A Control Matching Model Predictive Control Approach to String Stable Vehicle Platooning

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Lecture content

1. Modeling and notation
2. Control objective, constraints and requirements
3. MPC problem formulation
4. Control matching
5. Experimental results

Note: The following notes have been extracted from "A Control Matching Model Predictive Control Approach to String Stable Vehicle Platooning", Roozbeh Kianfar, Paolo Falcone, Jonas Fredriksson. *Submitted to Control Engineering Practice.*

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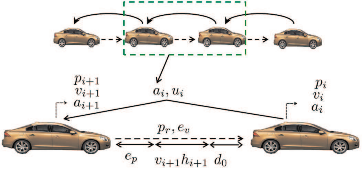
Modeling and notation

Consider the platoon in the figure. Model the i-th vehicle as $(i = 1, \ldots, N)$:

$$\begin{aligned}\dot{e}_{p,i} &= e_{v,i} - a_i h_i,\\ \dot{e}_{v,i} &= a_{i-1} - a_i,\\ a_i &= \frac{K_i}{\tau_i s+1} e^{-\theta_i s} a_i^{\text{des}},\end{aligned}$$



The resulting model is $\dot{x}(t) = Ax(t) + B_u u(t-\theta) + B_\omega \omega(t)$, with

$$\begin{aligned} x &= \begin{bmatrix} e_{p,i} & e_{v,i} & a_i & v_i \end{bmatrix}^{\mathrm{T}},\\ u_i &= a_i^{\text{des}},\\ \omega &= a_{i-1}, \end{aligned}$$

$$A = \begin{bmatrix} 0 & 1 & -h_i & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & -1/\tau_i & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B_u = \begin{bmatrix} 0 \\ 0 \\ \frac{K_i}{\tau_i} \\ 0 \end{bmatrix}, B_\omega = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix},$$

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Control objective, constraints and requirements

Problem statement. Minimizing the position and velocity errors while satisfying a performance (e.g., string stability and smooth driving) and safety requirements (e.g., rear-end collision avoidance).

1. ***Safety***. A safe *minimum* distance must be maintained from the preceding vehicle in order to prevent rear-end collisions

$$e_{p,\min} \leq e_{p,i}(t) \leq e_{p,\max}, \qquad \forall t \geq 0.$$

2. ***Performance (comfort)***.
   - Bounds on ***relative velocities***

$$e_{v,\min} \leq e_{v,i}(t) \leq e_{v,\max}, \qquad \forall t \geq 0,$$

   - ***Max velocity*** $0 \leq v_i(t) \leq v_{\max}, \qquad \forall t \geq 0,$
   - ***Acceleration***

$$a_{\min} \leq a_i(t) \leq a_{\max}, \qquad \forall t \geq 0,$$

3. ***Physical limitation***. Max acceleration demands imposed by the powertrain

$$u_{\min} \leq u_i(t) \leq u_{\max}, \qquad \forall t \geq 0.$$

Control objective, constraints and requirements

Constraints 1-3 can be compactly written as

$$\begin{bmatrix} H_x & H_u \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \leq h_x.$$

Definition

A vehicle platoon is predecessor-follower string stable w.r.t. an acceleration disturbance if

$$\|\Gamma_i(s)\|_\infty = \sup_{a_{i-1} \neq 0} \frac{\|a_i(t)\|_{\mathcal{L}_2}}{\|a_{i-1}(t)\|_{\mathcal{L}_2}}.$$

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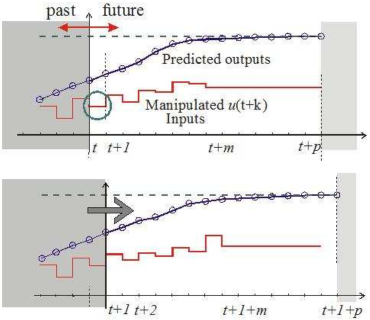
MPC problem formulation. Background



1. Every sampling time solve a ***CFTOC, based on the current state***
2. Apply the optimal input ***only in*** $[t, t+1]$
3. At $t+1$ solve a CFTOC over a ***shifted horizon*** based on new state measurements

4. The resulting controller is referred to as ***Receding Horizon Controller (RHC)***.
5. If the finite time optimal control law is computed by solving an optimization problem on-line, the RHC is usually referred to as ***Model Predictive Control (MPC)***.

MPC problem formulation

$$
\begin{aligned}
J(x(t),\omega(t)) = \min_{U_t} \quad & \|Px(N|t)\|_2 \\
& + \sum_{k=0}^{N-1} \|Qx(t+k|t)\|_2 + \|Ru(t+k|t)\|_2
\end{aligned}
$$

subj. to

$$
\begin{aligned}
& x_{t+k+1,t} = F_i x_{t+k,t} + G_{1,i} u_{t+k,t} + G_{2,i}\omega_{t+k,t}, \\
& \begin{bmatrix} H_x & H_u \end{bmatrix} \begin{bmatrix} x_{t+k,t} \\ u_{t+k,t} \end{bmatrix} \leq h_x, \\
& x_{t+N,t} \in \mathcal{X}_f = \mathbb{R}^n \\
& \omega_{t+k|t} = \omega_{t|t} \\
& \qquad\qquad\qquad\qquad k = [0, \ldots, N-1], \\
& \omega_{t,t} = \omega(t), \\
& x_{t,t} = x(t),
\end{aligned}
$$

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Control matching

Basic idea

1. Design a string stable linear controller
2. "Tune" the weighting matrices Q, P, R in order to retrieve the behavior of the string stable linear controller.

Step 1. Consider the feedback/feedforward control policy,

$$u_{ss,i} = \quad K^{ss}\begin{bmatrix} x \\ \omega \end{bmatrix} = \quad [K^{ss}_{\mathrm{FB}}\ K^{ss}_{\mathrm{FF}}]\begin{bmatrix} x \\ \omega \end{bmatrix},$$

where K^{ss}_{FB} and K^{ss}_{FF} are static state feedback and feedforward gains.

The closed-loop dynamics are $\dot{x}(t) = A^{\mathrm{cl}}x(t) + E^{\mathrm{cl}}_{\omega}\omega(t)$, where $A^{\mathrm{cl}} = A + B_u K^{ss}_{\mathrm{FB}}$ and $E^{\mathrm{cl}}_{\omega} = B_u K^{ss}_{\mathrm{FF}} + B_{\omega}$. Consider the following output signals,

$$\begin{bmatrix} a_i \\ e_{p,i} \\ u_{ss,i} \end{bmatrix} = \begin{bmatrix} C_{\Gamma} & D_{\Gamma} \\ C_H & D_H \end{bmatrix}\begin{bmatrix} x \\ \omega \end{bmatrix}.$$

Control matching

Define the transfer functions

1. $\Gamma = C_\Gamma \left(sI - A^{\mathrm{cl}}\right)^{-1} E_\omega^{\mathrm{cl}} + D_\Gamma$, the transfer function from ω to a_i,
2. $H = C_H \left(sI - A^{\mathrm{cl}}\right)^{-1} E_\omega^{\mathrm{cl}} + D_H$, the transfer function from ω to $[e_{p,i}\ u_{ss,i}]^T$.

Calculate K^{ss} as,

$$\begin{aligned} K^{ss} = \underset{K^{ss}}{\operatorname{argmin}} \quad & \alpha\|\Gamma\|_\infty + \beta\|H\|_2, \\ \text{subj.to} \quad & \|\Gamma\|_\infty \leq\ 1, \end{aligned}$$

Step 2. Rewrite the MPC problem as

$$\begin{aligned} J(x(k), \omega(k), U(k)) = \min_{U(k)} & U(k)^{\mathrm{T}} H U(k) \\ & + 2x(k)^{\mathrm{T}} F U(k) + x(k)^{\mathrm{T}} Y x(k) \\ \text{subj. to} & \\ & MU(k) \leq W(k) + Ex(k), \end{aligned}$$

Control matching

Solve the "unconstrained" problem

$$J(x(k), \omega(k), U(k)) = \min_{U(k)} U(k)^{\mathrm{T}} H U(k) \\ + 2x(k)^{\mathrm{T}} F U(k) + x(k)^{\mathrm{T}} Y x(k)$$

The solution is $U^{\star}(k) = -H^{-1} F^{\mathrm{T}} x(k)$ and the feedback control law is

$$u^{\star}(k) = u(0|k) = -\Lambda H^{-1} F^{\mathrm{T}} x(k),$$

where $\Lambda = [\begin{array}{cccc} I & 0 & \cdots & 0 \end{array}]$.

"Match" the controller by finding the weighting matrices $P,\ Q,\ R$ (hidden in $H,\ F$) such that

$$u_{ss}(k) = [K_{FB} K_{FF}]\tilde{x}(k) = -\Lambda H^{-1} F^{\mathrm{T}} \tilde{x}(k)$$

Note that, the obtained MPC controller can guarantee string stability as long as constraints are not active.

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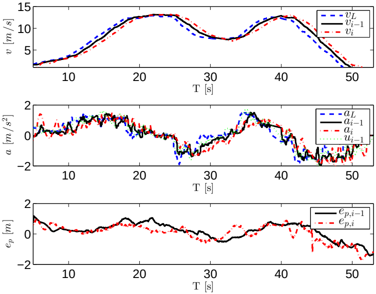

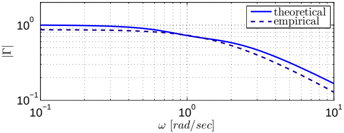

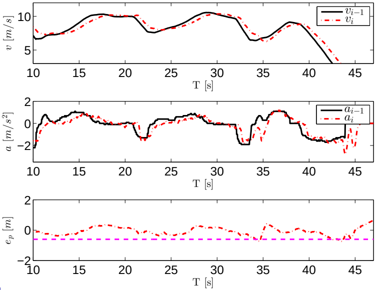

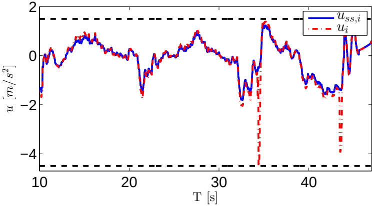

Experimental results