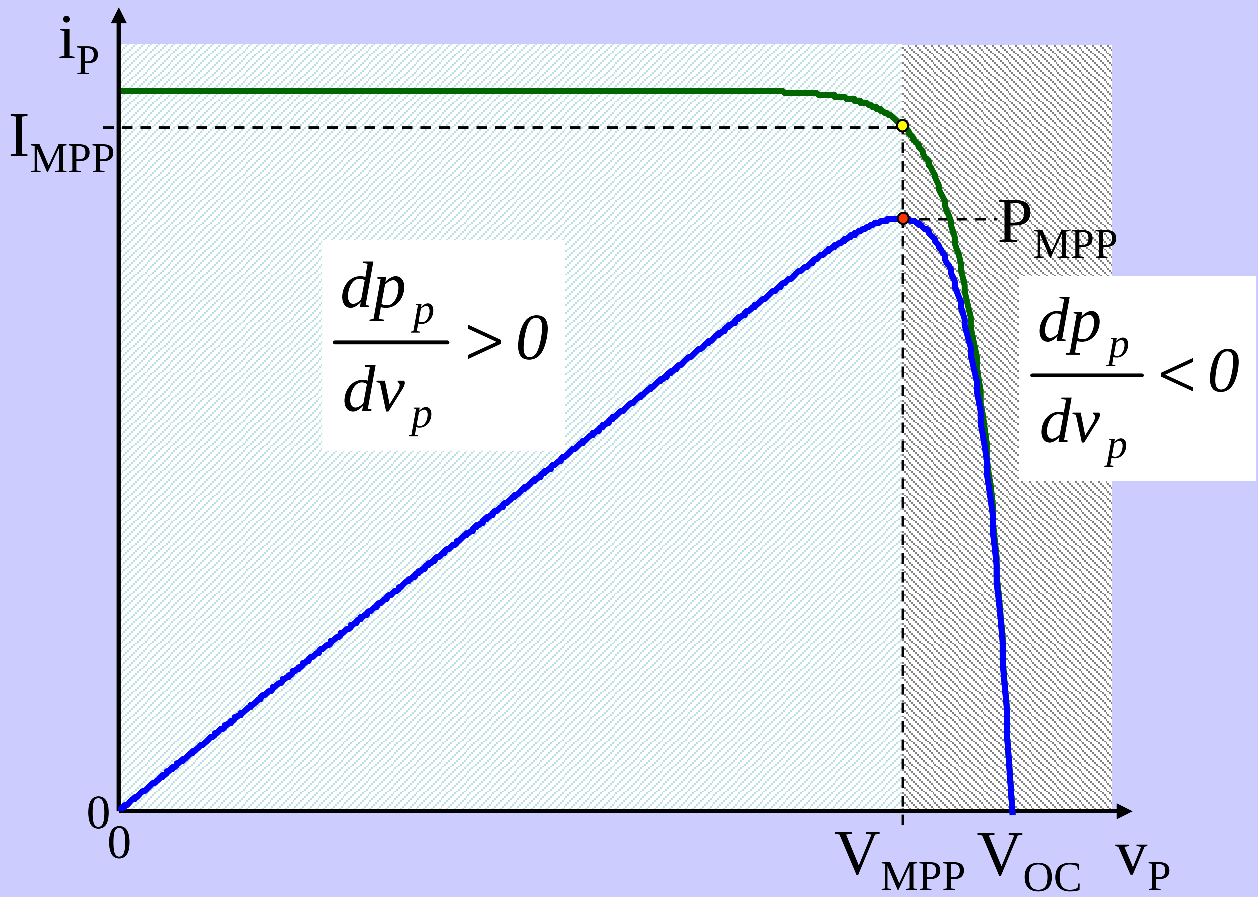
Maximum Power Point Tracking

- The photovoltaic module maximum power point changes with time and operating conditions like illumination level and temperature.
- All modern photovoltaic systems include a switching converter aimed to control the photovoltaic module operating point, i.e. that implements a Maximum Power Point Tracking (**MPPT**) function.
- The effectiveness of an MPPT technique is defined as the ratio between the extracted power and the maximum available power, i.e.:

$$\eta_{MPPT} = \frac{P_P}{P_{MPP}}$$

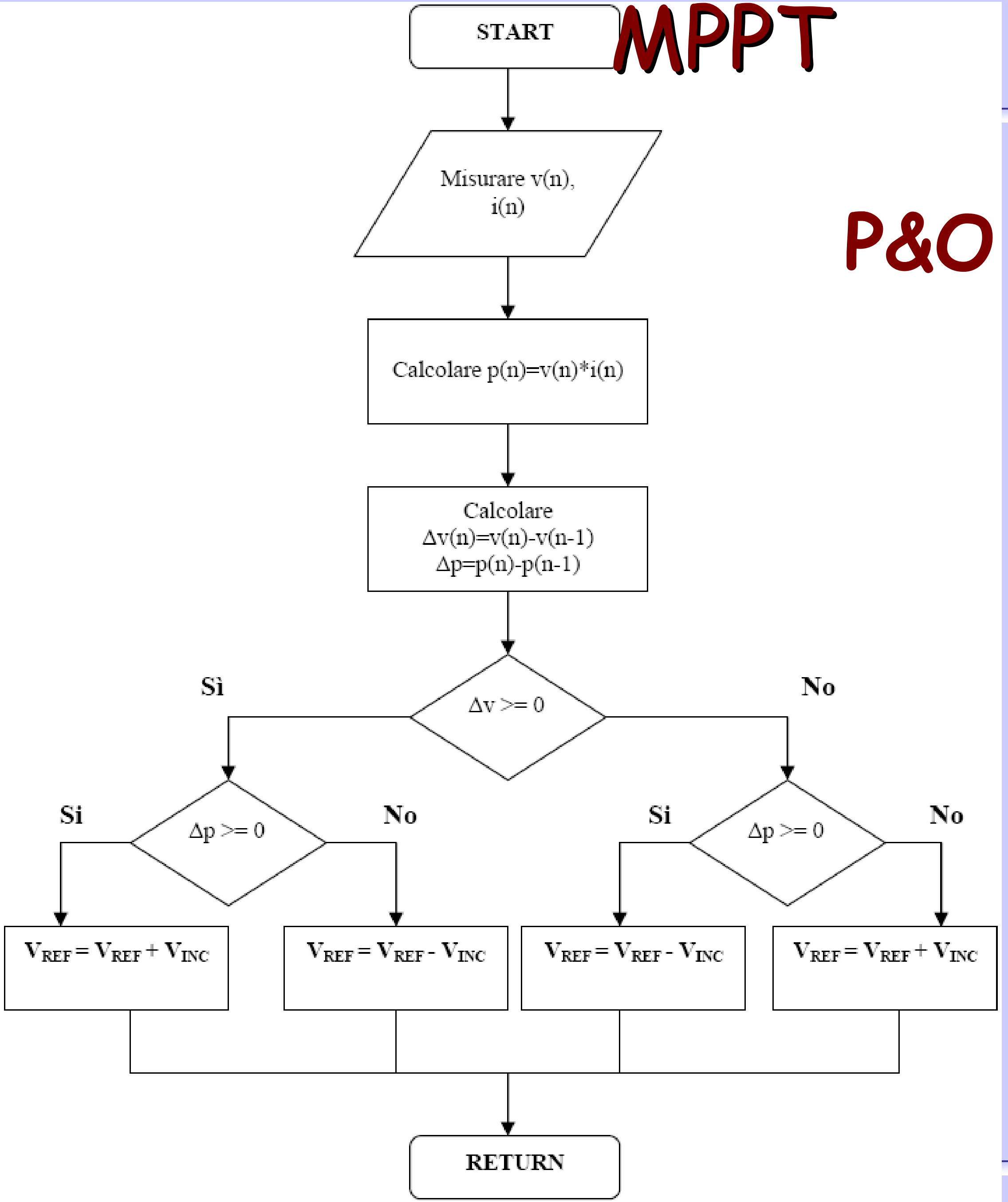
MPPT

- i_p-v_p and p_p-v_p static characteristics



MPPT

P&O (Hill Climbing)



MPPT: P&O (Hill Climbing)

- Easily implemented in digital form (microcontroller).
- Existence of limit cycle oscillations (LCO) around the MPP
- Constant or variable perturbation amplitude (high perturbation amplitude means high speed of response but lower effectiveness due to limit cycle oscillations).

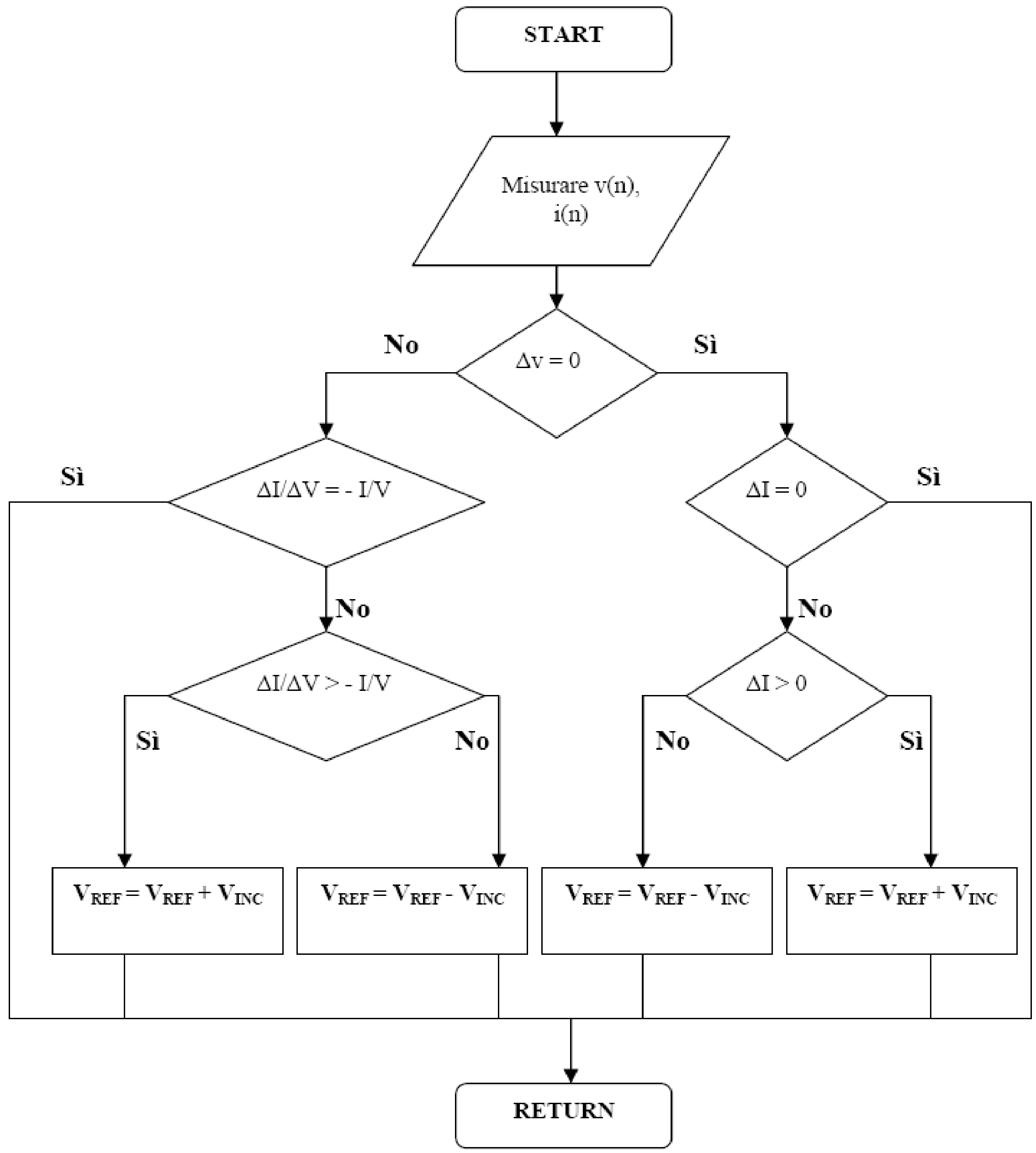
MPPT: Incremental Conductance

$$\left.\frac{dp_P}{dv_P}\right|_{MPP} = \left.\frac{d(v_P i_P)}{dv_P}\right|_{MPP} = \left. v_P\frac{di_P}{dv_P} + i_P\right|_{MPP} = V_{MPP}\left.\frac{di_P}{dv_P}\right|_{MPP} + I_{MPP}$$

$$\left.\frac{dp_P}{dv_P}\right|_{MPP} = 0 \quad\Rightarrow\quad \left.\frac{di_P}{dv_P}\right|_{MPP} = -\frac{I_{MPP}}{V_{MPP}}$$

$$\begin{cases} \dfrac{di_P}{dv_P} = -\dfrac{I_{MPP}}{V_{MPP}} & \text{at MPP} \\ \dfrac{di_P}{dv_P} < -\dfrac{I_{MPP}}{V_{MPP}} & \text{right of MPP} \\ \dfrac{di_P}{dv_P} > -\dfrac{I_{MPP}}{V_{MPP}} & \text{left of MPP} \end{cases}$$

MPPT: Incremental Conductance



MPPT: Incremental Conductance

- Easily implemented in digital form (microcontroller).
- Able, in theory, to reach the exact MPP. In practice, due to quantization and limited resolution, limit cycle oscillations exist
- Constant or variable perturbation amplitude (high perturbation amplitude means high speed of response but lower effectiveness due to limit cycle oscillations).

MPPT: Ripple Correlation

- Based on correlation between AC components of output power and voltage (or current) of PV panel

$$\overline{x}(t) = k\int_0^t \frac{dp_p(x)}{dx}\,d\tau$$

where x stands for v_P or i_P

MPPT: Ripple Correlation

$$\overline{v}(t) = k_v \int_0^t \frac{dp(v)}{dv} d\tau \qquad \overline{i}(t) = k_i \int_0^t \frac{dp(i)}{di} d\tau$$

The use of a positive weighting function allows to consider time derivatives of variables

$$\overline{v}(t) = k \int_0^t \frac{dp(v)}{dv} \left(\frac{dv}{d\tau} \right)^2 d\tau = k \int_0^t \frac{dp(v)}{d\tau} \left(\frac{dv}{d\tau} \right) d\tau$$

Weighting function

MPPT: Ripple Correlation

If a switching converter is used to process the PV power, the duty-cycle can be used to control PV voltage or current....

$$d(t) = -k_d \int_0^t \frac{dp(v)}{d\tau}\left(\frac{dv}{d\tau}\right)d\tau = k_d \int_0^t \frac{dp(i)}{d\tau}\left(\frac{di}{d\tau}\right)d\tau$$

......and the power and voltage derivatives can be substituted by switching ripple components

$$d(t) = -k_d \int_0^t \tilde{p}\,\tilde{v}\,d\tau = k_d \int_0^t \tilde{p}\,\tilde{i}\,d\tau$$

MPPT: Ripple Correlation

Alternative approach: only sign information can be used

$$d(t) = -k_d \int_0^t \mathrm{sign}\left(\frac{dp}{d\tau}\right)\mathrm{sign}\left(\frac{dv}{d\tau}\right)d\tau = k_d \int_0^t \mathrm{sign}\left(\frac{dp}{d\tau}\right)\mathrm{sign}\left(\frac{di}{d\tau}\right)d\tau$$

$$d(t) = -k_d \int_0^t \mathrm{sign}(\tilde{p})\,\mathrm{sign}(\tilde{v})\,d\tau = k_d \int_0^t \mathrm{sign}(\tilde{p})\,\mathrm{sign}(\tilde{i})\,d\tau$$

MPPT: Current Sweep

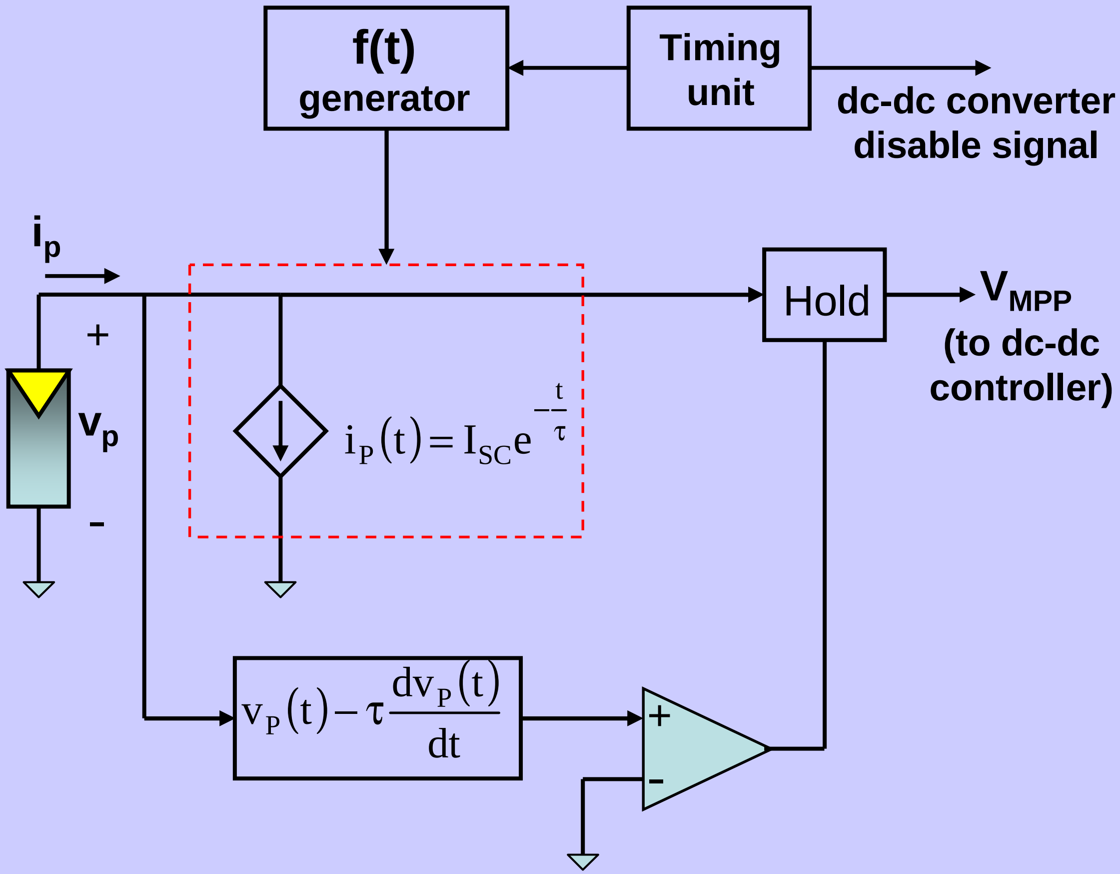
- Based on periodical measurement of i_p-v_p characteristic through a current sweep
- Some loss of effectiveness due to the measurement process

$$i_P(t) = f(t) = k\frac{df(t)}{dt} \qquad \left(f(t) = I_{SC} e^{-\frac{t}{\tau}} \right)$$

$$\frac{dp_P(t)}{dt} = v_P(t)\frac{df(t)}{dt} + f(t)\frac{dv_P(t)}{dt} = 0$$

$$\frac{dp_P(t)}{dt} = \left(v_P(t) + k\frac{dv_P(t)}{dt} \right)\frac{df(t)}{dt} = 0 \quad \Rightarrow \quad v_P(t) + k\frac{dv_P(t)}{dt} = 0$$

MPPT: Current Sweep



Indirect MPPT Methods

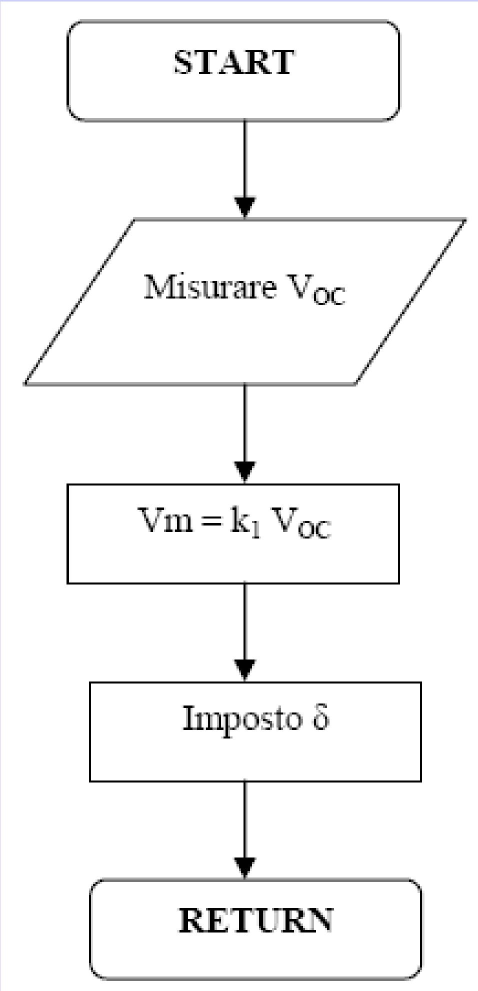
- Based on correlation between V_{MPP} and V_{OC} or between I_{MPP} and I_{SC}
- Effectiveness loss due to uncertainty of proportionality factors k_1 and k_2

$$V_{MPP} = k_1 V_{OC} \qquad I_{MPP} = k_2 I_{SC}$$

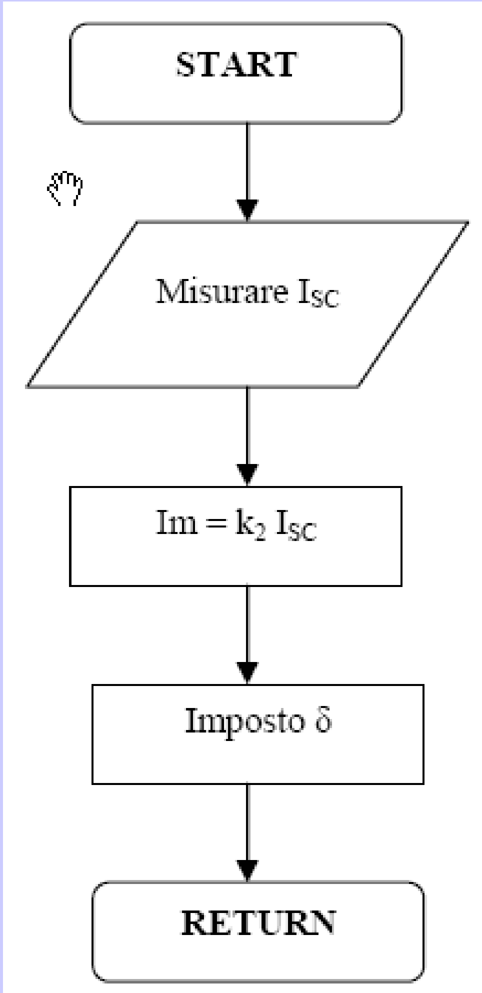
$0.71 \geq k_1 \geq 0.78$ and $0.78 \geq k_2 \geq 0.92$

Indirect MPPT Methods

Constant voltage



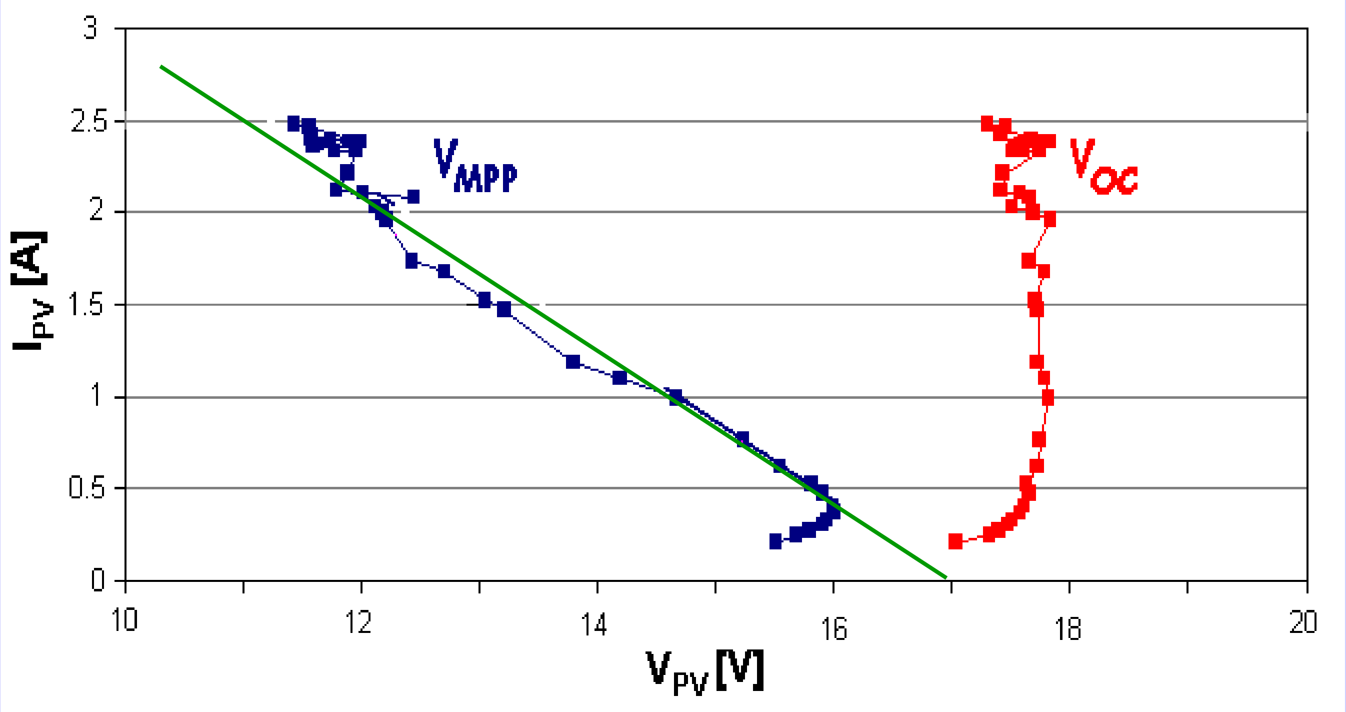
Constant current



Indirect MPPT Methods

Voltage Linear Reference (VLR)

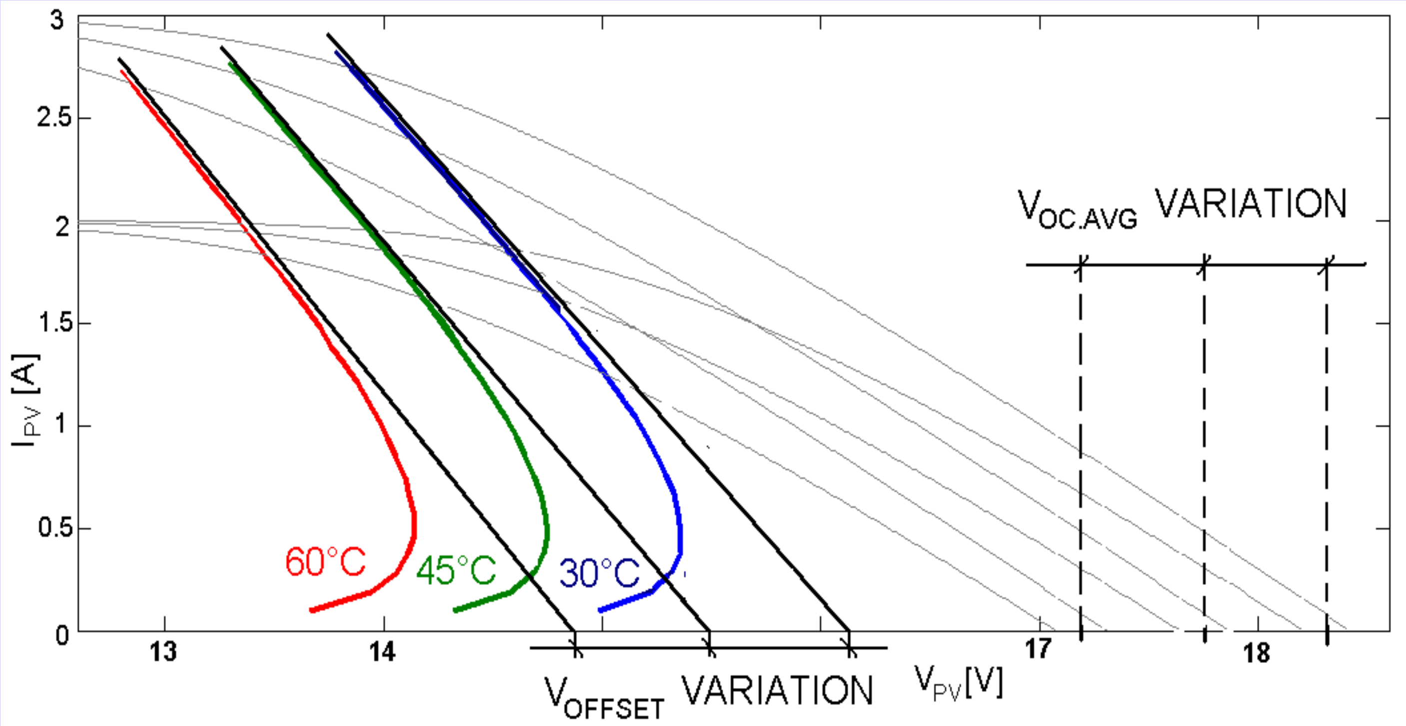
Measurements on a 55W PV module



NOTE: at high irradiation levels the MPP locus resembles a straight line

Indirect MPPT Methods: VLR

- Effect of temperature variation: the VLR intercept on the horizontal axis has a temperature coefficient similar to the open circuit voltage V_{OC}



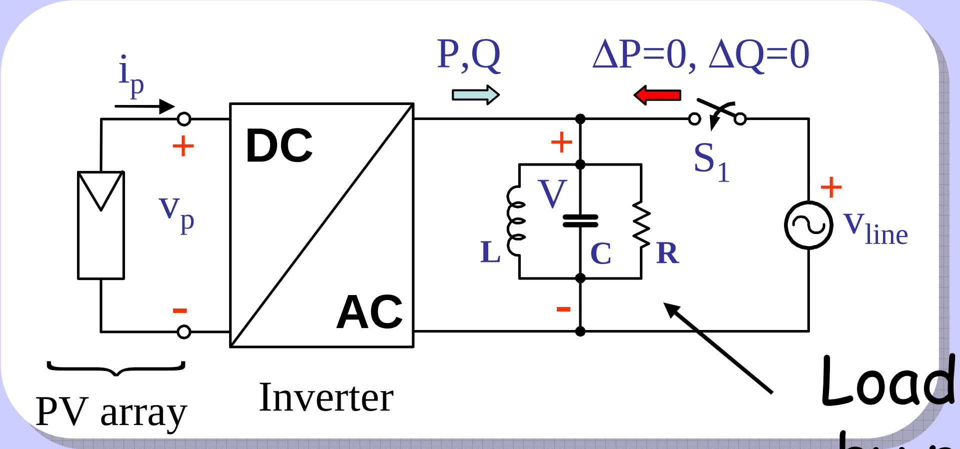
Anti-islanding

- Islanding: a continuous operation of an inverter (or other generator) connected to the utility grid when the latter is disconnected

To be avoided because:

- It can be harmful to people (particularly for the grid management personnel)
- It can cause malfunction and/or failure of connected loads due to voltage parameters out of range (RMS value, frequency, etc.)

Anti-islanding

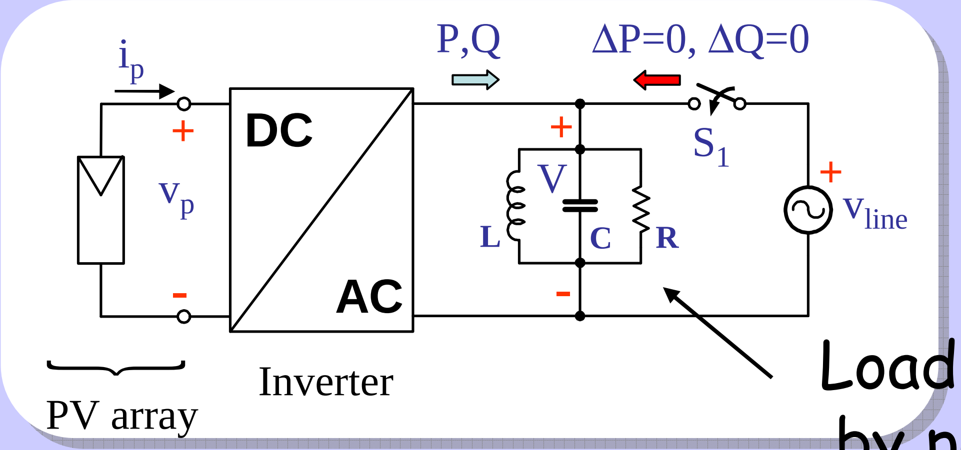


Load represented by parallel R-L-C

Conditions for islanding operation:

- Load matching: the active and reactive power generated by the inverter should match the load needs (active and reactive power delivered by the grid should be zero just before disconnection)

Anti-islanding



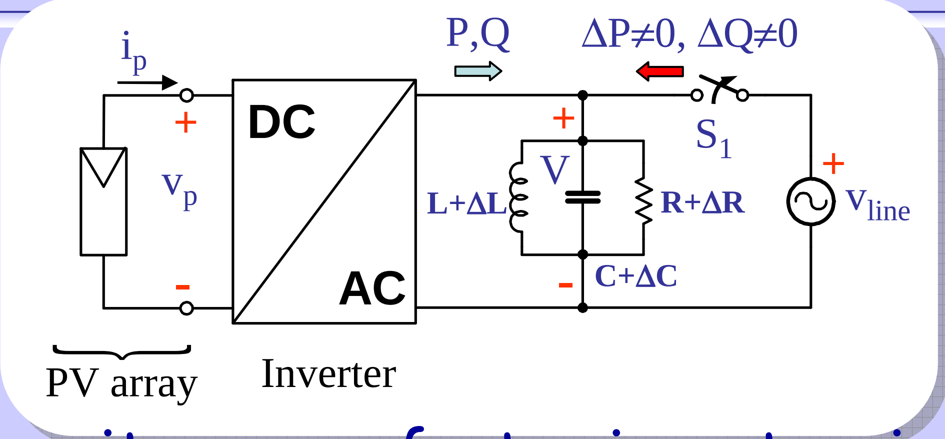
Load represented by parallel R-L-C

- In practice, it is not strictly necessary for the active and reactive power coming from the utility grid to be zero
- Anti-islanding methods differ in the amplitude and shape of the *Non detection Zone (NDF)* in the ΔP-ΔQ plane

Anti-islanding Methods

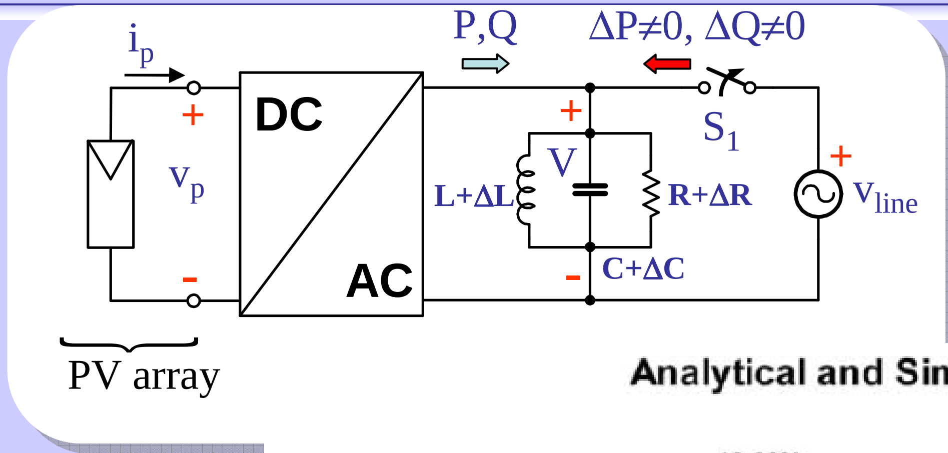
- *Passive Methods*: some grid parameters (voltage amplitude, frequency, etc.) are monitored at the Point of Common Coupling (PCC) (no degradation of power quality, but poor effectiveness)
- *Active Methods*: perturbations are introduced so as to detect the presence of the utility grid (some degradation of power quality is expected)
- *Methods taken by the utility grid manager* (measurements, communications, etc.)

Passive Methods: OUV - OUF

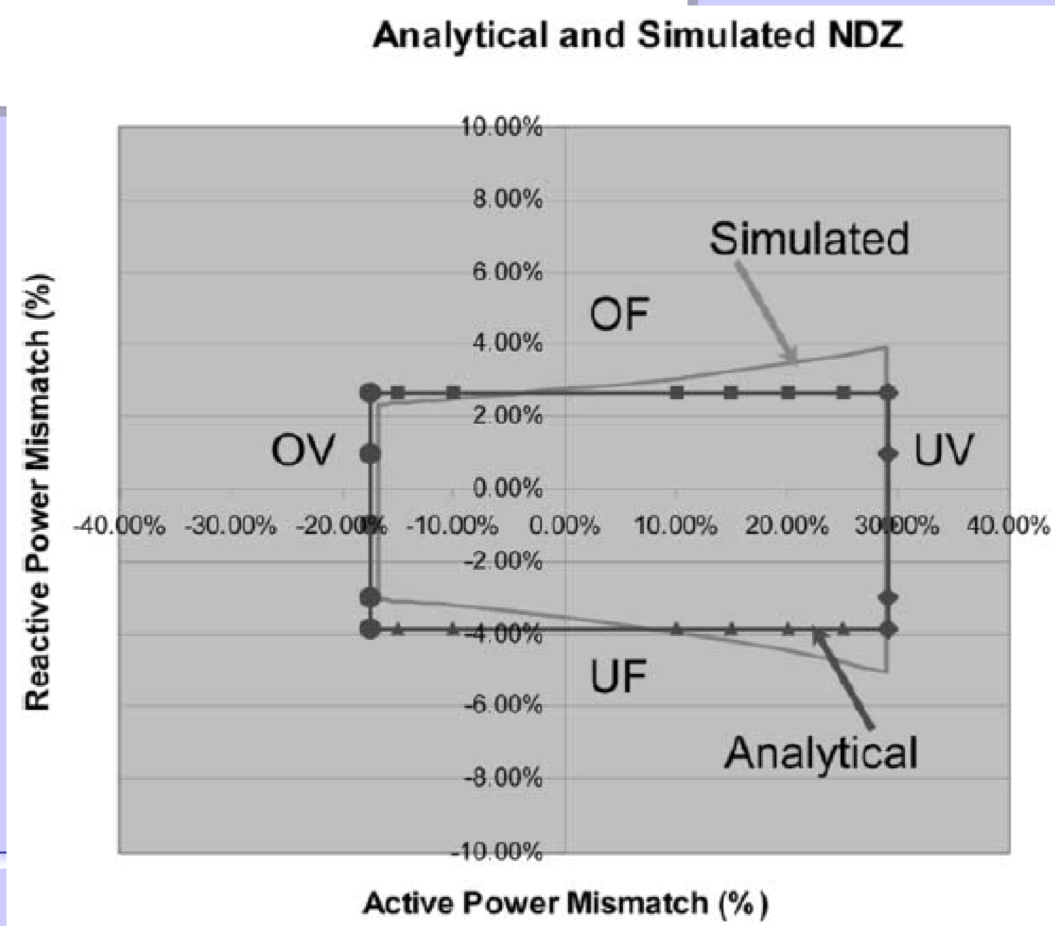


- Assuming a unity power factor inverter, i.e. Q = 0, if before disconnection $\Delta P \neq 0$, the RMS voltage at PCC will vary, thus triggering the Over/Under Voltage protection circuitry
- Assuming a unity power factor inverter, i.e. Q = 0, if before disconnection $\Delta Q \neq 0$, the current-to-voltage phase shift induced by the load will force the inverter to move the frequency toward the load resonant value so as to achieve Q = 0, thus triggering the Over/Under Frequency protection circuitry

Passive Methods: OUV - OUF

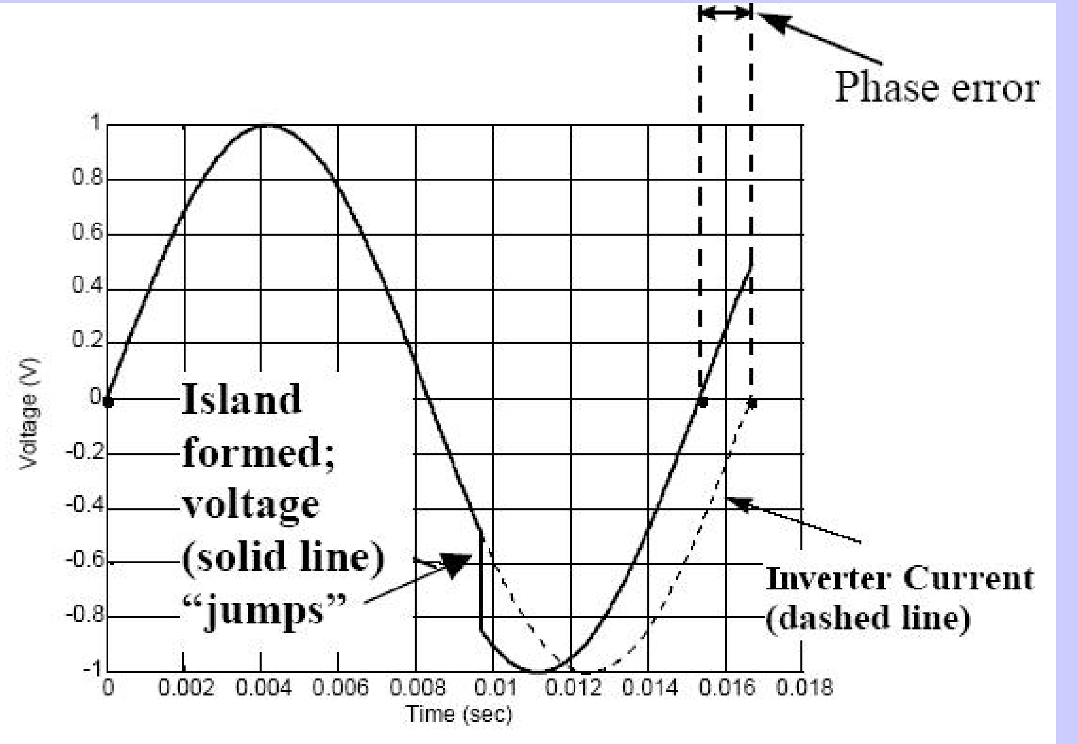


- Pros:
  - Low cost
  - Easy to implement
- Cons:
  - Wide Non Detection Zone



Passive Methods: Phase Jump Detection

- *For current-controlled inverters:* the internal PLL synchronize the current with the voltage zero crossing. When the island is formed, the voltage at PCC undergoes a jump (due to the reactive nature of the load) that will lead to a different zero crossing that can be detected



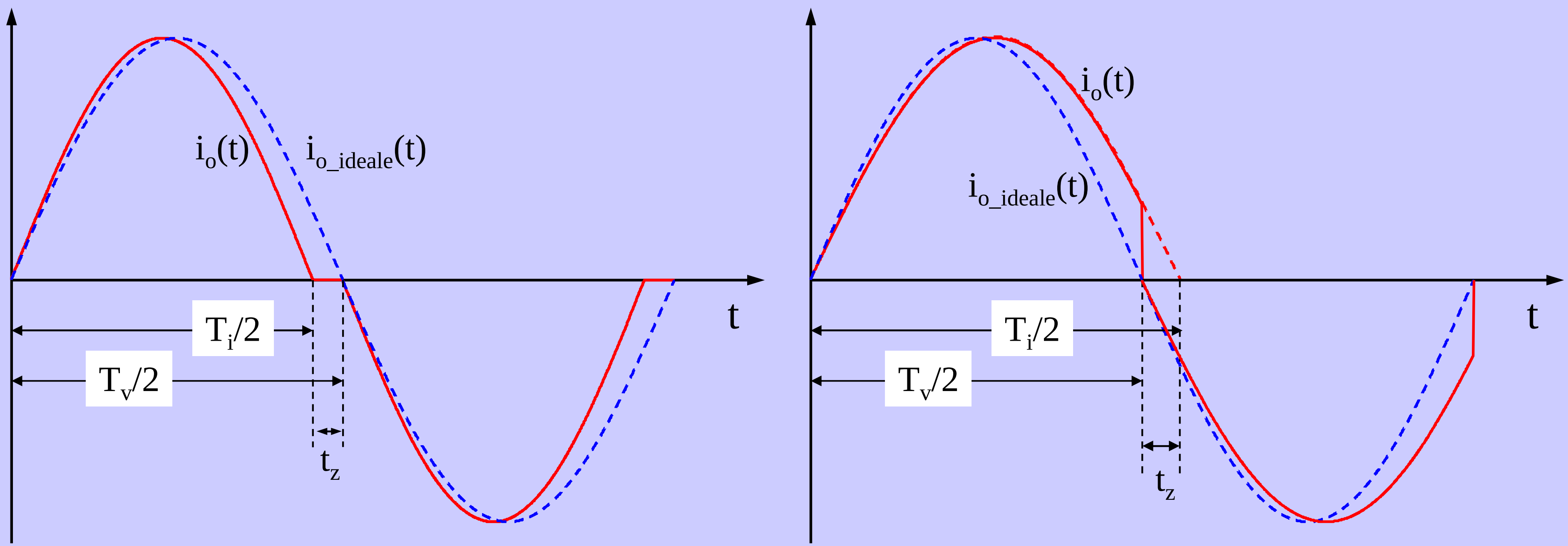
- Pros:
  - Easy to implement
- Cons:
  - Difficult to select the threshold

Passive Methods: Harmonic Detection

- *It is based on the measurement of the Total Harmonic Distortion of the voltage at PCC* (in the presence of the grid, the current harmonics produced by non linear loads and the inverter itself are bypassed by the small line impedance)
- Pros:
  - Easy to implement
- Cons:
  - Affected by the line impedance
  - Difficult to select the threshold
  - Effectiveness reduction due to loads with low-pass characteristics

Active Methods: Active Frequency Drift

- *A current-controlled inverter forces a frequency shift in the output current:* in the presence of the grid, the line frequency zero crossing of the voltage at PCC reset the process



Active Methods: Active Frequency Drift

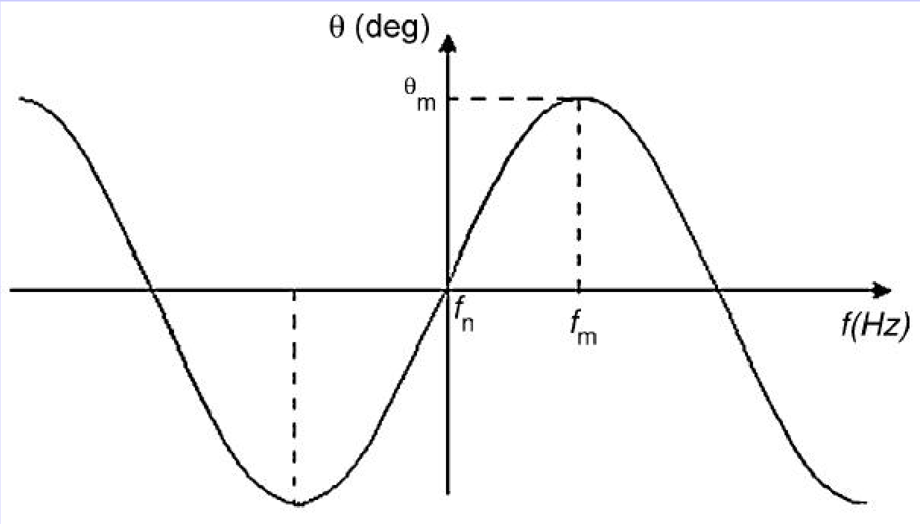
- *A current-controlled inverter forces a frequency shift in the output current:* in the presence of the grid, the line frequency zero crossing of the voltage at PCC reset the process

- Pros:
  - Easy to implement in a microprocessor
- Cons:
  - Increase of current harmonic distortion
  - Affected by the presence of other inverters connected to the same PCC

Active Methods: Slip-Mode Frequency Shift

- *It is based on a positive feedback on the phase of the voltage at PCC*

Current-to-voltage phase displacement

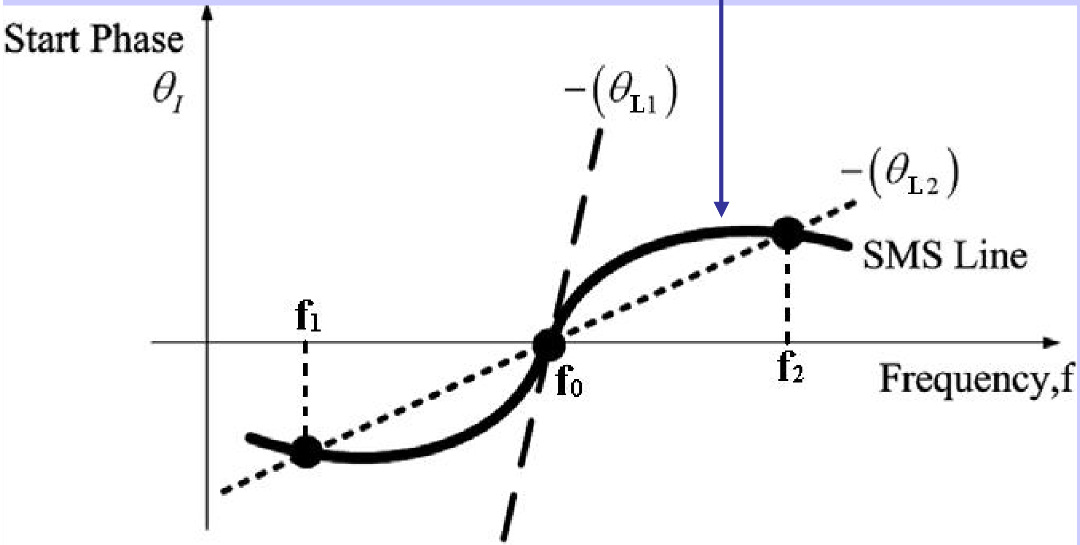


$$\theta[k] = \theta_m \mathrm{sen}\left(\frac{\pi(f[k-1]-f_n)}{2(f_m - f_n)}\right)$$

θ_m = maximum phase angle (example: 10°)
f_n = nominal frequency
f_m = frequency corresponding to θ_m

Active Methods: Slip-Mode Frequency Shift

Inverter current-to-voltage phase displacement



Positive feedback condition:

$$\left.\frac{d\theta}{df}\right|_{f=f_n} = \frac{\pi\theta_m}{2(f_m - f_n)} > \frac{d\theta_L}{df}$$

θ_L = load phase

Active Methods: Slip-Mode Frequency Shift

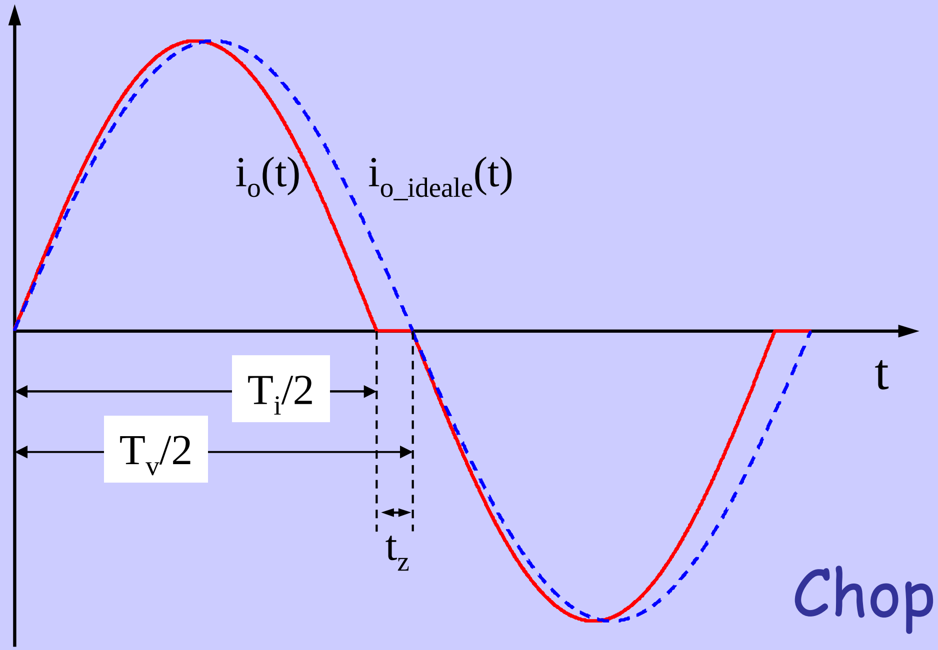
- *It is based on a positive feedback on the phase of the voltage at PCC*

- Pros:
  - Easy to implement in a microprocessor
- Cons:
  - Slight degradation of power quality
  - NDZ caused by high Q resonant loads

Active Methods: Sandia Frequency Shift

- *It is based on a positive feedback on the frequency of the voltage at PCC*

$$cf = cf_0 + K(f - f_n)$$



cf_0 = chopping fraction at zero frequency error

f_n = nominal frequency

K = gain

Chopping fraction: $$cf = \frac{2t_z}{T_v}$$

Active Methods: Sandia Frequency Shift

- *It is based on a positive feedback on the frequency of the voltage at PCC*

- Pros:
  - Easy to implement in a microprocessor
  - Good trade-off between effectiveness and power quality
- Cons:
  - Slight degradation of power quality
  - NDZ caused by high Q resonant loads

Active Methods: Sandia Voltage Shift

- *It is based on a positive feedback on the RMS value of the voltage at PCC:* if the voltage increases the inverter increases the injected current (i.e. the power), and viceversa. If the grid is connected, the power variation is absorbed by the grid itself.

- Pros:
  - Easy to implement in a microprocessor
- Cons:
  - Slight degradation of system effectiveness because the variation of the inverter output power moves the PV operating point out of MPP

Anti-Islanding Methods

- No one of the proposed anti-islanding methods can be considered satisfactory in terms of Non Detection Zone and/or dynamic response
- The best approach could be to implement two or more different methods so as to minimize the probability of false triggering and/or lack of intervention

Renewable Energy Sources for Distributed Generation in Smart Grids: the role of Power Electronics

Speaker: G. Spiazzi

P. Tenti, L. Rossetto, G. Spiazzi, S. Buso, P. Mattavelli, L. Corradini
Dept. of Information Engineering – DEI
University of Padova

Seminar Outline

- From “Traditional” to “Smart Grids”
  - Challenges & potentialities
- Power electronics for renewable energy sources
  - Principles of photovoltaic energy generation
  - Power electronics for interfacing PV panels with the utility grid
    - Inverter operation
    - Maximum power point tracking techniques
    - Anti-islanding methods
- Traditional power theories
  - Budeanu power theory
  - Fryze power theory
  - Kusters & Moore theory (time domain)
  - Czarnecki theory (frequency domain)
- Conservative power theory
  - Mathematical and physical foundations of the theory
  - Examples

Need for a revision of power terms ?

- In a situation where:
  - the grid is weak,
  - frequency may change,
  - voltages are asymmetrical,
  - distortion affects voltages and currents,
- are the usual definitions of reactive, unbalance and distortion power still valid?
- Which is the physical meaning of such terms?
- Are they useful for compensation?
- To which extent are power measurements affected by source non-ideality?
- It is possible to identify supply and load responsibility on voltage distortion and asymmetry at a given network port?

Power theory objectives

Goals:

- Power transfer analysis in presence of voltage and current distortion
- Identification of the sources of grid distortion
- Reactive and harmonic compensation
- Definition of suitable measurement methods for a correct accountability

Some power theories

- In the frequency domain
  - Budeanu (1927)
  - Sheperd & Zakikhani (1971)
  - Czarnecki (1984 …)

- In the time domain
  - Fryze (1931)
  - Kusters & Moore (1975)
  - Depenbrock (1993 …)
  - Akagi & Nabae (1983 …)
  - Tenti, Mattavelli & Tedeschi (2003 …)

Mathematical background

Internal product $\langle f,g\rangle = \frac{1}{T}\int_0^T f(t)\cdot g(t)\,dt$ $P = \langle u,i\rangle = \frac{1}{T}\int_0^T u(t)\cdot i(t)\,dt$

Norm $\|f\| = \sqrt{\langle f,f\rangle}$ $\|u\| = U_{RMS} = \sqrt{\frac{1}{T}\int_0^T u^2(t)\,dt}$

Cauchy-Schwartz inequality $|\langle f,g\rangle| \le \|f\|\|g\|$ $P \le \|U_{RMS}\|\|I_{RMS}\| = S$

Mathematical background

Orthogonality $f_1 \perp f_2 \leftrightarrow \langle f_1, f_2 \rangle = 0$

$f = f_1 + f_2 \qquad \| f \|^2 = \| f_1 \|^2 + \| f_2 \|^2$ **Sum of orthogonal quantities**

Fundamental properties

$S = 0 \leftrightarrow U_{RMS} = 0 \vee I_{RMS} = 0$ Law of null product

$S \geq P$ Cauchy – Schwartz inequality

$S \equiv P \leftrightarrow i(t) = \alpha u(t)$ Given U_{rms} and I_{rms} the apparent power is the maximum deliverable active power (resistive load R= U_{rms}/ I_{rms})

Budeanu Theory (1927)

- Derivation of reactive power definition in distorted regimes from the classical sinusoidal one.

$$Q = Q_B \stackrel{\Delta}{=} \sum_{n=1}^{\infty} U_n I_n \sin\varphi_n$$

- Definition of Distortion Power:

$$P^2 + Q_B^2 \leq S^2 \quad \Rightarrow \quad D \stackrel{\Delta}{=} \sqrt{S^2 - (P^2 + Q_B^2)}$$

$$S^2 = P^2 + Q_B^2 + D^2$$

Why Q_B definition is wrong?

$$u(t) = \sqrt{2}[U_1 \sin(\omega_1 t) + U_3 \sin(3\omega_1 t)]$$

$$\dot{Y}_n = Y_n e^{j\varphi_n} = Y_n \cos(\varphi_n) + jY_n \sin(\varphi_n)$$

$$\dot{Y}_1 = jB_1 = j\left(\omega_1 C_a - \frac{1}{\omega_1 L_a}\right)$$

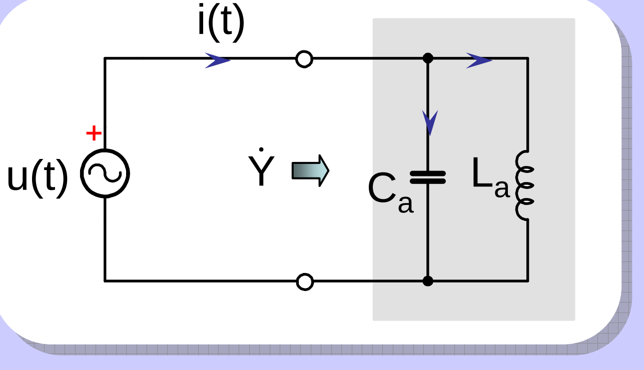
$$\dot{Y}_3 = jB_3 = j\left(3\omega_1 C_a - \frac{1}{3\omega_1 L_a}\right)$$

Let us impose a zero reactive power: $Q = -\sum_{n=1}^{\infty} U_n I_n \sin(\varphi_n) = -U_1^2 B_1 - U_3^2 B_3 = 0$

$$\begin{cases} \dfrac{U_1^2}{U_3^2}\left(\omega_1 C_a - \dfrac{1}{\omega_1 L_a}\right) + \left(3\omega_1 C_a - \dfrac{1}{3\omega_1 L_a}\right) = 0 \\ \omega_1 C_a - \dfrac{1}{\omega_1 L_a} = B_1 \end{cases}$$

$$\begin{cases} \left(3 + \dfrac{U_1^2}{U_3^2}\right)\omega_1 C_a - \left(\dfrac{U_1^2}{U_3^2} + \dfrac{1}{3}\right)\dfrac{1}{\omega_1 L_a} = 0 \\ \omega_1 C_a - \dfrac{1}{\omega_1 L_a} = B_1 \end{cases}$$

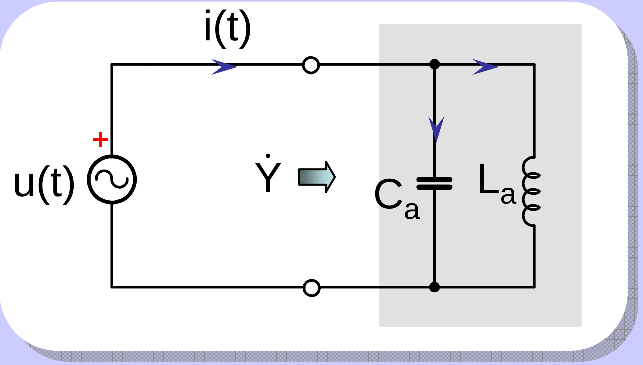
$$\begin{cases} L_a = -\dfrac{1}{3B_1\omega_1}\left(\dfrac{8}{3 + \dfrac{U_1^2}{U_3^2}}\right) \\ C_a = -\dfrac{B_1}{8\omega_1}\left(1 + 3\dfrac{U_1^2}{U_3^2}\right) \end{cases}$$

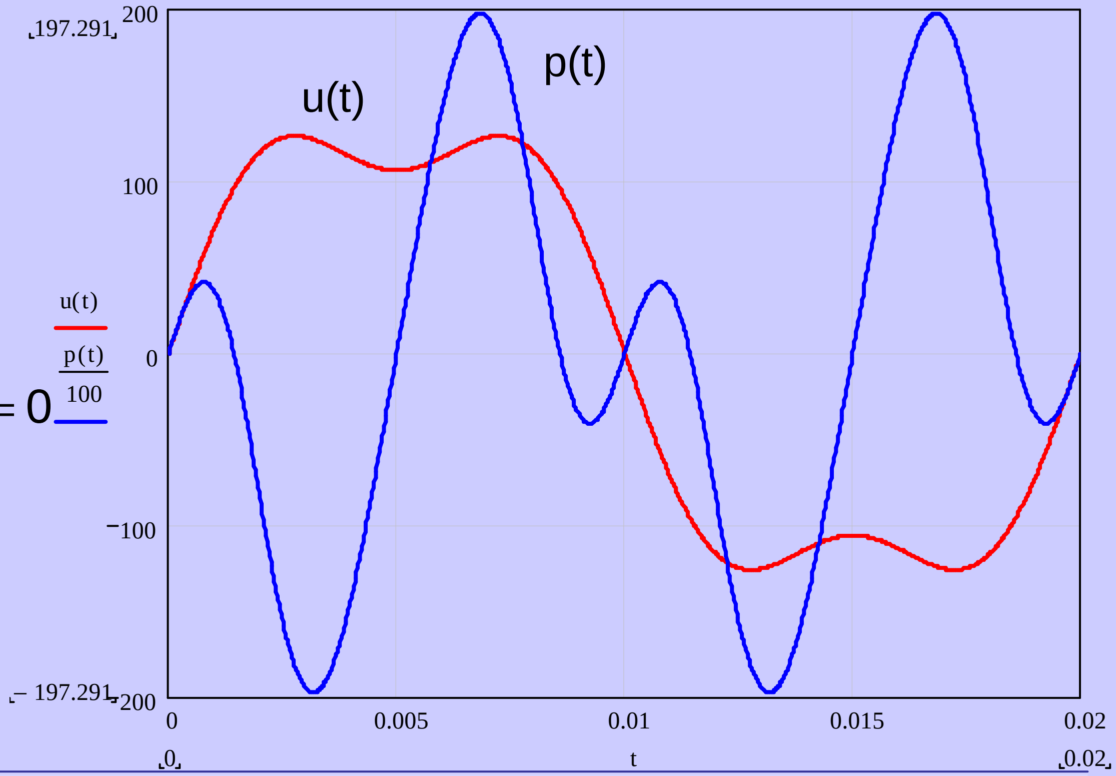


Why Q_B definition is wrong?

Example: $u(t) = \sqrt{2}[U_1 \sin(\omega_1 t) + U_3 \sin(3\omega_1 t)]$ $\quad U_1 = 100V$, $U_3 = 25V$

$$\omega_1 = 1\frac{rad}{s}, \quad B_1 = -\frac{1}{4}S \Rightarrow \begin{cases} L_a = \frac{32}{57}H \\ C_a = \frac{49}{32}F \end{cases} \Rightarrow i(t) = \sqrt{2}\left[I_1 \sin\left(\omega_1 t - \frac{\pi}{2}\right) + I_3 \sin\left(3\omega_1 t + \frac{\pi}{2}\right)\right] \quad I_1 = 25A, \; I_3 = 100A$$





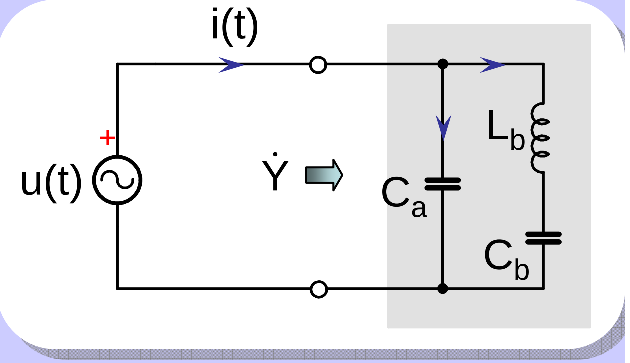
$$Q_B = -\sum_{n=1}^{\infty} U_n I_n \sin(\varphi_n) = -U_1^2 B_1 - U_3^2 B_3 = 0$$

There are energy oscillations in spite of zero Budeanu's reactive power Q_B

Why D definition is wrong?

Generic distorted line voltage: $u(t) = \sqrt{2}\sum_{n=1}^{\infty} U_n \sin(n\omega_1 t)$

Generic linear load: $\dot{Y}_n = Y_n e^{j\varphi_n} = Y_n \cos(\varphi_n) + jY_n \sin(\varphi_n)$



$$i(t) = \sqrt{2}\sum_{n=1}^{\infty} I_n \sin(n\omega_1 t + \varphi_n) = \sqrt{2}\sum_{n=1}^{\infty} U_n Y_n \sin(n\omega_1 t + \varphi_n)$$

Apparent power: $S^2 = \left(\sum_{n=1}^{\infty} U_n^2\right)\left(\sum_{n=1}^{\infty} Y_n^2 U_n^2\right) = \sum_{n=1}^{\infty} Y_n^2 U_n^4 + \sum_{n=1}^{\infty}\sum_{k\neq n}^{\infty} U_n^2 U_k^2 Y_k^2 = \sum_{n=1}^{\infty} Y_n^2 U_n^4 + \sum_{n=1}^{\infty}\sum_{k=n+1}^{\infty} U_n^2 U_k^2 \left(Y_n^2 + Y_k^2\right)$

Active power: $P = \sum_{n=1}^{\infty} U_n I_n \cos(\varphi_n) = \sum_{n=1}^{\infty} U_n^2 Y_n \cos(\varphi_n)$

Reactive power: $Q = -\sum_{n=1}^{\infty} U_n I_n \sin(\varphi_n) = -\sum_{n=1}^{\infty} U_n^2 Y_n \sin(\varphi_n)$

Distortion power: $D^2 = S^2 - \left(P^2 + Q^2\right) = \sum_{n=1}^{\infty}\sum_{k=n+1}^{\infty} U_n^2 U_k^2 \left(Y_n^2 - 2Y_n Y_k \cos(\varphi_n - \varphi_k) + Y_k^2\right)$

$$= \sum_{n=1}^{\infty}\sum_{k=n+1}^{\infty} U_n^2 U_k^2 \left|\dot{Y}_n - \dot{Y}_k\right|^2$$

Why D definition is wrong?

D = 0 if: $\dot{Y}_n = \dot{Y}_k \quad n \neq k$

$$\dot{Y}_n = jB_n = j\left(n\omega_1 C_a - \frac{1}{n\omega_1 L_{eqn}}\right) \qquad L_{eq1} = L_b\left(1-\left(\frac{\omega_b}{\omega_1}\right)^2\right) \quad L_{eq3} = L_b\left(1-\left(\frac{\omega_b}{3\omega_1}\right)^2\right) \quad \omega_b = \frac{1}{\sqrt{L_b C_b}}$$

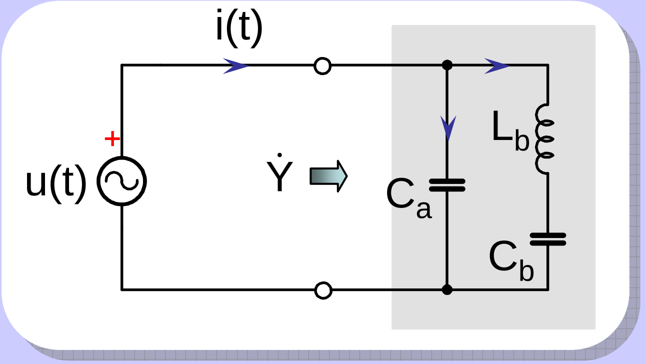
$$u(t) = \sqrt{2}[U_1 \sin(\omega_1 t) + U_3 \sin(3\omega_1 t)]$$

By imposing:
$$\begin{cases} \omega_1 C_a - \dfrac{1}{\omega_1 L_{eq1}} = 3\omega_1 C_a - \dfrac{1}{3\omega_1 L_{eq3}} \\ \omega_1 C_a - \dfrac{1}{\omega_1 L_{eq1}} = B_1 \end{cases}$$

Example: $U_1 = 100\text{V}$, $U_3 = 50\text{V}$

Selecting: $\omega_1 = 1\dfrac{\text{rad}}{\text{s}}$, $B_1 = B_3 = 1\text{S}$, $L_1 = 1\text{H}$

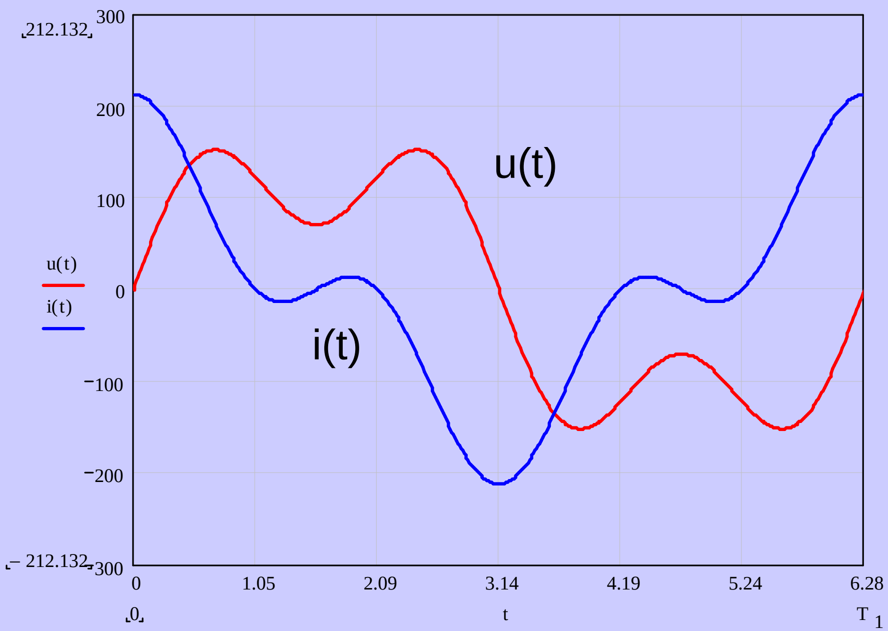
$$\Rightarrow \begin{cases} C_b = \dfrac{1}{3}\text{F} \\ C_a = \dfrac{1}{2}\text{F} \end{cases}$$

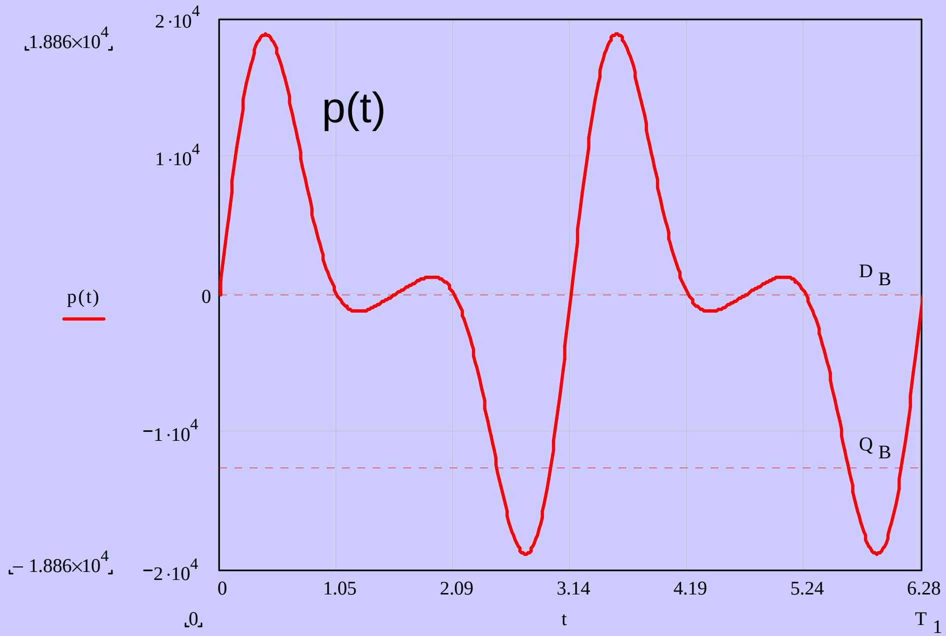


Why D definition is wrong?

$$i(t) = \sqrt{2}\left[I_1 \sin\left(\omega_1 t + \frac{\pi}{2}\right) + I_3 \sin\left(3\omega_1 t + \frac{\pi}{2}\right)\right]$$

$$I_1 = 100A$$
$$I_3 = 50A$$





The load current is distorted in spite of zero distortion power D

Why D definition is wrong?

Non distorted current: $i(t) = \alpha u(t-\tau)$

$$u(t) = \sqrt{2}\sum_n U_n \sin(n\omega_1 t) = \sqrt{2}\sum_n \mathrm{Im}\left\{\dot{U}_n e^{jn\omega_1 t}\right\}$$

$$i(t) = \alpha\sqrt{2}\sum_n \mathrm{Im}\left\{\dot{U}_n e^{jn\omega_1 (t-\tau)}\right\} = \sqrt{2}\sum_n \mathrm{Im}\left\{\dot{Y}_n \dot{U}_n e^{jn\omega_1 t}\right\} \Rightarrow \dot{Y}_n = \alpha e^{-jn\omega_1 \tau}$$

$$u(t) = \sqrt{2}[U_1 \sin(\omega_1 t) + U_3 \sin(3\omega_1 t)] \qquad \omega_1\tau = \frac{\pi}{2} \Rightarrow i(t) = \sqrt{2}\left[I_1 \sin\left(\omega_1 t - \frac{\pi}{2}\right) + I_3 \sin\left(3\omega_1 t - \frac{3\pi}{2}\right)\right]$$

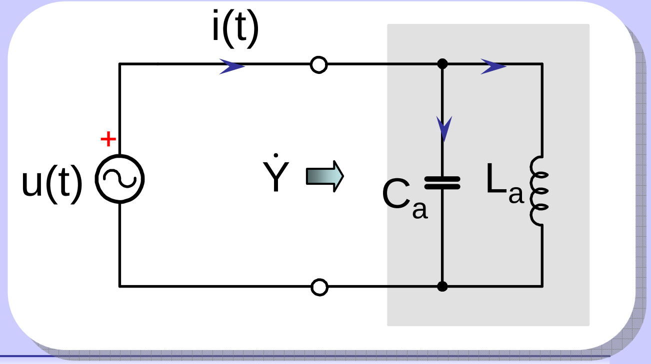
$$\begin{cases} \omega_1 C_a - \dfrac{1}{\omega_1 L_a} = B_1 \\ \omega_1 C_a - \dfrac{1}{\omega_1 L_a} = -\left(3\omega_1 C_a - \dfrac{1}{3\omega_1 L_a}\right) \end{cases} \qquad \begin{cases} \omega_1 C_a - \dfrac{1}{\omega_1 L_a} = B_1 \\ \omega_1 C_a - \dfrac{1}{3\omega_1 L_a} = 0 \end{cases} \qquad \begin{cases} C_a = -\dfrac{B_1}{2\omega_1} \\ L_a = -\dfrac{2}{3B_1\omega_1} \end{cases}$$

Example:

$$U_1 = 100\,\mathrm{V} \qquad U_3 = 30\,\mathrm{V}$$

$$\omega_1 = 1\frac{\mathrm{rad}}{\mathrm{s}} \qquad B_1 = -\frac{1}{2}\mathrm{S} \qquad \alpha = |B_1|$$

$$\Rightarrow \begin{cases} L_a = \dfrac{4}{3}\mathrm{H} \\ C_a = \dfrac{1}{4}\mathrm{F} \end{cases}$$

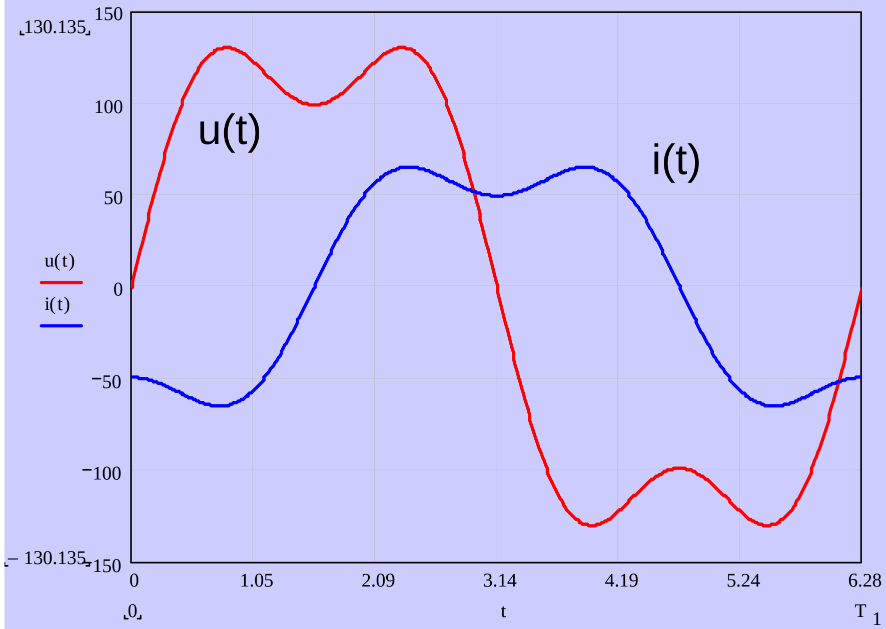


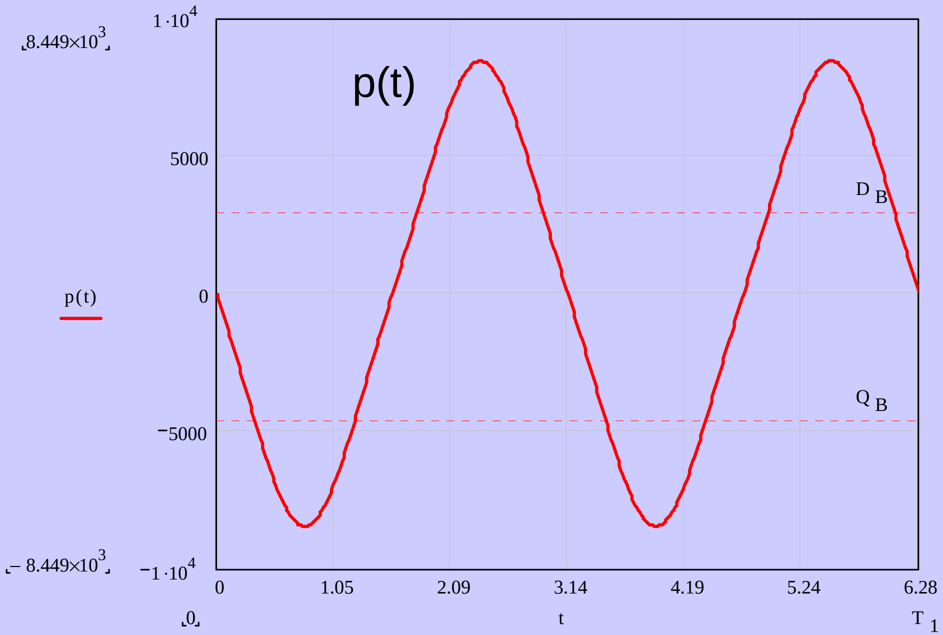
Why D definition is wrong?

$$i(t) = \sqrt{2}\left[I_1 \sin\left(\omega_1 t - \frac{\pi}{2}\right) + I_3 \sin\left(3\omega_1 t - \frac{3\pi}{2}\right)\right]$$

$$I_1 = 50A$$
$$I_3 = 15A$$

$$D = \sqrt{S^2 - \left(P^2 + Q^2\right)} = U_1 U_3 \left|\dot{Y}_1 - \dot{Y}_3\right| = 3kVA$$





The load current is not distorted in spite of a non zero distortion power D

Compensation?

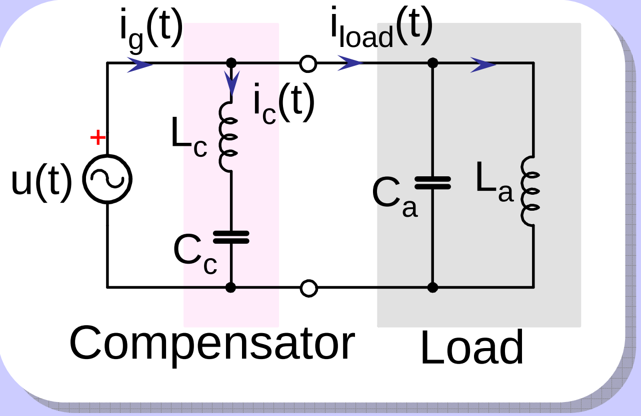
$$u(t) = \sqrt{2}\left[U_1 \sin(\omega_1 t) + U_3 \sin(3\omega_1 t)\right]$$

Example: $U_1 = 100V$ $\quad U_3 = 25V$ $\quad \begin{cases} L_a = \frac{32}{57} H \\ C_a = \frac{49}{32} F \end{cases}$ $\Rightarrow$ $Q = -\sum_{n=1}^{\infty} U_n I_n \sin(\varphi_n) = -U_1^2 B_1 - U_3^2 B_3 = 0$

$$i_{load}(t) = \sqrt{2}\left[I_1 \sin\left(\omega_1 t - \frac{\pi}{2}\right) + I_3 \sin\left(3\omega_1 t + \frac{\pi}{2}\right)\right]$$

$I_1 = 25A$

$I_3 = 100A$



For compensation, we want: $i_g(t) = i_c(t) + i_{load}(t) = 0$

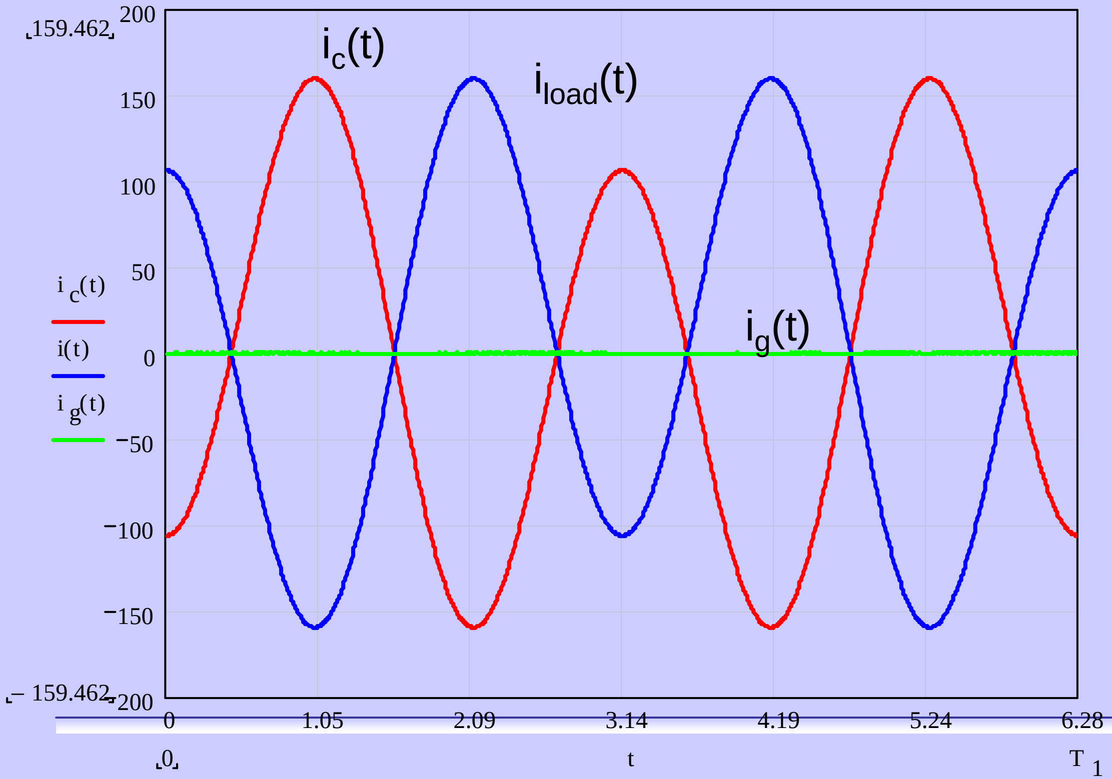
$$i_c(t) = \sqrt{2}\left[I_{c1} \sin\left(\omega_1 t + \frac{\pi}{2}\right) + I_{c3} \sin\left(3\omega_1 t - \frac{\pi}{2}\right)\right]$$

with $\quad I_{c1} = I_1$, $\quad I_{c3} = I_3$

Compensation?

$$\dot{Z}_{1c}(t) = -jX_1 = -j\frac{U_1}{I_1} \qquad \dot{Z}_{3c}(t) = jX_3 = j\frac{U_3}{I_3}$$

$$\begin{cases} \omega_1 L_c - \dfrac{1}{\omega_1 C_c} = -X_1 \\ 3\omega_1 L_c - \dfrac{1}{3\omega_1 C_c} = X_3 \end{cases} \qquad \begin{cases} C_c = \dfrac{8}{3}\dfrac{1}{\omega_1 (X_1 + 3X_3)} \\ L_c = \dfrac{X_1 + 3X_3}{8\omega_1} \end{cases} \Rightarrow \begin{cases} C_c = \dfrac{32}{147}F \\ L_c = \dfrac{19}{32}H \end{cases}$$



Budeanu's Q_B definition is Useless for compensation

Budeanu's theory limitations

- The defined reactive power Q_B has no correlation with energy exchange
- The defined reactive power Q_B is useless for compensation
- The distortion power D is not related with the current and voltage distortions

1987

L.S. Czarnecki: **What is Wrong With the Budeanu's Concept of Reactive and Distortion Powers and Why it Should be Abandoned,**
IEEE Trans. on Instrumentation and Measurements

Fryze Theory (1931)

- Time domain theory, (single phase)
- Current decomposition into two terms

Active current:

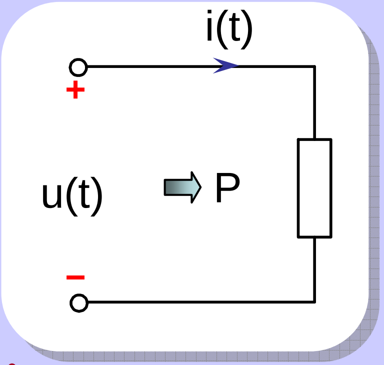
$$i_a(t) = Gu(t) \qquad \langle u, i_a \rangle = \frac{1}{T}\int_0^T u(t) i_a(t)\,dt = G\frac{1}{T}\int_0^T u^2(t)\,dt = G\|u\|^2 = P \quad \Rightarrow \quad G = \frac{P}{\|u\|^2}$$

NON active current

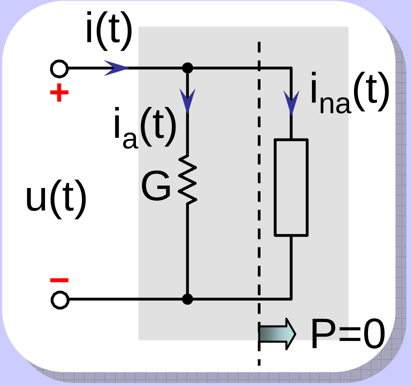
$$i_{na}(t) = i(t) - i_a(t)$$

The active current is the minimum current needed to deliver a given power P to the load

Meaning of Fryze's decomposition



$$i(t) = i_a(t) + i_{na}(t)$$



Properties:

- $\langle u, i_{na} \rangle = \frac{1}{T}\int_0^T u(t)\big(i(t) - i_a(t)\big)dt = 0$

- $\langle i_a, i_{na} \rangle = \frac{1}{T}\int_0^T G u(t) i_{na}(t)dt = 0$

Orthogonality:

$$\boxed{\|i\|^2 = \|i_a\|^2 + \|i_{na}\|^2}$$

Power balance:

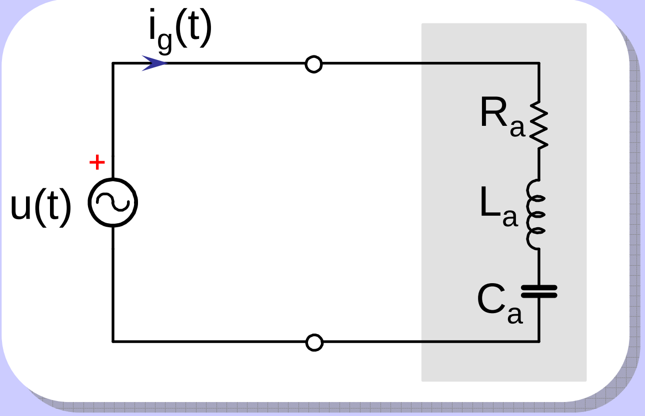
$$S^2 = \|u\|^2\|i\|^2 = \|u\|^2\|i_a\|^2 + \|u\|^2\|i_{na}\|^2 = \|u\|^2 G^2\|u\|^2 + \|u\|^2\|i_{na}\|^2 = P^2 + Q_F^2$$

where:

$$\boxed{Q_F^2 = \|u\|^2\|i_{na}\|^2}$$

Fryze's Theory limitations

Let's consider the following linear load:



With $\omega_1 = 1\frac{\text{rad}}{\text{s}}$ we want

$$\dot{Z}_1 = 1 - j$$

$$\dot{Z}_3 = 1 + j$$

Thus, choosing $R_a = 1\Omega$:

$$\begin{cases} \omega_1 L_a - \dfrac{1}{\omega_1 C_a} = -1 \\ 3\omega_1 L_a - \dfrac{1}{3\omega_1 C_a} = +1 \end{cases} \Rightarrow \begin{cases} C_a = \dfrac{2}{3}F \\ L_a = \dfrac{1}{2}H \end{cases}$$

Let's consider the following voltage:

$$u(t) = \sqrt{2}[U_1 \sin(\omega_1 t) + U_3 \sin(3\omega_1 t)]$$

$$U_1 = 100V \qquad U_3 = 100V$$

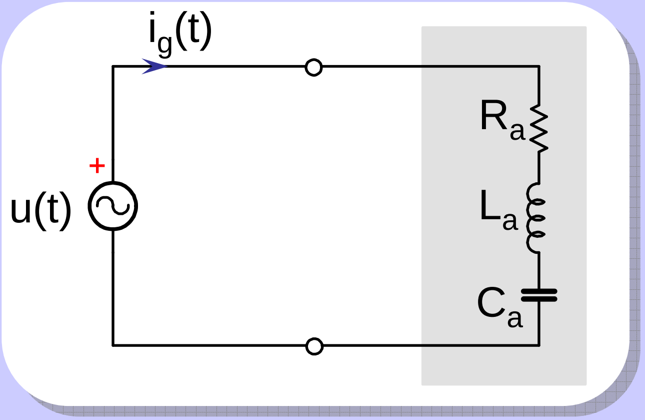
$$\|u\| = \sqrt{U_1^2 + U_3^2} = 141.4V$$

$$i(t) = \sqrt{2}\left[I_1 \sin\left(\omega_1 t + \frac{\pi}{4}\right) + I_3 \sin\left(3\omega_1 t - \frac{\pi}{4}\right)\right]$$

$$I_1 = 70.7A \qquad I_3 = 70.7A$$

Fryze's Theory limitations

$$\|i\| = \sqrt{I_1^2 + I_3^2} = 100\text{A}$$



Active power: $$P = \sum_{n=1,3} U_n I_n \cos(\varphi_n) = 10\,\text{kW}$$

Active power is also equal to the power dissipated in R_a:

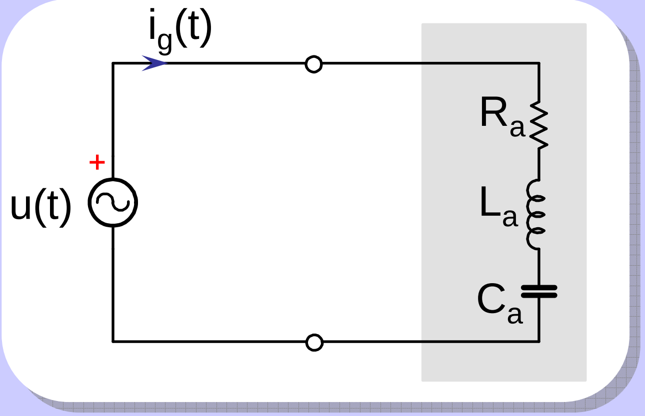
$$P = P_R = R_a \|i\|^2 = 10\,\text{kW}$$

Apparent power: $$S = \|u\|\|i\| = 14.14\ \text{kVA}$$

Reactive power: $$Q_F = \sqrt{S^2 - P^2} = 10\ \text{kVAr}$$

Power Factor: $$PF = \frac{P}{S} = 0.707$$

Fryze's Theory limitations



Active current:

$$G = \frac{P}{\|u\|^2} = 0.5S$$

$$i_a(t) = \sqrt{2}[I_{1a}\sin(\omega_1 t) + I_{3a}\sin(3\omega_1 t)]$$

$$I_{1a} = 50A$$

$$I_{3a} = 50A$$

$$\|i_a\| = G\|u\| = 70.7A$$

Non active current:

$$i_{na}(t) = i(t) - i_a(t)$$

$$\|i_{na}\| = \sqrt{\|i\|^2 - \|i_a\|^2} = 70.7\,A$$

Now, if we change only the reactive elements so as to draw the same total current RMS value, all power values remain unchanged!

Fryze's Theory limitations

$$\|i\|^2 = I_1^2 + I_3^2 = U_1^2\left(\frac{1}{|\dot{Z}_1|^2} + \frac{1}{|\dot{Z}_3|^2}\right)$$

Case A: $\begin{cases} R_a = 1\Omega \\ L_a = \frac{1}{2}H \\ C_a = \frac{2}{3}F \end{cases}$

$$\frac{1}{|\dot{Z}_{1A}|^2} + \frac{1}{|\dot{Z}_{3A}|^2} = \frac{1}{|\dot{Z}_{1B}|^2} + \frac{1}{|\dot{Z}_{3B}|^2}$$

But: $\frac{1}{|\dot{Z}_{1A}|^2} + \frac{1}{|\dot{Z}_{3A}|^2} = 1$

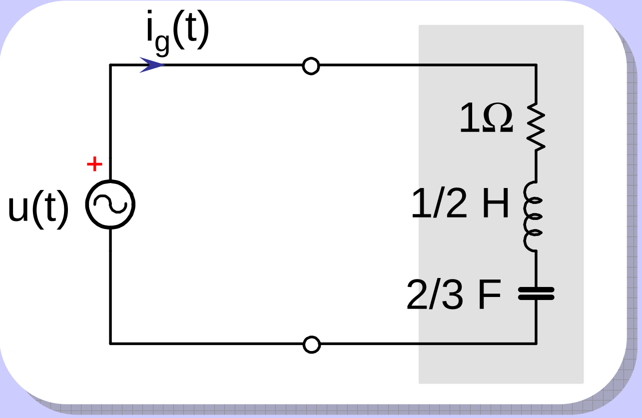
$$1 = \frac{1}{1 + X_{1B}^2} + \frac{1}{1 + X_{3B}^2}$$

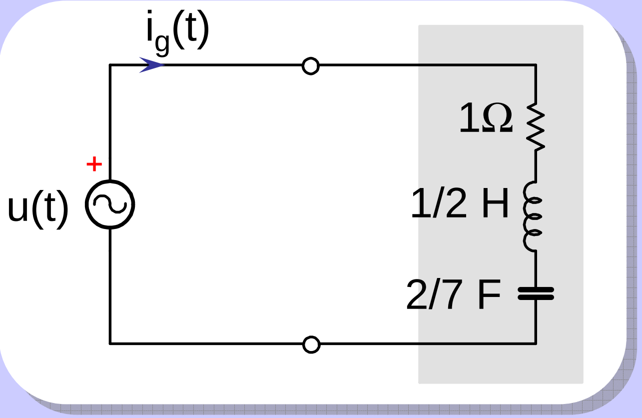
Let's choose X_{1b} = -3Ω ➡ $X_{3B} = \frac{1}{3}\Omega$

$$\begin{cases} \omega_1 L_b - \frac{1}{\omega_1 C_b} = -3 \\ 3\omega_1 L_b - \frac{1}{3\omega_1 C_b} = +\frac{1}{3} \end{cases} \Rightarrow \begin{cases} C_b = \frac{2}{7}F \\ L_b = \frac{1}{2}H \end{cases}$$

➡ **Case B:** $\begin{cases} R_b = 1\Omega \\ L_b = \frac{1}{2}H \\ C_b = \frac{2}{7}F \end{cases}$

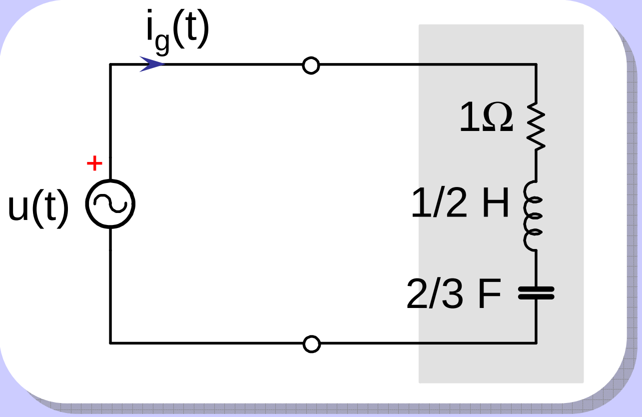
Fryze's Theory limitations





These loads **cannot be distinguished** with respect to Fryze's powers. They differ as to the possibility of their compensation

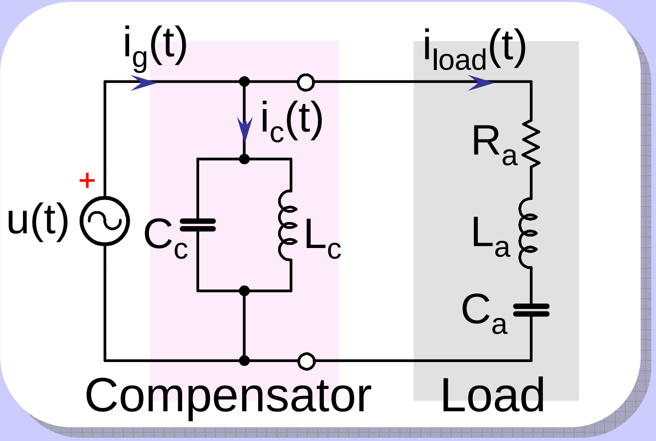
Fryze's Theory limitations



$$\dot{Y}_a = \frac{1}{R_a + jX_a} = \frac{R_a - jX_a}{R_a^2 + X_a^2} = G_a - jB_a$$

$$\dot{Y}_{a1} = \frac{1}{2} + j\frac{1}{2} \qquad \dot{Y}_{a3} = \frac{1}{2} - j\frac{1}{2}$$

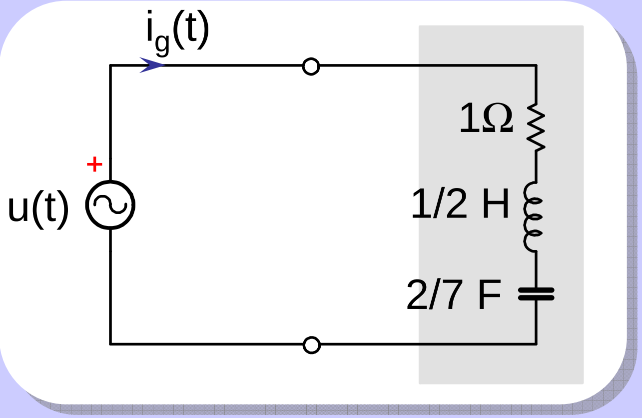
For reactive compensation select
$B_{c1} = -B_{a1}$ and $B_{c3} = -B_{a3}$



$$\begin{cases} \omega_1 C_c - \dfrac{1}{\omega_1 L_c} = -\dfrac{1}{2} \\ 3\omega_1 C_c - \dfrac{1}{3\omega_1 L_c} = +\dfrac{1}{2} \end{cases} \qquad \begin{cases} C_c = \dfrac{1}{4}\text{F} \\ L_c = \dfrac{4}{3}\text{H} \end{cases}$$

P = 10kW, Q_F = 0VAr, PF = 1

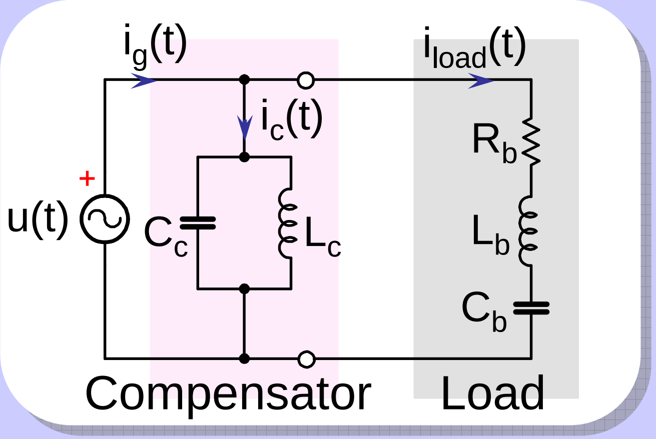
Fryze's Theory limitations



$$\dot{Y}_b = \frac{1}{R_b + jX_b} = \frac{R_b - jX_b}{R_b^2 + X_b^2} = G_b - jB_b$$

$$\dot{Y}_{b1} = \frac{1}{10} + j\frac{3}{10} \qquad \dot{Y}_{b3} = \frac{9}{10} - j\frac{3}{10}$$

For reactive compensation select
$B_{c1} = -B_{b1}$ and $B_{c3} = -B_{b3}$

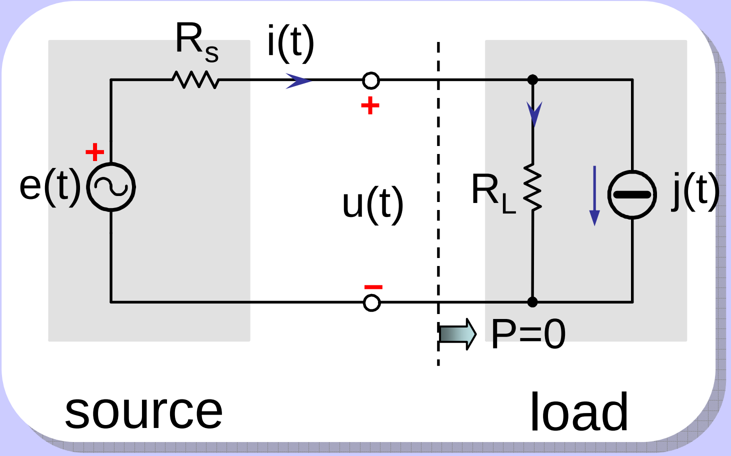


$$\begin{cases} \omega_1 C_c - \dfrac{1}{\omega_1 L_c} = -\dfrac{3}{10} \\ 3\omega_1 C_c - \dfrac{1}{3\omega_1 L_c} = +\dfrac{3}{10} \end{cases} \qquad \begin{cases} C_c = \dfrac{3}{20}\,\text{F} \\ L_c = \dfrac{20}{9}\,\text{H} \end{cases}$$

P = 10kW, Q_F = 8kVAr,
PF = 0.781

Harmonic compensation

Let us consider the following situation:



With $e(t)=\sqrt{2}E\sin(\omega_1 t),\quad E=100V$

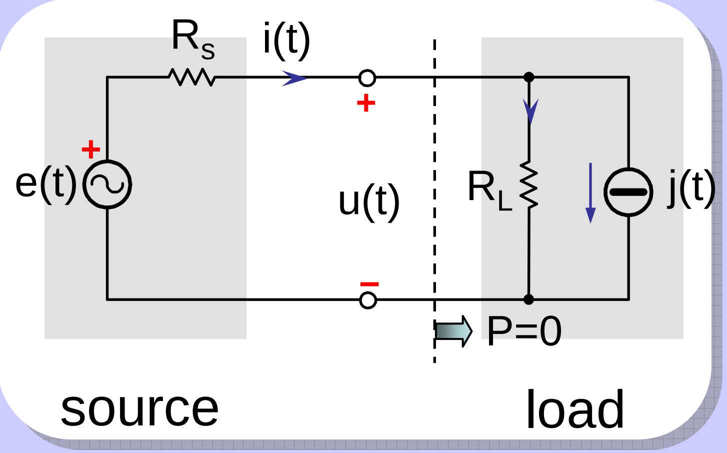
$$j(t)=\sqrt{2}J\sin(3\omega_1 t),\quad J=50A$$

$$e(t)=R_s i(t)+R_L\left(i(t)-j(t)\right)$$

$$i(t)=\frac{1}{R_s+R_L}e(t)+\frac{R_L}{R_s+R_L}j(t)=\sqrt{2}\left(I_1\sin(\omega_1 t)+I_3\sin(3\omega_1 t)\right)$$

$$u(t)=\frac{R_L}{R_s+R_L}e(t)-\frac{R_L R_s}{R_s+R_L}j(t)=\sqrt{2}\left(U_1\sin(\omega_1 t)-U_3\sin(3\omega_1 t)\right)$$

Harmonic compensation



Let us calculate the load resistance by imposing a zero active power into the load:

$$P = U_1 I_1 \cos(\phi_1) + U_3 I_3 \cos(\phi_3)$$

$$= U_1 I_1 - U_3 I_3 = \frac{R_L}{(R_s + R_L)^2} E^2 - \frac{R_s R_L^2}{(R_s + R_L)^2} J^2 = 0$$

$$R_L = \frac{E^2}{R_s J^2}$$ **With** $R_s = 1\Omega$ ⇒ $R_L = 4\Omega$

$$i(t) = \sqrt{2}\left(I_1 \sin(\omega_1 t) + I_3 \sin(3\omega_1 t)\right)$$

$I_1 = 20A$

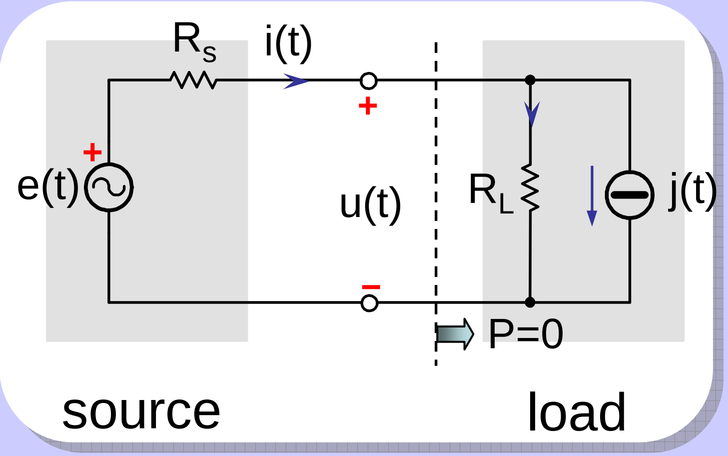
$I_3 = 40A$

$$u(t) = \sqrt{2}\left(U_1 \sin(\omega_1 t) - U_3 \sin(3\omega_1 t)\right)$$

$U_1 = 80V$

$U_3 = 40V$

Harmonic compensation



Active power:

$$P = 1600 - 1600 = 0W$$

Active current: $G = \frac{P}{\|u\|^2} = 0$

$$i_a(t) = Gu(t) = 0$$

Non active current: $i_{na}(t) = i(t) - i_a(t) = i(t)$

According to Fryze's Power Theory,
total compensation requires that the current i_{na}
is reduced to zero

This is a wrong conclusion: only the 3rd order current harmonic should be compensated

Fryze theory limitations

- Fryze's power theory is not able of an efficient load characterization
- It does not allow a load compensation using passive components

Kusters & Moore theory (1975)

- Time domain
  - An active current is defined as Fryze's theory

$$i_a(t) = \frac{P}{\|u\|^2} u(t) = \frac{\langle u,i \rangle}{\|u\|^2} u(t)$$

  - A capacitive (inductive) current component is introduced

$$i_{rC} = C\dot{u}(t) = \frac{\langle \dot{u},i \rangle}{\|\dot{u}\|^2} \dot{u}(t) \quad \text{with } \dot{u}(t) = \frac{d}{dt} u(t)$$

$$i_{rL} = L^{-1}\hat{u}(t) = \frac{\langle \hat{u},i \rangle}{\|\hat{u}\|^2} \hat{u}(t) \quad \text{with } \hat{u}(t) = \int_0^t u(\tau)\, d\tau$$

Kusters & Moore theory (1975)

- Residual current $i = i_a + i_r + i_{nr}$ $\quad i_{nr} = i - (i_a + i_r)$

$$i_r = i_{rC} \quad \text{or} \quad i_r = i_{rL}$$

- Orthogonal current decomposition $\|i\|^2 = \|i_a\|^2 + \|i_r\|^2 + \|i_{nr}\|^2$

- $\langle i_a, i_r \rangle = G\langle u, i_r \rangle = 0$ $\qquad \langle u, i_{nr} \rangle = \frac{1}{T}\int_0^T u(i - i_a - i_r)dt = P - P - 0 = 0$

- $\langle i_a, i_{nr} \rangle = G\langle u, i_{nr} \rangle = 0$ $\qquad \langle u, i_{rC} \rangle = C\frac{1}{T}\int_0^T u\dot{u}dt = 0$

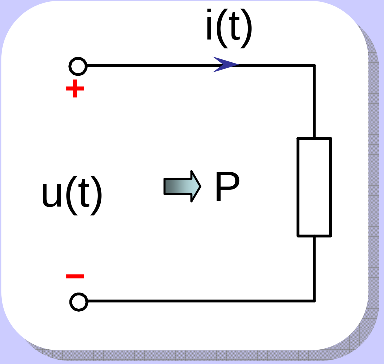
- $\langle i_{rC}, i_{nr} \rangle = C\langle \dot{u}, i_{nr} \rangle = C(\langle \dot{u}, i \rangle - \langle \dot{u}, i_a \rangle - \langle \dot{u}, i_{rC} \rangle)$ $\qquad \langle u, i_{rL} \rangle = \frac{1}{L}\frac{1}{T}\int_0^T u\hat{u}dt = 0$

$$= C\left(C\|\dot{u}\|^2 - G\langle \dot{u}, u \rangle - C\|\dot{u}\|^2\right) = 0$$

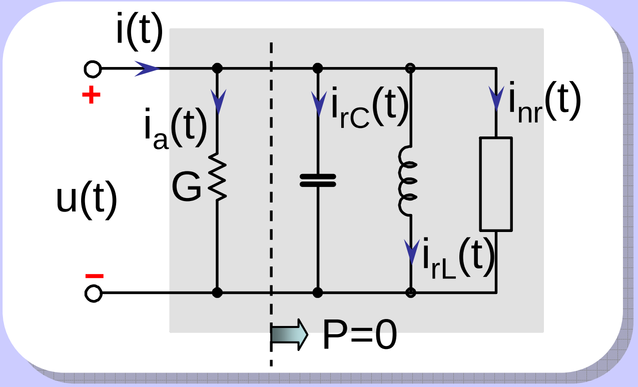
- $\langle i_{rL}, i_{nr} \rangle = L^{-1}\langle \hat{u}, i_{nr} \rangle = L^{-1}(\langle \hat{u}, i \rangle - \langle \hat{u}, i_a \rangle - \langle \hat{u}, i_{rL} \rangle)$

$$= L^{-1}\left(L^{-1}\|\hat{u}\|^2 - G\langle \hat{u}, u \rangle - L^{-1}\|\hat{u}\|^2\right) = 0$$

Load decomposition (single phase)



$$i = i_a + i_r + i_{nr}$$



- Current component proportional to the voltage: $i_a(t)$
- Current component proportional to the voltage derivative: $i_{rC}(t)$
- Current component proportional to the voltage integral: $i_{rL}(t)$
- Residual current component: $i_{nr}(t)$

Kusters & Moore Theory (1975)

- Power balance

$$S^2 = \|u\|^2\|i\|^2 = \|u\|^2\|i_a\|^2 + \|u\|^2\|i_r\|^2 + \|u\|^2\|i_{nr}\|^2 = P^2 + Q_K^2 + Q_{nr}^2$$

where: $Q_k = \|u\|\|i_r\|$ and: $Q_{nr} = \sqrt{S^2 - P^2 - Q_K^2}$

Reactive compensation:

in case of negative load reactive terms C or L, reactive compensation can be easily achieved by inserting reactive components of equal value (but opposite sign)

K & M Theory limits

- Current decomposition uncorrelated with the physical meaning of each term
- Reactive currents $i_{rL}(t)$ and $i_{rC}(t)$ non orthogonal
- Decomposition valid for zero source impedance

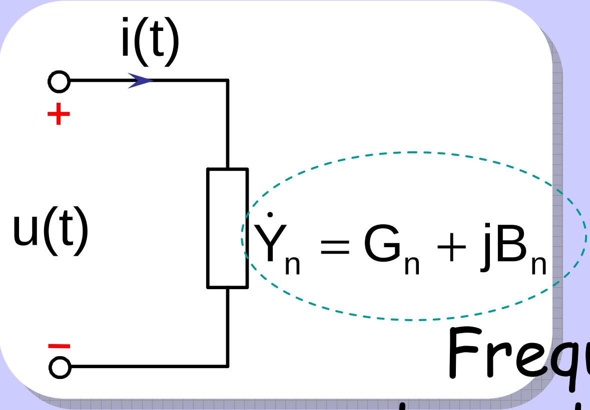
Czarnecki Theory (1984)

- Frequency domain

Single-phase, linear, time-invariant load

$$u = U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} \dot{U}_n e^{jn\omega_1 t}$$

$$i = G_0 U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} \dot{Y}_n \dot{U}_n e^{jn\omega_1 t}$$



Frequency dependent load

- **Proposed decomposition**

$$i_a = G_e u \qquad G_e = \frac{P}{\|u\|^2}$$

Active current

Czarnecki Theory (1984)

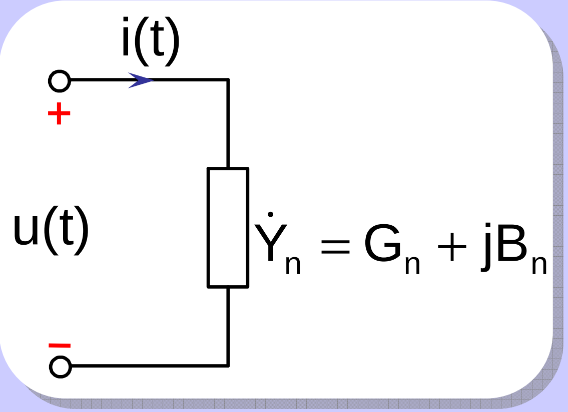
- Proposed decomposition:

$$i_a = G_e u \qquad G_e = \frac{P}{\|u\|^2}$$

Active current

$$i_r = \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} jB_n \dot{U}_n e^{jn\omega_1 t}$$

Reactive current

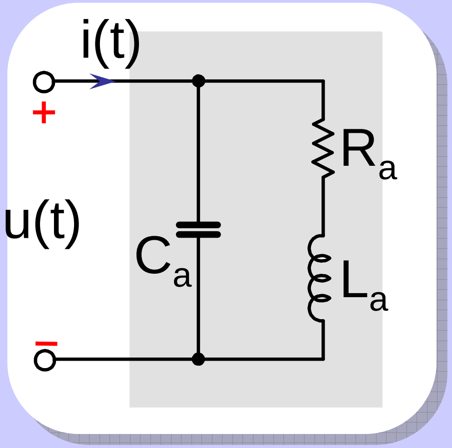


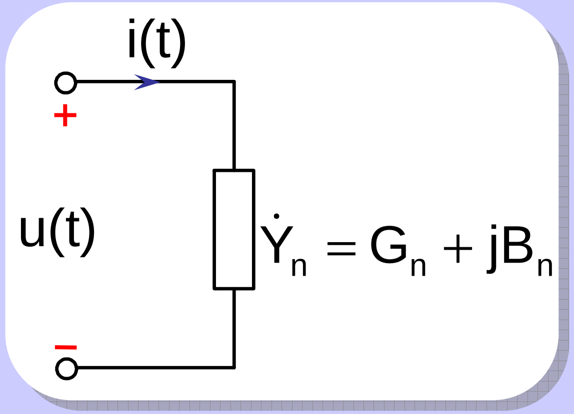
$$i = i_a + i_r + i_s$$ → "scattering" current

$$i_s = i - i_a - i_r = (G_0 - G_e)U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty}(G_n - G_e)\dot{U}_n e^{jn\omega_1 t}$$

New current component that appears when the load conductance changes with frequency

Numerical example





$\omega_1 = 1\frac{\text{rad}}{\text{s}}$

$R_a = 1\Omega,\ L_a = 1H,\ C_a = 0.5F$

$G_0 = 1\ S$

$\dot{Y}_1 = G_1 + jB_1 = 0.5\,S$

$\dot{Y}_5 = G_5 + jB_5 = 0.04 + j2.31S$

$$u(t) = U_0 + \sqrt{2}\Re e\left\{U_1 e^{j\omega_1 t} + U_5 e^{j5\omega_1 t}\right\} \qquad U_0 = 50V,\quad U_1 = 100V,\quad U_5 = 20V$$

$$\|u\| = \sqrt{U_0^2 + U_1^2 + U_5^2} = 113.58V$$

Active power: $P = \sum\limits_{n=0,1,5} G_n U_n^2 = 7515W \qquad G_e = \frac{P}{\|u\|^2} = 0.583\,S$

Total current:

$$i(t) = I_0 + \sqrt{2}\Re e\left\{I_1 e^{j\omega_1 t} + I_5 e^{j5\omega_1 t}\right\} = G_0 U_0 + \sqrt{2}\Re e\left\{Y_1 U_1 e^{j(\omega_1 t + \varphi_1)} + Y_5 U_5 e^{j(5\omega_1 t + \varphi_5)}\right\}$$

$$I_0 = 50A,\quad I_1 = 50A,\quad I_5 = 46.16A,\quad \varphi_1 = 0^\circ,\quad \varphi_5 = 89^\circ$$

Numerical example

Current Physical Components (CPC):

Active current:

$$i_a(t) = G_e u(t) = I_{0a} + \sqrt{2}\Re e\{I_{1a} e^{j\omega_1 t} + I_{5a} e^{j5\omega_1 t}\} = G_e U_0 + \sqrt{2}\Re e\{G_e U_1 e^{j\omega_1 t} + G_e U_5 e^{j5\omega_1 t}\}$$

$$I_{0a} = 29.13\text{A}, \quad I_{1a} = 58.26\text{A}, \quad I_{5a} = 11.65\text{A}$$

Reactive current:

$$i_r(t) = \sqrt{2}\Re e\{jI_{1r} e^{j\omega_1 t} + jI_{5r} e^{j5\omega_1 t}\} = \sqrt{2}\Re e\{jB_1 U_1 e^{j\omega_1 t} + jB_5 U_5 e^{j5\omega_1 t}\}$$

$$I_{1r} = 0\text{A}, \quad I_{5r} = 46.15\text{A}$$

Scattering current:

$$i_s(t) = I_{0s} + \sqrt{2}\Re e\{I_{1s} e^{j\omega_1 t} + I_{5s} e^{j5\omega_1 t}\} = (G_0 - G_e)U_0 + \sqrt{2}\Re e\{(G_1 - G_e)U_1 e^{j\omega_1 t} + (G_5 - G_e)U_5 e^{j5\omega_1 t}\}$$

$$I_{0s} = 20.87\text{A}, \quad I_{1s} = -8.26\text{A}, \quad I_{5s} = -10.88\text{A}$$

Czarnecki Theory (1984)

- Orthogonal current decomposition

$$\langle i_a, i_r \rangle = G_e \langle u, i_r \rangle = 0 \qquad \langle i_s, i_r \rangle = 0$$

$$\langle i_a, i_s \rangle = G_e \langle u, i_s \rangle = G_e \langle u, (i - i_a - i_r) \rangle = G_e (\langle u, i \rangle - \langle u, i_a \rangle - \langle u, i_r \rangle)$$
$$= P - P - 0 = 0$$

- Power balance

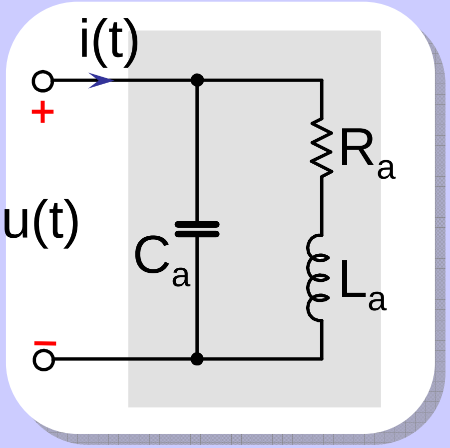
$$S^2 = \|u\|^2 \|i\|^2 = \|u\|^2 \|i_a\|^2 + \|u\|^2 \|i_r\|^2 + \|u\|^2 \|i_s\|^2 = P^2 + Q_C^2 + D_s^2$$

Reactive power $Q_C = \|u\| \|i_r\|$ **Definition:** $B_e = \dfrac{Q_C}{\|u\|^2}$

Scattering power $D_s = \sqrt{S^2 - P^2 - Q_C^2}$

Numerical example

$$u(t)=U_0+\sqrt{2}\,\Re e\left\{U_1 e^{j\omega_1 t}+U_5 e^{j5\omega_1 t}\right\} \qquad U_0=50\text{V},\quad U_1=100\text{V},\quad U_5=20\text{V}$$

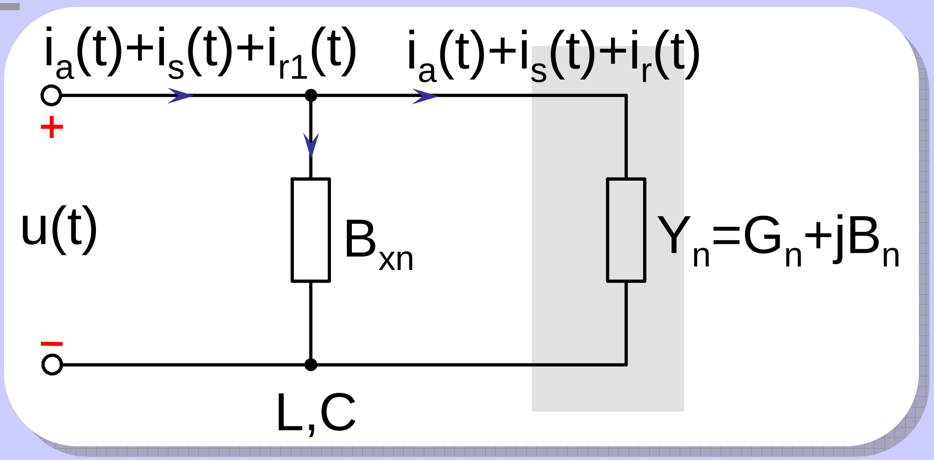


Active power: $$P=\sum_{n=0,1,5} G_n U_n^2 = 7515\text{W}$$

Reactive power: $$Q_C=\|u\|\|i_r\|=5.24\text{kVAr}$$

Scattering power: $$D_s=\sqrt{S^2-P^2-Q_C^2}=2.83\text{kVA}$$

Compensation of LTI loads



Lossless shunt reactive compensators do not change active power P and conductance G_n.

$$G_e = \frac{P}{\|u\|^2} = \text{constant}$$

$$\|i_a\| = G_e\|u\| = \text{constant} \qquad \|i_s\| = \sqrt{\sum_{n=0}^{\infty}(G_n - G_e)^2 U_n^2} = \text{constant}$$

The RMS value of the reactive current changes to:

$$\|i_{r1}\| = \sqrt{\sum_{n=1}^{\infty}(B_n + B_{xn})^2 U_n^2}$$

Total compensation of the reactive current:

$$\|i_{r1}\| = 0 \quad \text{if for each n such that } U_n \neq 0 \quad B_{xn} = -B_n$$

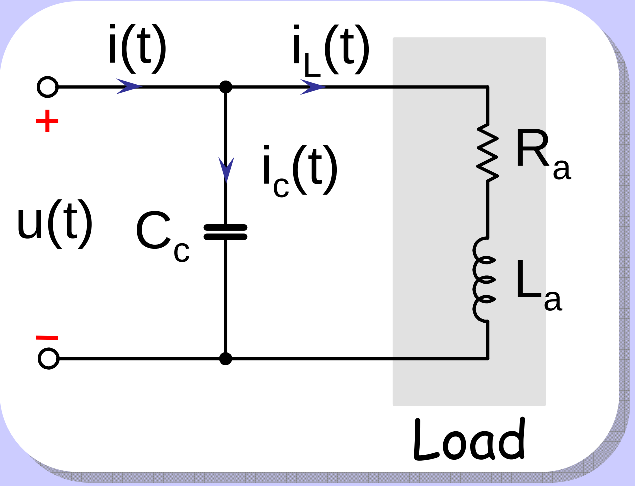
Compensation example

$$u(t)=\sqrt{2}\Re e\left\{U_1 e^{j\omega_1 t}+U_5 e^{j5\omega_1 t}\right\}$$

$U_1 = 100\text{V}, \quad U_5 = 5\text{V}$

$\omega_1 = 1\dfrac{\text{rad}}{\text{s}}$

$\|u\| = \sqrt{U_1^2 + U_5^2} = 100.12\text{V}$



Load admittance:

$R_a = 1\Omega,\ L_a = 2\text{H},\ C_c = 0.4\text{F}$

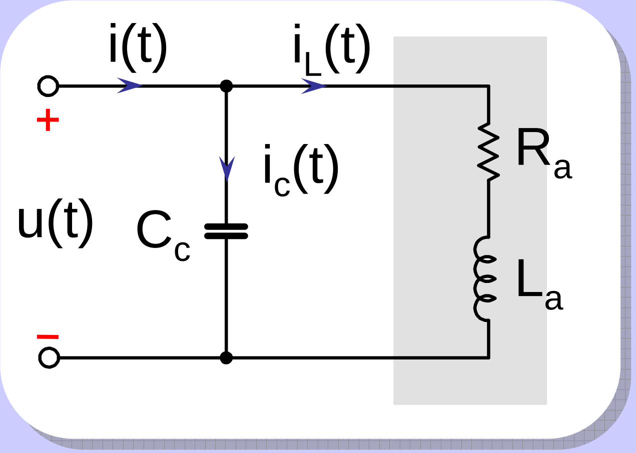
$\dot{Y}_1 = G_1 + jB_1 = 0.2 - j0.4\,\text{S}$

$\dot{Y}_5 = G_5 + jB_5 = 0.01 - j0.1\text{S}$

First harmonic compensation: $jB_{c1} = j0.4\ \text{S}$

Active power: $P = \sum_{n=0,1,5} G_n U_n^2 = 2\text{kW}$ $\qquad G_e = \dfrac{P}{\|u\|^2} = 0.2\,\text{S}$

Compensation example



Total current:

$$i(t) = \sqrt{2}\Re e\left\{I_1 e^{j\omega_1 t} + I_5 e^{j5\omega_1 t}\right\}$$
$$= \sqrt{2}\Re e\left\{Y_1 U_1 e^{j(\omega_1 t+\varphi_1)} + Y_5 U_5 e^{j(5\omega_1 t+\varphi_5)}\right\}$$

$$I_1 = 20\text{A}, \quad I_5 = 9.5\text{A}, \quad \varphi_1 = 0°, \quad \varphi_5 = 89.7°$$

$$\|i\| = 22.14\text{A}$$

Active current: $\|i_a\| = G_e\|u\| = 19.98\text{A}$

Scattering current: $\|i_s\| = 0.95\text{A}$

Reactive current: $\|i_r\| = 9.5\text{A}$

Power factor:

$$PF = \frac{P}{S} = \frac{\|i_a\|}{\sqrt{\|i_a\|^2 + \|i_r\|^2 + \|i_s\|^2}} = 0.902 \qquad PF_{MAX} = \frac{\|i_a\|}{\sqrt{\|i_a\|^2 + \|i_s\|^2}} \leq 1$$

Czarnecki Theory (1984)

Harmonic generator load: single-phase

N_u -> voltage harmonic set

N_i -> current harmonic set

$$N_u \subset N_i$$

$$i = i_o + i_g \qquad i_o = \sum_{n \in N_u} i_n \qquad i_g = i - i_o$$

i_o -> homolog current

i_g -> independently generated current

For each homolog current component:

$$G_{ne} = \frac{P_n}{\|u_n\|^2} \qquad B_{ne} = \frac{Q_n}{\|u_n\|^2}$$

Czarnecki Theory (1984)

Harmonic generator load: three-phase

$$\underline{i} = \boxed{\underline{i}_o} + \boxed{\underline{i}_g} \qquad \underline{i}_o = \sum_{n \in N_u} \underline{i}_n \qquad \underline{i}_g = \underline{i} - \underline{i}_o$$

Current decomposition into:

- Homolog current
- Independently generated current

- Load harmonic characterization

$$G_{ne} = \frac{P_n}{\left\|\underline{u}_n\right\|^2} \qquad B_{ne} = \frac{Q_n}{\left\|\underline{u}_n\right\|^2}$$

Extension to three-phase systems: previous definitions employ vector quantities with the same meaning

Czarnecki Theory (1984)

- Homolog current harmonic decomposition

$$\underline{i}_o = \sum_{n \in N_u} \underline{i}_n = \sum_{n \in N_u} (\underline{i}_{na} + \underline{i}_{nr} + \underline{i}_{nu})$$

- Current decomposition for each harmonic

$$\underline{i}_{na} = G_{ne} \underline{u}_n \qquad \underline{i}_{nr} = B_{ne} \frac{d}{d(n\omega_1 t)} \underline{u}_n \qquad \underline{i}_{nu} = \underline{i}_n - (\underline{i}_{na} + \underline{i}_{nr})$$

- Total homolog current components

$$\underline{i}_a = G_e \underline{u} \qquad \underline{i}_r = \sum_{n \in N_u} \underline{i}_{nr} \qquad \underline{i}_s = \left(\sum_{n \in N_u} \underline{i}_{na} \right) - \underline{i}_a \qquad \underline{i}_u = \sum_{n \in N_u} \underline{i}_{nu}$$

- Total current decomposition

$$\underline{i} = \underline{i}_o + \underline{i}_g = \sum_{n \in N_u} \underline{i}_{na} + \sum_{n \in N_u} \underline{i}_{nr} + \sum_{n \in N_u} \underline{i}_{nu} + \underline{i}_g = \underline{i}_a + \underline{i}_s + \underline{i}_r + \underline{i}_u + \underline{i}_g$$

Czarnecki Theory (1984)

- Orthogonality property:

$$\|\underline{\mathbf{i}}\|^2 = \|\underline{\mathbf{i}}_a\|^2 + \|\underline{\mathbf{i}}_s\|^2 + \|\underline{\mathbf{i}}_r\|^2 + \|\underline{\mathbf{i}}_u\|^2 + \|\underline{\mathbf{i}}_g\|^2$$

- Power balance:

$$S^2 = P^2 + D_s{}^2 + Q_r{}^2 + D_u{}^2 + D_g{}^2$$

Czarnecki Theory limitations

- Reactive power uncorrelated with energy exchanges
- Computational complexity
- Frequency domain theory
- Poor usefulness for compensation

Renewable Energy Sources for Distributed Generation in Smart Grids: the role of Power Electronics

Speaker: G. Spiazzi

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Seminar Outline

- From "Traditional" to "Smart Grids"
  - Challenges & potentialities
- Power electronics for renewable energy sources
  - Principles of photovoltaic energy generation
  - Power electronics for interfacing PV panels with the utility grid
    - Inverter operation
    - Maximum power point tracking techniques
    - Anti-islanding methods
- Traditional power theories
  - Budeanu power theory
  - Fryze power theory
  - Kusters & Moore theory (time domain)
  - Czarnecki theory (frequency domain)
- Conservative power theory
  - Mathematical and physical foundations of the theory
  - Examples

Conservative Power Theory

- Definition of current and power terms in single-phase networks under non-sinusoidal conditions
  - Orthogonal current decomposition into active, reactive and void terms
  - Physical meaning of current terms
  - Apparent power decomposition into active, reactive and void terms
  - Physical meaning of power terms
  - Application examples

Mathematical operators

Periodic scalar quantities

Let T be the period of variables x and y, we define:

- Average value $\bar{x} = \langle x \rangle = \frac{1}{T}\int_0^T x(t)\,dt$
- Time derivative $\breve{x} = \frac{dx}{dt}$
- Time integral $x_\int = \int_0^t x(\tau)\,d\tau$
- Unbiased time integral $\hat{x} = x_\int - \bar{x}_\int$
- Internal product $\langle x, y \rangle = \frac{1}{T}\int_0^T x \cdot y\,dt$
- Norm (rms value) $X = \|x\| = \sqrt{\langle x, x \rangle}$
- Orthogonality $\langle x, y \rangle = 0$

Mathematical operators

Periodic vector quantities

Let x and y be vector quantities of size N, we define:

- Scalar product $$\underline{x} \circ \underline{y} = \sum_{n=1}^{N} x_n y_n$$
- Magnitude $$|\underline{x}| = \sqrt{\underline{x} \circ \underline{x}} = \sqrt{\sum_{n=1}^{N} x_n^2}$$
- Internal product $$\langle \underline{x}, \underline{y} \rangle = \sum_{n=1}^{N} \langle x_n, y_n \rangle$$
- Norm $$\mathbf{X} = \|\underline{x}\| = \sqrt{\sum_{n=1}^{N} \langle x_n, x_n \rangle} = \sqrt{\sum_{n=1}^{N} X_n^2}$$
- Orthogonality $$\langle \underline{x}, \underline{y} \rangle = 0$$
- The vector norm is also called collective rms value

Properties of mathematical operators

Remember: $$\int_a^b f'(t)g(t)\,dt = f(t)g(t)\big|_a^b - \int_a^b f(t)g'(t)\,dt$$

- **Equivalences:**

$$\langle x, \breve{y} \rangle = \frac{1}{T}\int_0^T x y' dt = \frac{1}{T}\left[xy\right]_0^T - \frac{1}{T}\int_0^T x' y dt = \frac{1}{T}\underbrace{\left[xy\right]_0^T}_{=\,0} - \langle \breve{x}, y \rangle = -\langle \breve{x}, y \rangle$$

(Periodic quantities)

$$\langle x, \hat{y} \rangle = \frac{1}{T}\left[\hat{x}\hat{y}\right]_0^T - \frac{1}{T}\int_0^T \hat{x} y dt = \frac{1}{T}\underbrace{\left[\hat{x}\hat{y}\right]_0^T}_{=\,0} - \langle \hat{x}, y \rangle = -\langle \hat{x}, y \rangle$$

(Periodic quantities)

Properties of mathematical operators

Remember:

$$\int_a^b f'(t)g(t)\,dt = f(t)g(t)\Big|_a^b - \int_a^b f(t)g'(t)\,dt$$

- **Equivalences:**

$$\left\langle \overset{\frown}{x}, \breve{y} \right\rangle = \frac{1}{T}\left[\overset{\frown}{x}y\right]_0^T - \frac{1}{T}\int_0^T xy\,dt = \underbrace{\frac{1}{T}\left[xy\right]_0^T}_{=\,0} - \left\langle x, y \right\rangle = -\left\langle x, y \right\rangle$$

(Periodic quantities)

$$\left\langle \breve{x}, \overset{\frown}{y} \right\rangle = \frac{1}{T}\left[x\overset{\frown}{y}\right]_0^T - \frac{1}{T}\int_0^T xy\,dt = \underbrace{\frac{1}{T}\left[x\overset{\frown}{y}\right]_0^T}_{=\,0} - \left\langle x, y \right\rangle = -\left\langle x, y \right\rangle$$

(Periodic quantities)

Properties of mathematical operators

Summarizing:

The above operators have the following properties:

- **Orthogonality**

$$\langle x, \breve{x} \rangle = 0 \qquad \qquad \langle \underline{x}, \underline{\breve{x}} \rangle = 0$$
$$\Rightarrow$$
$$\langle x, \overset{\frown}{x} \rangle = 0 \qquad \qquad \langle \underline{x}, \underline{\overset{\frown}{x}} \rangle = 0$$

- **Equivalences**

$$\langle x, \breve{y} \rangle = -\langle \breve{x}, y \rangle$$
$$\langle x, \overset{\frown}{y} \rangle = -\langle \overset{\frown}{x}, y \rangle$$
$$\langle x, y \rangle = -\langle \breve{x}, \overset{\frown}{y} \rangle = -\langle \overset{\frown}{x}, \breve{y} \rangle$$

$$\Rightarrow$$

$$\langle \underline{x}, \underline{\breve{y}} \rangle = -\langle \underline{\breve{x}}, \underline{y} \rangle$$
$$\langle \underline{x}, \underline{\overset{\frown}{y}} \rangle = -\langle \underline{\overset{\frown}{x}}, \underline{y} \rangle$$
$$\langle \underline{x}, \underline{y} \rangle = -\langle \underline{\breve{x}}, \underline{\overset{\frown}{y}} \rangle = -\langle \underline{\overset{\frown}{x}}, \underline{\breve{y}} \rangle$$

Properties of mathematical operators

$$x(t) = \sqrt{2}X\sin(\omega t)$$

- **For sinusoidal quantities:**

$$\hat{x}(t) = -\frac{\sqrt{2}X}{\omega}\cos(\omega t)$$

$$X = \|x\| = \omega\|\hat{x}\| = \frac{1}{\omega}\|\breve{x}\|$$

$$\breve{x}(t) = \omega\sqrt{2}X\cos(\omega t)$$

$$x^2 + \omega^2\hat{x}^2 = 2X^2\sin^2(\omega t) + \omega^2\frac{2X^2}{\omega^2}\cos^2(\omega t) = 2X^2$$

$$x^2 + \frac{\breve{x}^2}{\omega^2} = 2X^2\sin^2(\omega t) + \frac{\omega^2 2X^2}{\omega^2}\cos^2(\omega t) = 2X^2$$

$$\langle x, y\rangle = \frac{2XY}{2\pi}\int_0^{2\pi}\sin(\theta)\sin(\theta-\varphi)d\theta = XY\cos(\varphi)$$

$$\langle \hat{x}, y\rangle = -\frac{2XY}{2\pi\omega}\int_0^{2\pi}\cos(\theta)\mathrm{sen}(\theta-\varphi)d\theta = \frac{XY}{\omega}\mathrm{sen}(\varphi)$$

Instantaneous power definitions

Given the vectors of the *N* phase currents i_n and voltages u_n measured at a generic network port we define:

Instantaneous (active) power:
$$p = \underline{u} \cdot \underline{i} = \sum_{n=1}^{N} u_n i_n = \sum_{n=1}^{N} p_n$$

Instantaneous reactive energy:
$$w = \underline{\hat{u}} \cdot \underline{i} = \sum_{n=1}^{N} \hat{u}_n i_n = \sum_{n=1}^{N} w_n$$

- Instantaneous power and reactive energy do not depend on the voltage reference
- Instantaneous power and reactive energy are conservative quantities in every real network

Conservation low

For every real network π, let $\underline{u}$ and $\underline{i}$ be **the vectors of the L branch voltages and currents**, we affirm that:

- ✓ Branch voltages, *their time derivative and unbiased integral* are consistent with network π, i.e. they comply with KLV (Kirchhoff's law for voltages)
- ✓ Branch currents, *their time derivative and unbiased integral* are consistent with network π, i.e. they comply with KLC (Kirchhoff's law for currents)

Thus, according to Tellegen's Theorem:

$$\underline{u}\cdot\underline{i}=\underline{\widehat{u}}\cdot\underline{\breve{i}}=\underline{\breve{u}}\cdot\underline{\widehat{i}}=0$$

$$\underline{\widehat{u}}\cdot\underline{i}=\underline{u}\cdot\underline{\widehat{i}}=0$$

$$\underline{\breve{u}}\cdot\underline{i}=\underline{u}\cdot\underline{\breve{i}}=0$$

Average power definitions

Active power: $P = \overline{p} = \langle \underline{u}, \underline{i} \rangle$

Reactive energy: $W = \overline{w} = \langle \underline{\hat{u}}, \underline{i} \rangle = -\langle \underline{u}, \underline{\hat{i}} \rangle$

Apparent power: $A = \|\underline{u}\| \|\underline{i}\| = \boldsymbol{U}\,\boldsymbol{I}$

Power factor: $\lambda = \dfrac{P}{A}$

Remark: Unlike active power *P* and reactive energy *W*, apparent power *A* depends on the voltage reference

Selection of voltage reference

Cauchy-Schwartz inequality:

$$\left|\langle \underline{u}, \underline{i} \rangle\right| \le \|\underline{u}\| \|\underline{i}\| \quad \Rightarrow |\lambda| = \frac{|P|}{A} \le 1$$

The equal sign is possible if:

$$\|\underline{u}\| \propto \|\underline{i}\| \quad \Rightarrow \quad \left|\langle \underline{u}, \underline{i} \rangle\right| = \|\underline{u}\| \|\underline{i}\| \quad \Rightarrow \quad |\lambda| = 1$$

We select the voltage reference so as to ensure unity power factor in case of symmetrical resistive load. This gives a physical meaning to the apparent power, which is the maximum active power that a supply line rated for V_{rms} Volts and I_{rms} Amperes can deliver to a (purely resistive and symmetrical) load.

Selection of voltage reference

N-phase systems without neutral wire

The proportionality condition between phase voltages and currents for symmetrical resistive load determines voltage reference

$$\underline{u} = R\,\underline{i}$$

$$\sum_{n=1}^{N} i_n = 0 \quad \Rightarrow \quad \sum_{n=1}^{N} u_n = 0$$

In fact, the voltage reference must be selected to comply with the zero-sum condition:

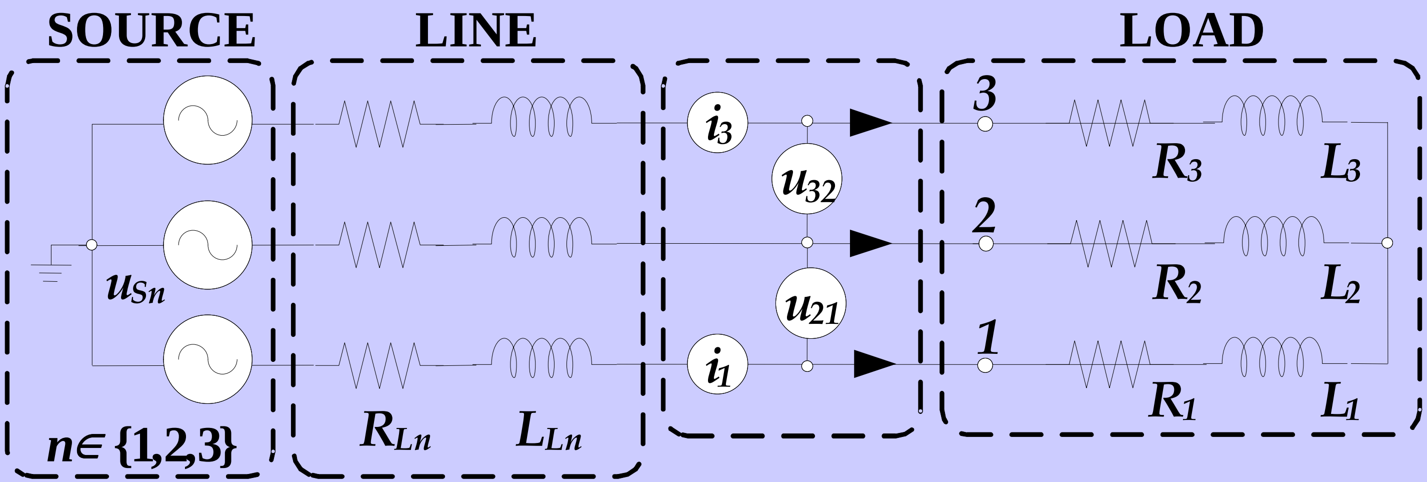
$$\sum_{n=1}^{N} u_n = 0 \quad \Rightarrow \quad \sum_{n=1}^{N} \underbrace{\left(u_{n_{measure}} - u_{ref}\right)}_{u_n} = 0 \quad \Rightarrow \quad u_{ref} = \frac{1}{N}\sum_{n=1}^{N} u_{n_{measure}}$$

This choice minimizes the norm of the voltage vector

Selection of voltage reference

N-phase systems **without** neutral wire

Measurement of voltages and currents



Derivation of phase voltages from line-to-line voltages

$$u_n = \frac{1}{N}\sum_{j=1}^{N}{}_{j\neq n} u_{nj}$$

$$|\underline{u}|^2 = \frac{1}{2N}\sum_{n=1}^{N}\sum_{j=1}^{N}{}_{j\neq n} u_{nj}^2 \qquad u_n^2 = \frac{1}{N}\left(\sum_{j=1}^{N}{}_{j\neq n} u_{nj}^2 - |\underline{u}|^2\right), \quad n = 1 \div N$$

Selection of voltage reference

N-phase systems with neutral wire

In case of symmetrical resistive load the proportionality condition between phase voltages and currents holds only if the voltage reference is set to the neutral wire.

$$u_{ref} = u_{o_{measure}} \quad \Rightarrow \quad u_n = u_{n_{measure}} - u_{ref}, \; n = 0 \div N$$

$$u_n = R\, i_n, \; n = 1 \div N \qquad u_o = 0$$

Unity power factor occurs if only the phase quantities are considered to calculate the apparent power:

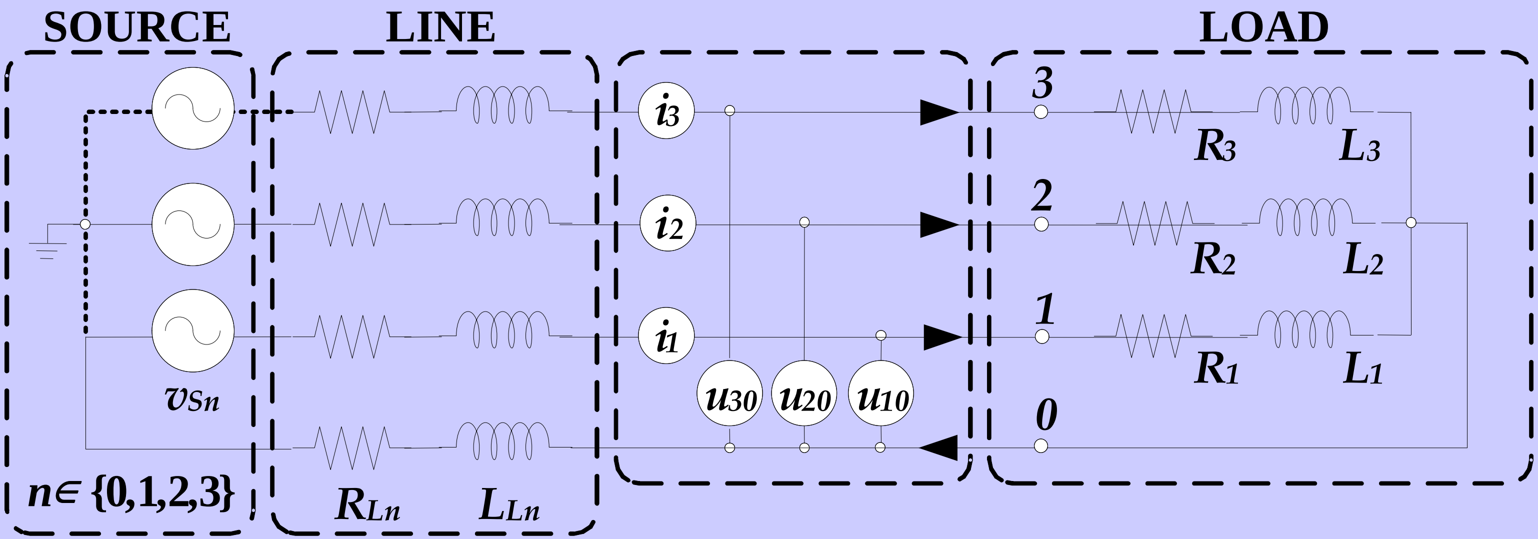
$$A = P = \boldsymbol{U}\,\boldsymbol{I}, \qquad \boldsymbol{U} = \sqrt{\sum_{n=1}^{N} U_n^2} \left(= \sqrt{\sum_{n=0}^{N} U_n^2} \right) \qquad \boldsymbol{I} = \sqrt{\sum_{n=1}^{N} I_n^2} \left(\neq \sqrt{\sum_{n=0}^{N} I_n^2} \right)$$

The neutral current is disregarded for apparent power computation

Selection of voltage reference

N-phase systems with neutral wire

Measurement of voltages and currents



Collective rms voltage and current

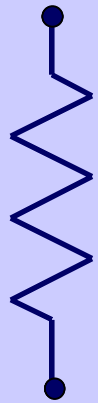
$$U = \sqrt{\sum_{n=1}^{N} U_n^2} \qquad I = \sqrt{\sum_{n=1}^{N} I_n^2}$$

Homopolar voltage and current

$$u^z = \frac{1}{N}\sum_{n=1}^{N} u_n \qquad i^z = \frac{1}{N}\sum_{n=1}^{N} i_n = -\frac{i_o}{N}$$

Power terms in passive networks

Resistor



$$u = R\, i$$

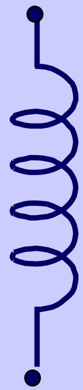
$$i = G u$$

$$P_R = \langle u, i \rangle = G \, \|u\|^2 = R \, \|i\|^2$$

$$W_R = \langle \widehat{u}, i \rangle = R \langle \widehat{i}, i \rangle = 0$$

Power terms in passive networks

Inductor



$$u = L\frac{di}{dt} = L\,\breve{i}$$

$$i = \frac{\overset{\frown}{u}}{L}$$

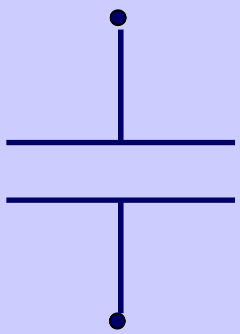
$$P_L = \langle u, i\rangle = \left\langle u, \frac{\overset{\frown}{u}}{L}\right\rangle = 0 \qquad W_L = \langle \overset{\frown}{u}, i\rangle = \langle L\,i, i\rangle = L\,\|i\|^2$$

Inductor energy

$$\varepsilon_L = \frac{1}{2}L\,i^2 \;\Rightarrow\; \bar{\varepsilon}_L = \mathrm{E}_L = \frac{1}{2}L\,\|i\|^2 = \frac{W_L}{2}$$

Power terms in passive networks

Capacitor



$$i = C\frac{du}{dt} = C\breve{u}$$

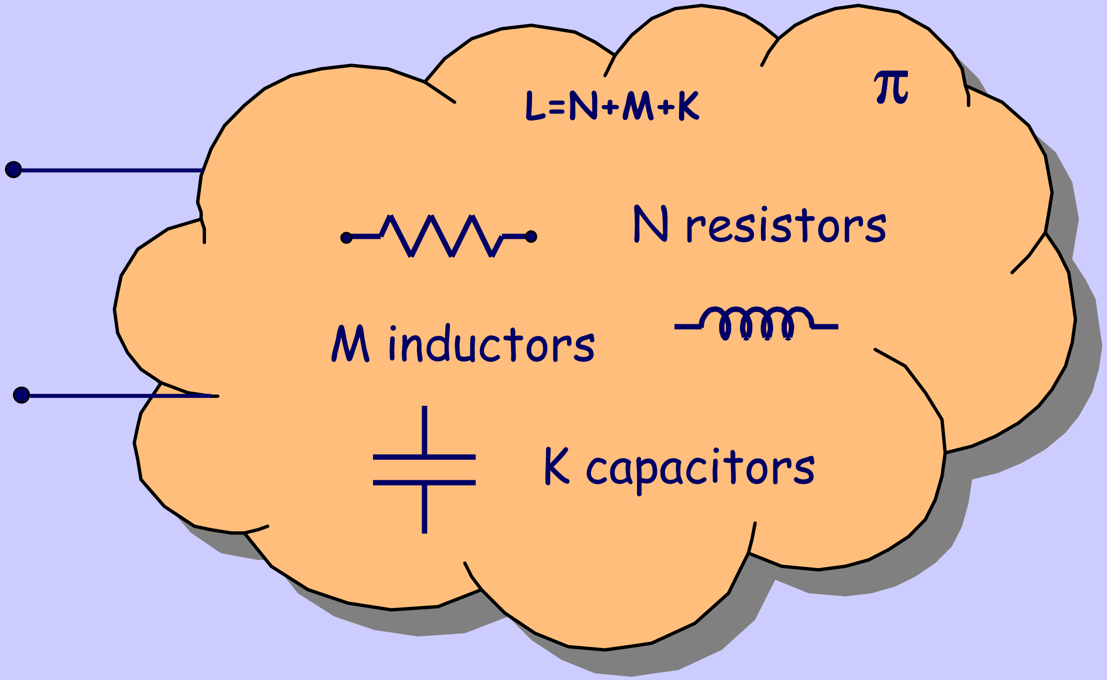
$$u = \frac{\widehat{i}}{C}$$

$$P_C = \langle u, i \rangle = \left\langle \frac{\widehat{i}}{C}, i \right\rangle = 0 \qquad W_C = -\langle u, \widehat{i} \rangle = -\langle u, C\,u \rangle = -C\|u\|^2$$

Capacitor energy

$$\varepsilon_C = \frac{1}{2}Cu^2 \quad \Rightarrow \quad \bar{\varepsilon}_C = \mathrm{E}_C = \frac{1}{2}C\|u\|^2 = -\frac{W_C}{2}$$

Linear passive network p



Remark: Whichever is the origin of reactive energy, including active and nonlinear loads, it can be compensated by reactive elements with proper energy storage capability

Total active power and reactive energy

$$P = \sum_{l=1}^{L} \langle u_l, i_l \rangle = \sum_{n=1}^{N} P_{R_n} = P_{R_{tot}}$$

$$W = \sum_{l=1}^{L} \langle \hat{u}_l, i_l \rangle = \sum_{m=1}^{M} W_{L_m} + \sum_{k=1}^{K} W_{C_k} = 2\left(\sum_{m=1}^{M} \mathrm{E}_{L_m} - \sum_{k=1}^{K} \mathrm{E}_{C_k}\right) = 2\left(\mathrm{E}_{L_{tot}} - \mathrm{E}_{C_{tot}}\right)$$

Orthogonal current decomposition

Voltage and current measured at a generic network port

✓ Desired current decomposition

$$\boxed{i = i_a + i_r + i_v = i_a + i_r + \underbrace{i_{sa} + i_{sr} + i_g}_{i_v}}$$

- i_a active current
- i_r reactive current
- i_v void current

- i_{sa} **scattered active current**
- i_{sr} **scattered reactive current**
- i_g **generated current**

✓ Orthogonality: all terms in the above equations are orthogonal

$$\|i\|^2 = \|i_a\|^2 + \|i_r\|^2 + \|i_v\|^2 = \|i_a\|^2 + \|i_r\|^2 + \|i_{sa}\|^2 + \|i_{sr}\|^2 + \|i_g\|^2$$

Orthogonal current decomposition

Voltage and current measured at a generic network port

- **Active current**: the minimum current (i.e., with minimum rms value) needed to convey active power *P* through the port

$$i_a = \frac{\langle u, i \rangle}{\|u\|^2} u = \frac{P}{U^2} u = G_e u$$

u = port voltage

U = rms value of port voltage

G_e = equivalent conductance

$$P_a = \langle u, i_a \rangle = G_e \langle u, u \rangle = G_e U^2 = P$$

$$W_a = \langle \hat{u}, i_a \rangle = G_e \langle \hat{u}, u \rangle = 0$$

Active current conveys **full active power** and **zero reactive energy**

Orthogonal current decomposition

Voltage and current measured at a generic network port

$$i_r = \frac{\langle \hat{u}, i \rangle}{\|\hat{u}\|^2}\hat{u} = \frac{W}{\hat{U}^2}\hat{u} = B_e \hat{u}$$

B_e = equivalent susceptance

- Reactive current: the minimum current needed to convey reactive energy *W* through the port

$$P_r = \langle u, i_r \rangle = B_e \langle u, \hat{u} \rangle = 0$$

$$W_r = \langle \hat{u}, i_r \rangle = B_e \langle \hat{u}, \hat{u} \rangle = B_e \hat{U}^2 = W$$

Reactive current conveys full reactive energy and no active power

$$\langle i_a, i_r \rangle = G_e B_e \langle u, \hat{u} \rangle = 0$$

Active and reactive current are orthogonal

Orthogonal current decomposition

Voltage and current measured at a generic network port

✓ **Void current:** is the remaining current component

$$i_v = i - i_a - i_r$$

Void current is not conveying active power or reactive energy

$$P_v = \langle u, i_v \rangle = \langle u, i \rangle - \langle u, i_a \rangle - \langle u, i_r \rangle = P - P_a - P_r = 0$$

$$W_v = \langle \hat{u}, i_v \rangle = \langle \hat{u}, i \rangle - \langle \hat{u}, i_a \rangle - \langle \hat{u}, i_r \rangle = W - W_a - W_r = 0$$

Void current is orthogonal to active and reactive terms

$$\langle i_v, i_a \rangle = G_e \langle i_v, u \rangle = G_e P_v = 0$$

$$\langle i_v, i_r \rangle = B_e \langle i_v, \hat{u} \rangle = B_e W_v = 0$$

Orthogonal current decomposition

Voltage and current measured at a generic network port

The **void current** reflects the presence of **scattered active**, **scattered reactive** and **load-generated** harmonic terms

$$i_v = i_{sa} + i_{sr} + i_g$$

Scattered current terms:
Account for different values of equivalent admittance at different harmonics

Load-generated current harmonics:
Harmonic terms that exist in currents only, not in voltages

$$\langle i_{sa}, i_{sr} \rangle = \langle i_{sa}, i_g \rangle = \langle i_{sr}, i_g \rangle = 0$$

Scattered and load-generated harmonic currents are orthogonal (they belong to separated harmonic sets)

Scattered active current

For each co-existing harmonic components of voltage and current we define:

- Harmonic active current terms

$$i_{ak} = \frac{\langle u_k, i_k \rangle}{\|u_k\|^2} u_k = \frac{P_k}{U_k^2} u_k = \frac{I_k \cos\varphi_k}{U_k} u_k = G_k u_k$$

✓ **Total harmonic active current**

$$i_{ha} = \sum_{k \in K} i_{ak} \qquad P_{ha} = \sum_{k \in K} P_k = P_a = P, \quad W_{ha} = 0$$

✓ **Scattered active current**

$$i_{sa} = i_{ha} - i_a = \sum_{k \in K} (G_k - G_e) u_k \qquad P_{sa} = P_{ha} - P_a = 0, \quad W_{sa} = 0$$

Scattered reactive current

For each co-existing harmonic components of voltage and current we define:

- Harmonic reactive current terms

$$i_{rk} = \frac{\langle \hat{u}_k, i_k \rangle}{\|\hat{u}_k\|^2}\hat{u}_k = \frac{W_k}{\hat{U}_k^2}\hat{u}_k = \frac{\omega k I_k \sin\varphi_k}{U_k}\hat{u}_k = B_k \hat{u}_k$$

✓ **Total harmonic reactive current**

$$i_{hr} = \sum_{k \in K} i_{rk} \qquad W_{hr} = \sum_{k \in K} W_k = W_r = W, \quad P_{hr} = 0$$

✓ **Scattered reactive current**

$$i_{sr} = i_{hr} - i_r = \sum_{k \in K} (B_k - B_e)\hat{u}_k \qquad W_{sr} = W_{hr} - W = 0, \quad P_{sr} = 0$$

Apparent power decomposition

$$A = \|u\|\|i\| = UI = \sqrt{P^2 + Q^2 + V^2}$$

- **Active power:** $P = \|u\|\,\|i_a\| = U\,I_a$
- **Reactive power:** $Q = \|u\|\,\|i_r\| = U\,I_r$
- **Void power:** $V = \|u\|\,\|i_v\| = U\,I_v = \sqrt{S_a^2 + S_r^2 + G^2}$
  - Scattered active power: $S_a = \|u\|\,\|i_{sa}\| = U\,I_{sa}$
  - Scattered reactive power: $S_r = \|u\|\,\|i_{sr}\| = U\,I_{sr}$
  - Load-generated harmonic power: $G = \|u\|\,\|i_g\| = U\,I_g$

Reactive Power

U and $\hat{U}$ can be decomposed in fundamental and harmonic components
(THD means total harmonic distortion)

$$U = \sqrt{U_f^2 + U_h^2} = U_f \sqrt{1 + [THD(u)]^2}$$

$$\hat{U} = \sqrt{\hat{U}_f^2 + \hat{U}_h^2} = \hat{U}_f \sqrt{1 + [THD(\hat{u})]^2}$$

Recalling that: $U_f / \hat{U}_f = \omega$

We have:

$$Q = U\, I_r = \frac{U}{\hat{U}} W = \omega W \frac{\sqrt{1 + [THD(u)]^2}}{\sqrt{1 + [THD(\hat{u})]^2}}$$

Note that, unlike reactive energy *W*, REACTIVE POWER *Q* IS NOT CONSERVATIVE. In fact, it depends on line frequency and (local) voltage distortion.

Void Power Terms

Void Power:

$$V = U\,I_v = \sqrt{S_a^2 + S_r^2 + G^2}$$

✓ **Scattered active power:**

$$S_a = UI_{sa} = U\sqrt{\langle i_{sa}, i_{sa}\rangle} = U\sqrt{\sum_{k\in\{K\}}(G_k - G_e)^2 U_k^2}$$

$$= \sqrt{U^2 \sum_{k\in\{K\}}\left(\frac{P_k}{U_k^2} - \frac{P}{U^2}\right)^2 U_k^2}$$

Void Power Terms

✓ Scattered reactive power:

$$S_r = UI_{sr} = U\sqrt{\langle i_{sr}, i_{sr}\rangle} = U\sqrt{\sum_{k\in\{K\}}(B_k - B_e)^2 \hat{U}_k^2} = \frac{U}{\hat{U}}\sqrt{\hat{U}^2 \sum_{k\in\{K\}}(B_k - B_e)^2 \hat{U}_k^2}$$

$$= \omega \frac{\sqrt{1+THD_u^2}}{\sqrt{1+THD_{\hat{u}}^2}} \sqrt{\hat{U}^2 \sum_{k\in\{K\}} \left(\frac{W_k}{\hat{U}_k^2} - \frac{W}{\hat{U}^2}\right)^2 \hat{U}_k^2}$$

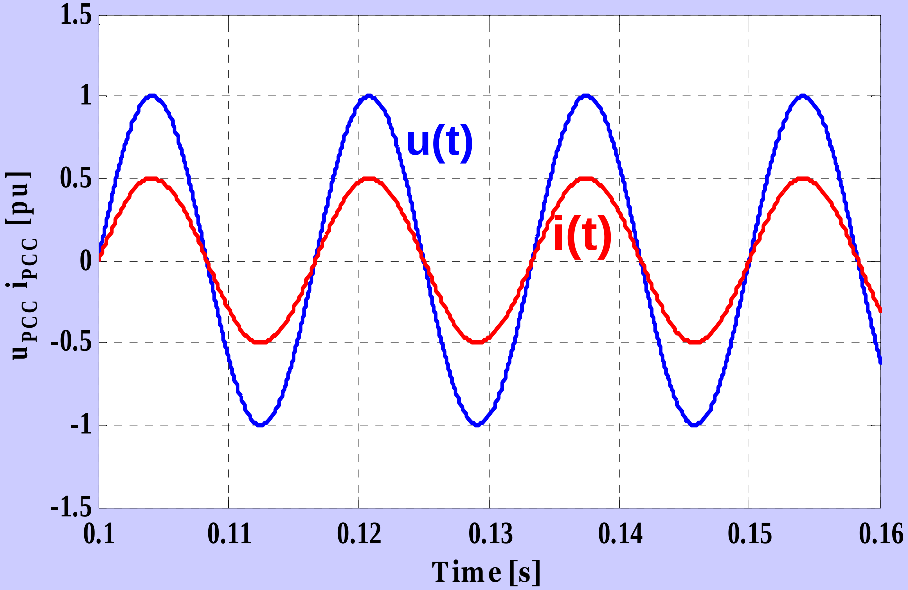
✓Load-generated harmonic power:

$$G = UI_g$$

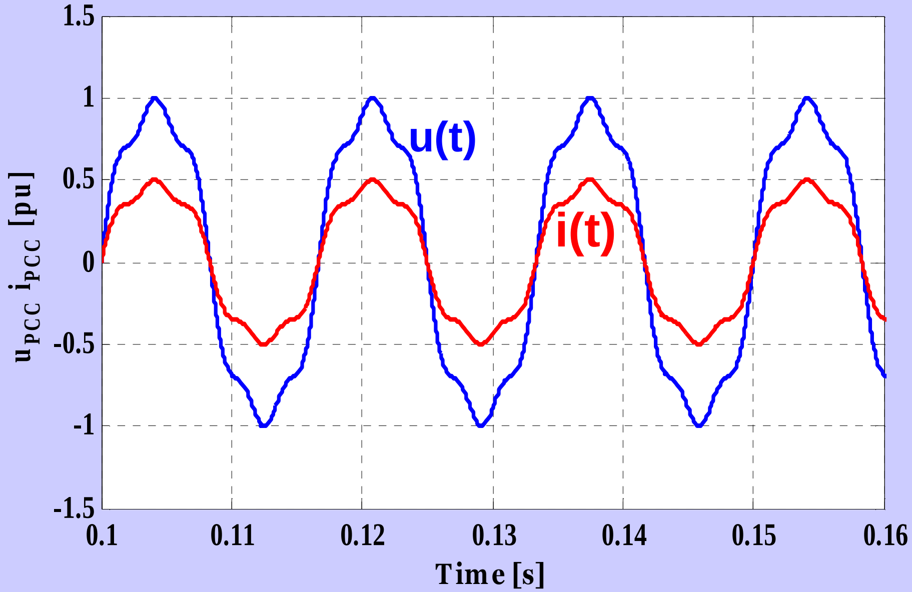
Application Examples

Example # 1

Voltage and Current : Resistive Load



Sinusoidal voltage



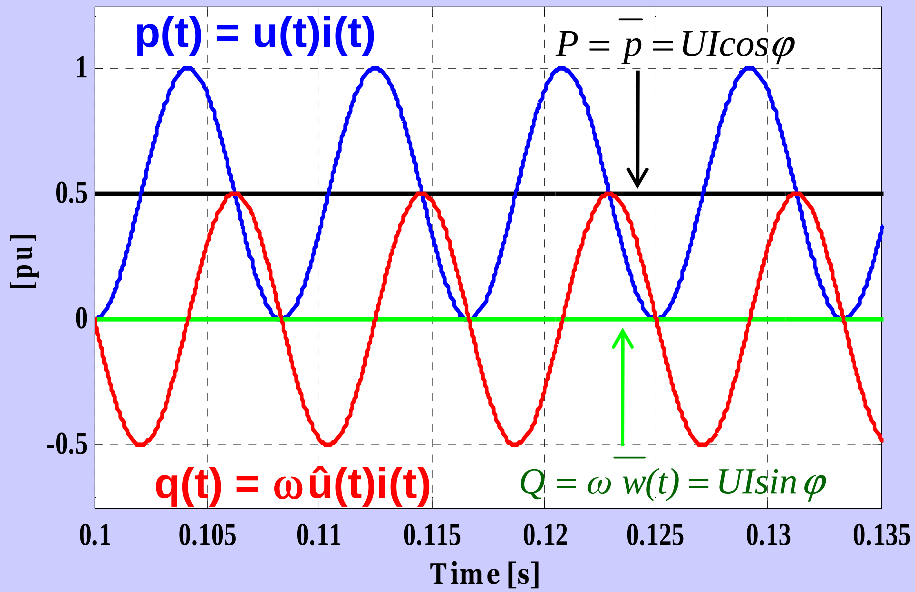
Non - sinusoidal voltage

Current = $i_{pu}(t)/2$

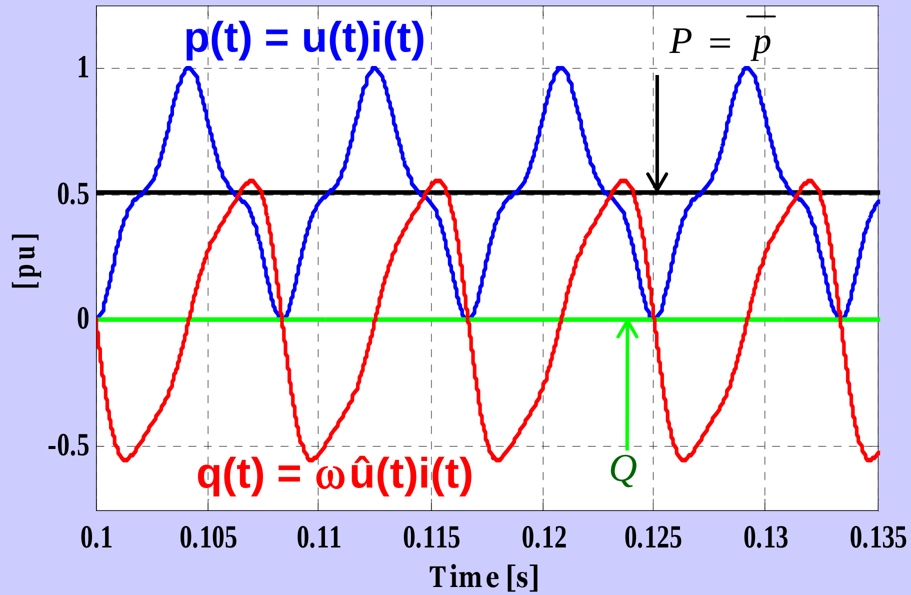
Application Examples

Example # 1

Conservative Power Terms: Resistive Load



Sinusoidal voltage



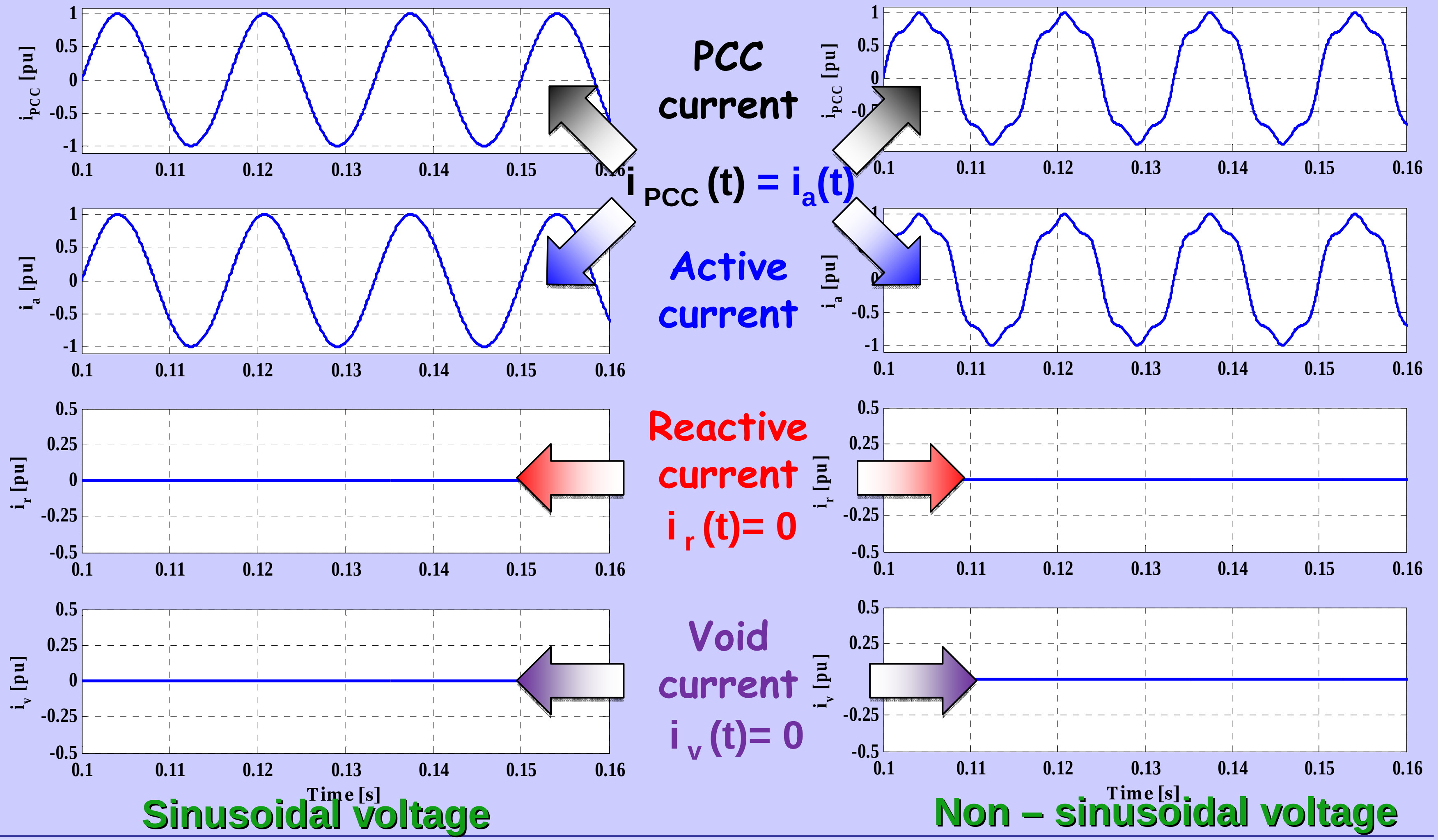
Non – sinusoidal voltage

This example shows the correspondences between the CPT theory and conventional theory

Application Examples

Example # 1 - Single-phase

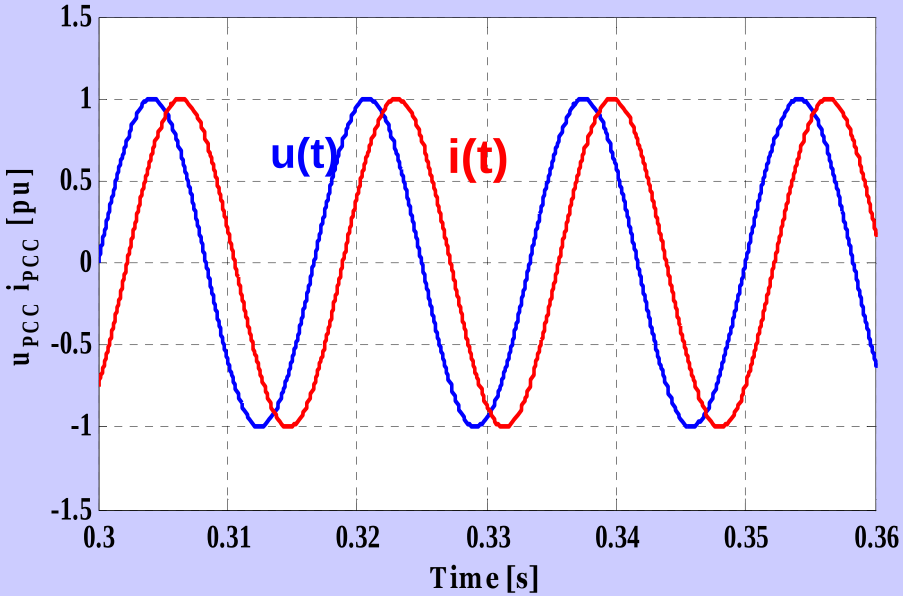
Current Terms: Resistive Load



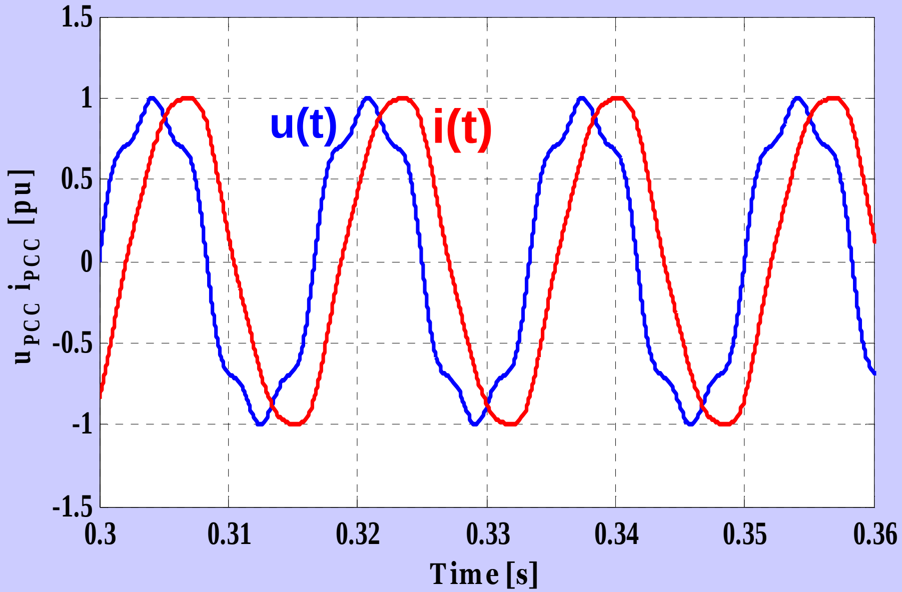
Application Examples

Example # 2

Voltage and Current : Ohmic-inductive Load



Sinusoidal voltage

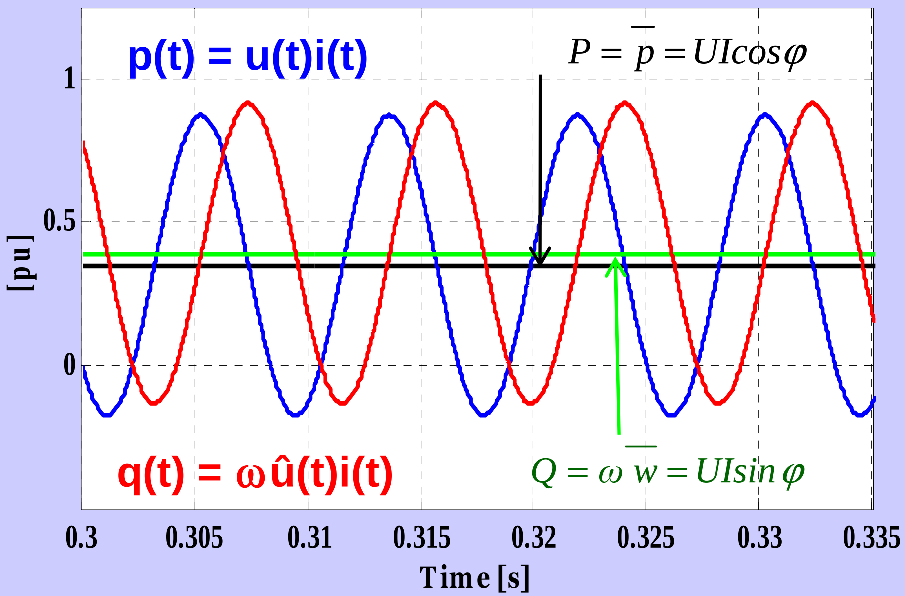


Non – sinusoidal voltage

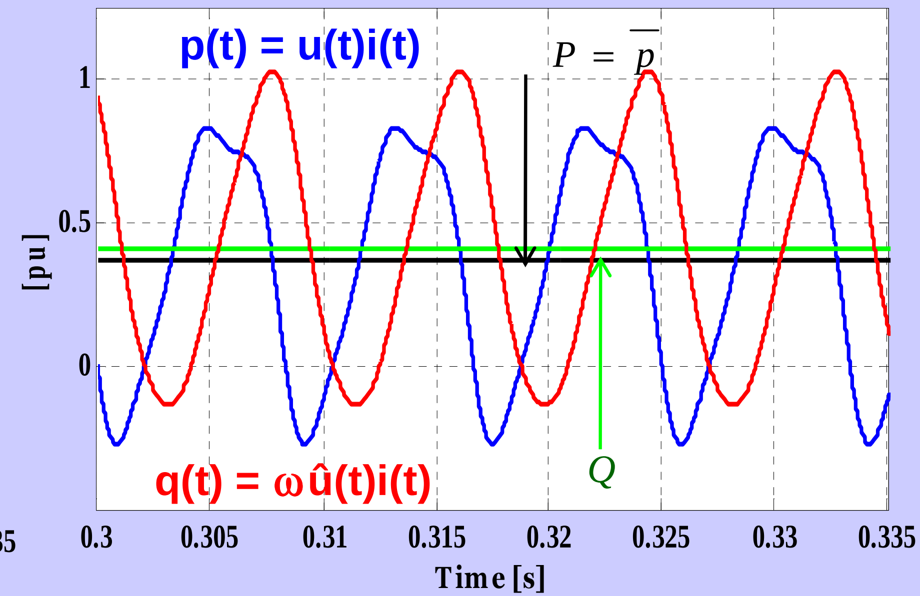
Application Examples

Example # 2

Conservative Power Terms: Ohmic-inductive Load



Sinusoidal voltage



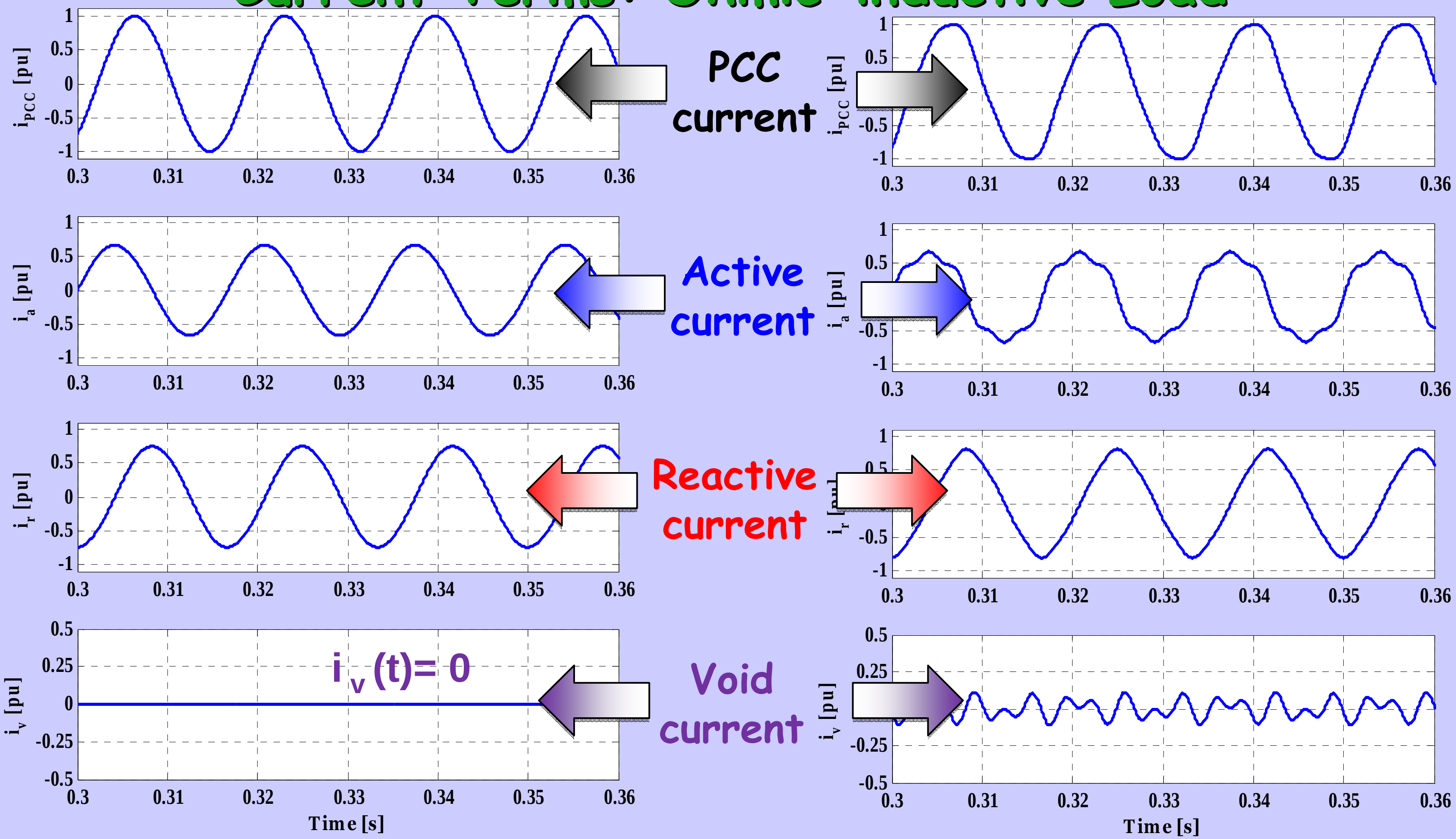
Non – sinusoidal voltage

This example shows the correspondence between CPT and conventional theory under sinusoidal conditions

Application Examples

Example # 2

Current Terms: Ohmic-inductive Load

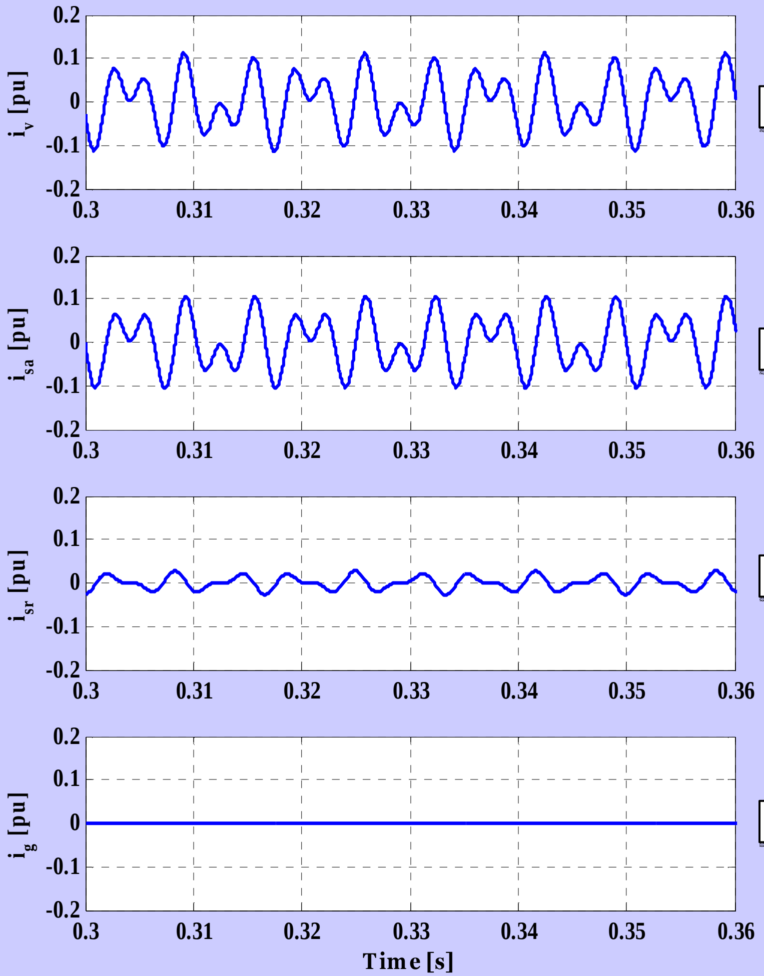


Sinusoidal voltage — Non – sinusoidal voltage

Physical meaning of void current

Example # 2: non-sinusoidal voltage

Void Current Terms: Ohmic-inductive Load



Void current

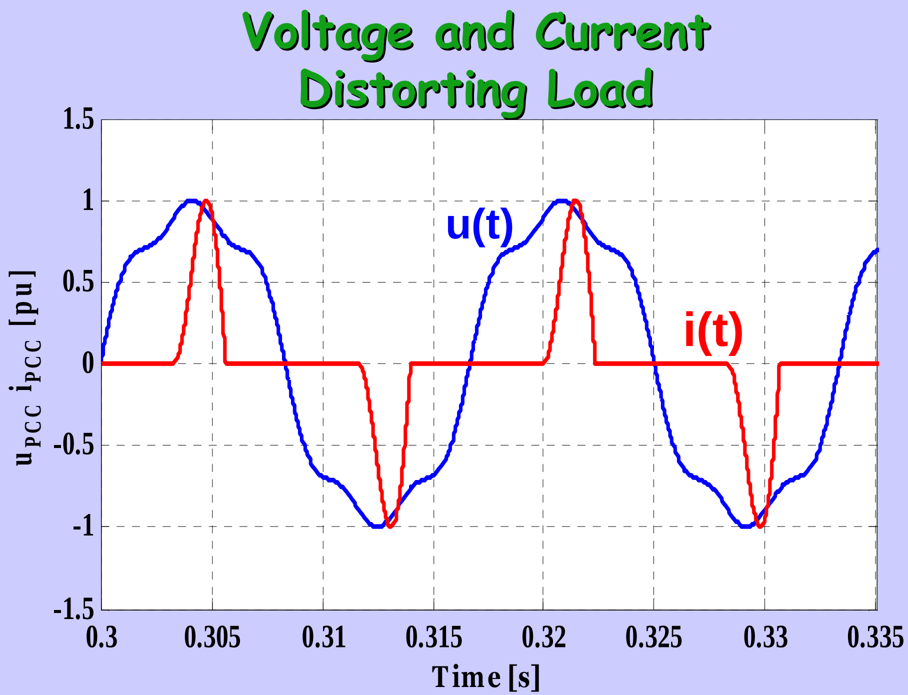
Scattered active current

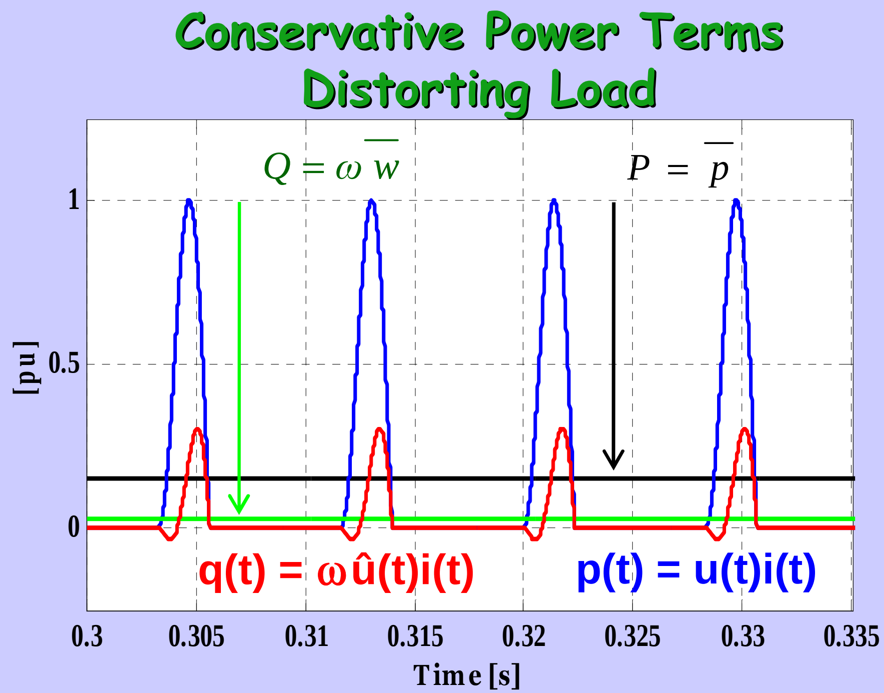
Scattered reactive current

Load-generated harmonic current

Application Examples

Example # 3

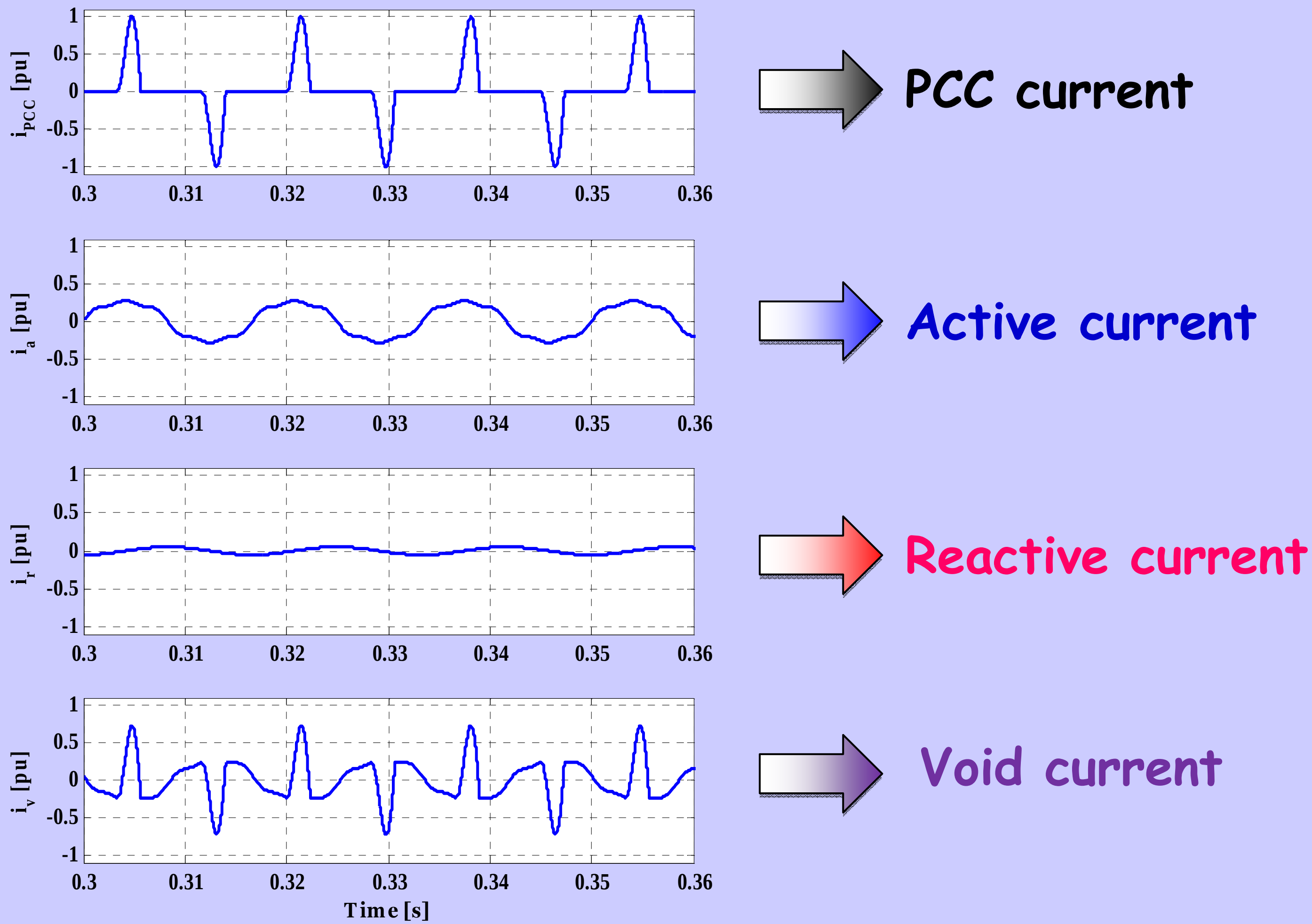




Non - sinusoidal voltage

Physical meaning of void current

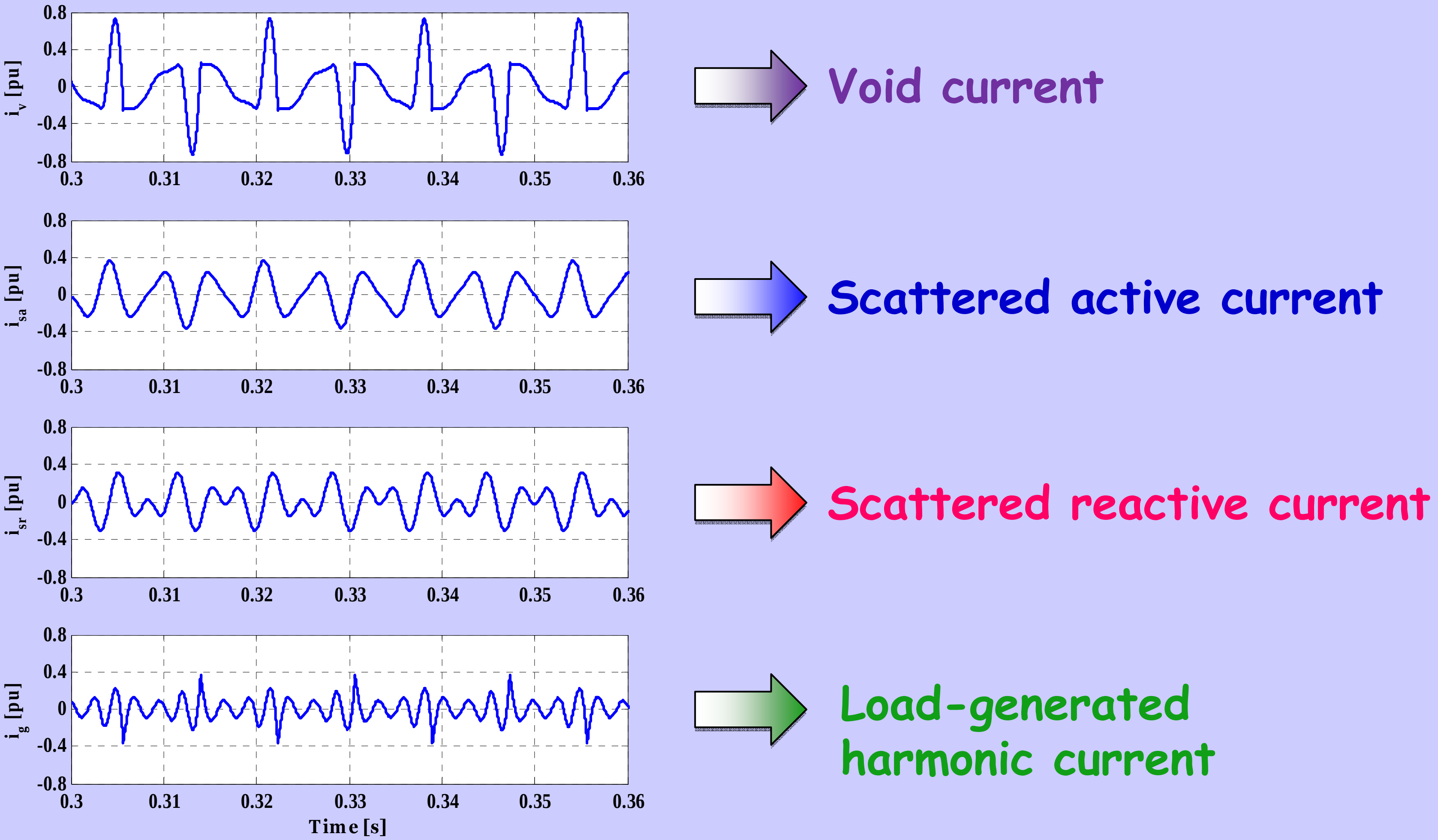
Void Current Terms: distorting Load



Non – sinusoidal voltage

Physical meaning of void current

Void Current Terms: distorting Load



Non – sinusoidal voltage

Conclusions

- The proliferation of alternative power generation systems based mainly on renewable energy sources is likely to change deeply the way the energy is generated and distributed
- The associated distributed power processors add many degrees of freedom that can be exploited for many purposes:
  - reduction of transmission & distribution losses;
  - improvement of conventional power sources utilization;
  - improvement of power quality (less distortion, less "useless" currents, etc.).

Conclusions

- If not properly managed, these DERs can cause instabilities in the grid, in the absence of a stiff equivalent voltage generator (weak grid)
- Proper grid control and operation requires a reliable communication infrastructure (Power Line Communications?) between different DPPs and enough on board intelligence
- A correct definition of different current and power terms is needed for a correct grid management