Linear Matrix Inequalities LMI-Based State Feedback Synthesis for Platoon Control

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1. Platoon Model under Predecessor Following
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4. Synthesis with Multiple Performance Objectives
5. String Stability under Predecessor Following
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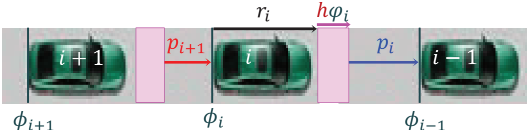
Platoon Model under Predecessor Following



- Kinematics (under **constant time headway** policy with a **time gap** of h):

	Position	Velocity	Acceleration
Absolute	ϕ_i	$\varphi_i = \dot{\phi}_i$	$\alpha_i = \dot{\varphi}_i$
Relative	$p_i \triangleq \phi_{i-1} - \phi_i - r_i - h\varphi_i$	$v_i \triangleq \varphi_{i-1} - \varphi_i$	$a_i \triangleq \alpha_{i-1} - \alpha_i$

- Vehicle Dynamics (with a first-order model identified by a **time constant** τ):

$$\dot{\alpha}_i = -\frac{1}{\tau}\alpha_i + \frac{1}{\tau}u_i$$

- Platoon Model:

$$\underbrace{\begin{bmatrix} \dot{p}_i \\ \dot{v}_i \\ \dot{\alpha}_i \end{bmatrix}}_{\dot{x}_i} = \underbrace{\begin{bmatrix} 0 & 1 & -h \\ 0 & 0 & -1 \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} p_i \\ v_i \\ \alpha_i \end{bmatrix}}_{x_i} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau} \end{bmatrix}}_{B} u_i + \underbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}_{H} \underbrace{\alpha_{i-1}}_{d_i}$$

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Formulation of a Synthesis Problem

- Consider the platoon model with fixed and known (A, B, H):
$$\dot{x}_i = Ax_i + Bu_i + Hd_i$$
- Introduce a **performance indicator** whose "**size**" is required to be "**small**". With c serving as a parameter to adjust comfort, a good choice would be
$$\underbrace{\begin{bmatrix} p_i \\ c\tau\dot{\alpha}_i \end{bmatrix}}_{z_i} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -c \end{bmatrix}}_{C} x_i + \underbrace{\begin{bmatrix} 0 \\ c \end{bmatrix}}_{D} u_i + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{E} d_i$$
String stability will be linked to $\alpha_i = \underbrace{\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}}_{S} x_i + 0 \cdot u_i + 0 \cdot d_i$
- **Problem:** Find gains F and G such that the controlled system
$$u_i = \underbrace{Fx_i}_{FB} - \underbrace{Gd_i}_{FF} \quad \Rightarrow \quad \begin{array}{rcl} \dot{x}_i & = & (A + BF)x_i + (H - BG)d_i \\ z_i & = & (C + DF)x_i + (E - DG)d_i \end{array}$$
is **stable** and respects the following constraint (with a "**desirably**" small γ):
$$\|z_i\| < \gamma\|d_i\|, \quad \text{for all } d_i \text{ with } 0 < \|d_i\| < \infty \text{ and } x_i(0) = 0$$
- Size $\|\cdot\|$ can be defined in alternative ways and differently for d_i and z_i.

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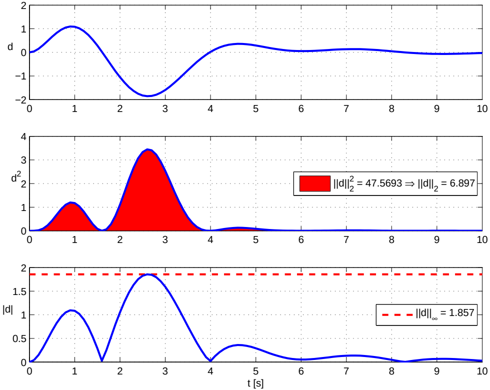
$\mathcal{L}_p$ Norms as Signal Sizes



- Size is usually measured with the common $\mathcal{L}_p$-norms ($p = 2, \infty$):

$$\|d\|_2 \triangleq \sqrt{\int_0^\infty d^T(t)d(t)dt} \qquad \|d\|_\infty \triangleq \sup_{t\geq 0} \sqrt{d^T(t)d(t)}$$

$\mathcal{H}_\infty$ versus Generalized $\mathcal{H}_2$ Performance Objectives

- For simplicity drop index i and introduce the transfer function from d to z:

$$\hat{z}(s) = \underbrace{\Big[(C + DF)(sI - A - BF)^{-1}(H - BG) + E - DG\Big]}_{\mathcal{S}(s)} \hat{d}(s)$$

- $\mathcal{H}_\infty$ and $\mathcal{H}_2$ norms are defined for **stable** transfer functions as follows:

$$\|\mathcal{S}\|_\infty \triangleq \sup_{\omega \in \mathbb{R}} \sqrt{\lambda_{\max}\left[\mathcal{S}^*(\jmath\omega)\mathcal{S}(\jmath\omega)\right]} \qquad \|\mathcal{S}\|_2 \triangleq \sqrt{\int_0^\infty \mathsf{trace}\left[\mathcal{S}^*(\jmath\omega)\mathcal{S}(\jmath\omega)\right] d\omega}$$

- **Energy-to-energy** gain bounding is an $\boldsymbol{\mathcal{H}_\infty}$ constraint:

$$\gamma > \|\mathcal{S}\|_\infty \overbrace{=}^{FACT} \sup_{d \neq 0} \frac{\|z\|_2}{\|d\|_2} \quad \Leftrightarrow \quad \|z\|_2 < \gamma \|d\|_2$$

- **Energy-to-squared-peak** gain bounding is a **generalized** $\boldsymbol{\mathcal{H}_2}$ constraint:

$$\textbf{When } \boldsymbol{z} \textbf{ is scalar: } \gamma > \|\mathcal{S}\|_2 \overbrace{=}^{FACT} \sup_{d \neq 0} \frac{\|z\|_\infty}{\|d\|_2} \quad \Leftrightarrow \quad \|z\|_\infty < \gamma \|d\|_2$$

LMI Conditions for $\mathcal{H}_\infty$ Constraints

- Construct a candidate Lyapunov function via $Y \succ 0$ and impose

$$\underbrace{\frac{d}{dt}x^T(t)Y^{-1}x(t) + \frac{1}{\gamma}z^T(t)z(t) - \gamma d^T(t)d(t)}_{\eta(t)} < 0 \Rightarrow \underbrace{\frac{1}{\gamma}\|z\|_2^2 - \gamma\|d\|_2^2}_{\int_0^\infty \eta(t)dt} < 0$$

- Introduce $\theta \triangleq Y^{-1}x$ ($\Rightarrow x = Y\theta$) and $N \triangleq FY$ ($\Rightarrow Fx = N\theta$) to obtain

$$\begin{aligned} \dot{x} &= (AY + BN)\theta + (H - BG)d \\ z &= (CY + DN)\theta + (E - DG)d \end{aligned}$$

- Let us now express η in terms of θ as well:

$$\eta = \underbrace{x^T Y^{-1}}_{\theta^T}\dot{x} + \dot{x}^T \underbrace{Y^{-1}x}_{\theta} + \frac{1}{\gamma}z^T z - \gamma d^T d$$

- We now derive the concise expression

$$\eta = 2\left(\theta^T\dot{x} + \frac{1}{2\gamma}z^T z - \frac{\gamma}{2}d^T d\right) = 2\underbrace{\begin{bmatrix} \theta \\ d \\ \frac{1}{\gamma}z \end{bmatrix}^T}_{\varkappa^T}\underbrace{\begin{bmatrix} \dot{x} \\ -\frac{\gamma}{2}d \\ \frac{1}{2}z \end{bmatrix}}_{\mu} = \varkappa^T\mu + \mu^T\varkappa$$

LMI Conditions for $\mathcal{H}_\infty$ Constraints (ctd.)

- Using the system dynamics, we relate μ to $\varkappa$ as

$$\mu = \begin{bmatrix} \dot{x} \\ -\frac{\gamma}{2} d \\ \frac{1}{2} z \end{bmatrix} = \underbrace{\begin{bmatrix} AY + BN & H - BG & 0 \\ 0 & -\frac{\gamma}{2} I & 0 \\ CY + DN & E - DG & -\frac{\gamma}{2} I \end{bmatrix}}_{\mathcal{M}} \underbrace{\begin{bmatrix} \theta \\ d \\ \frac{1}{\gamma} z \end{bmatrix}}_{\varkappa}$$

- We thus express $\eta < 0$ as follows:

$$\eta = \varkappa^T \underbrace{\mathcal{M}\varkappa}_{\mu} + \underbrace{\varkappa^T \mathcal{M}^T}_{\mu^T} \varkappa = \varkappa^T \underbrace{\left(\mathcal{M} + \mathcal{M}^T\right)}_{\mathcal{N} = \mathrm{He}\{\mathcal{M}\}} \varkappa < 0$$

- In order to ensure this condition at each time instant, we need to have

$$\eta(t) < 0, \forall t \geq 0 \quad \Leftrightarrow \quad \varkappa^T \mathcal{N} \varkappa < 0, \forall \varkappa \neq 0 \quad \Leftrightarrow \quad \mathcal{N} \prec 0$$

- LMI condition for $\eta(t) < 0$ and the resulting FB gain are obtained as follows:

$$\mathcal{N} = \begin{bmatrix} \mathrm{He}\{AY + BN\} & H - BG & * \\ * & -\gamma I & * \\ CY + DN & E - DG & -\gamma I \end{bmatrix} \prec 0, \quad Y \succ 0, \quad F = NY^{-1}$$

LMI Conditions for Generalized $\mathcal{H}_2$ Constraints

- In order for z to have finite peak for all finite-energy d, we need to have

$$E - DG = 0$$

- In this case candidate Lyapunov function is required to satisfy two conditions:

$$\underbrace{\frac{d}{dt}x^T(t)Y^{-1}x(t) - \gamma d^T(t)d(t)}_{\eta(t)} < 0 \Rightarrow x^T(t)Y^{-1}x(t) < \gamma \underbrace{\int_0^t d^T(\tau)d(\tau)d\tau}_{\leq \|d\|_2^2}$$

$$\underbrace{x^T(t)Y^{-1}x(t) - \frac{1}{\gamma}z^T(t)z(t)}_{\mu(t)} > 0 \Rightarrow \frac{1}{\gamma}z^T(t)z(t) - \gamma\|d\|_2^2 < 0, \forall t \geq 0$$

- LMI conditions for $\eta(t) < 0$ and $\mu(t) > 0$ are obtained in a similar fashion:

$$\begin{bmatrix} \mathsf{He}\{AY + BN\} & H - BG \\ * & -\gamma I \end{bmatrix} \prec 0, \qquad \begin{bmatrix} Y & * \\ CY + DN & \gamma I \end{bmatrix} \succ 0$$

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Synthesis with Multiple Performance Objectives

- Consider a synthesis problem in which constraints need to be imposed on

$$z^j = C^j x + D^j u + E^j d, \quad j = 1, \ldots, m$$

- Normally, LMI conditions for j'th constraint can be imposed with (Y^j, N^j).
- In order achieve the objectives with a common FB gain, we need to have

$$F = N^1 (Y^1)^{-1} = \ldots = N^m (Y^m)^{-1}$$

- The synthesis problem becomes nonlinear due to these coupling constraints.
- Suboptimal synthesis can be performed with identical (Y^j, N^j).
- To potentially improve the suboptimal synthesis (for $m \leq 3$), one can use

$$(Y^j, N^j) = \psi_j (Y, N)$$

  with $\psi_1 = 1$ and perform a search over the scalars $\psi_j > 0$ for $j = 2, \ldots, m$.
- A better alternative would be to use the so-called **dilated LMI conditions**, which also usually have bilinear dependance on scalar variables.

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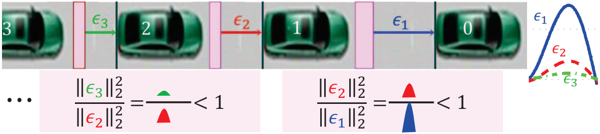
$\mathcal{L}_2$ String Stability under Predecessor Following



- It is important to ensure the string stability of acceleration signals α_i as well.
- $\mathcal{L}_2$ string stability conditions read as $\mathcal{H}_\infty$ constraints (on $\frac{\hat{\epsilon}_i(s)}{\hat{\epsilon}_{i-1}(s)}$, $\frac{\hat{\alpha}_i(s)}{\hat{\alpha}_{i-1}(s)}$, ...).
- Since $d_i = \alpha_{i-1}$ in predecessor following, string stability of α_i is expressed as

$$\|\mathcal{T}\|_\infty \leq 1, \quad \text{where} \quad \hat{\alpha}_i(s) = \underbrace{S(sI - A - BF)^{-1}(H - BG)}_{\mathcal{T}(s)} \hat{d}_i(s)$$

- In homogenous platoons, this ensures string stability for ϵ_i as well since

$$\frac{\hat{\epsilon}_i(s)}{\hat{\epsilon}_{i-1}(s)} = \frac{s^2 \hat{p}_i(s)}{s^2 \hat{p}_{i-1}(s)} = \frac{s[\hat{v}_i(s) + h\hat{\alpha}_i(s)]}{s[\hat{v}_{i-1}(s) + h\hat{\alpha}_{i-1}(s)]} = \frac{\hat{a}_i(s) + hs\hat{\alpha}_i(s)}{\hat{a}_{i-1}(s) + hs\hat{\alpha}_{i-1}(s)} = \mathcal{T}(s)$$

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An LMI-Based Synthesis for Predecessor Following

- One can formulate a synthesis based on two performance objectives:
  - an $\mathcal{H}_\infty$ (or generalized $\mathcal{H}_2$ objective) for $z_i = C \cdot x_i + D \cdot u_i + E \cdot d_i$:
  $$\|z_i\|_2 < \gamma \|d_i\|_2 \quad (\text{or } \|z_i\|_\infty < \gamma \|d_i\|_2)$$
  - an $\mathcal{H}_\infty$ objective for $\alpha_i = S \cdot x_i + 0 \cdot u_i + 0 \cdot d_i$ (related with string stability):
  $$\|\alpha_i\|_2 < \sigma \|d_i\|_2$$

- By a line search over $\psi > 0$, one can minimize γ (σ) for fixed σ (γ) under
  - $$\begin{bmatrix} \mathsf{He}\{AY + BN\} & H - BG & * \\ * & -\gamma I & * \\ CY + DN & E - DG & -\gamma I \end{bmatrix} \prec 0, \quad Y \succ 0$$

$$\left(\text{or } \begin{bmatrix} \mathsf{He}\{AY + BN\} & H - BG \\ * & -\gamma I \end{bmatrix} \prec 0, \quad \begin{bmatrix} Y & * \\ CY + DN & -\gamma I \end{bmatrix} \succ 0, \;\; E - DG = 0 \right)$$

  - (Adapt first condition by $C \to S, D \to 0, E \to 0, Y \to \psi Y$)
  $$\begin{bmatrix} \psi \mathsf{He}\{AY + BN\} & H - BG & * \\ * & -\sigma I & 0 \\ \psi S Y & 0 & -\sigma I \end{bmatrix} \prec 0$$

- The feedback gain can then be constructed as $F = NY^{-1}$.
- **Remark:** For given model, minimum σ will be 1 for sufficiently large h

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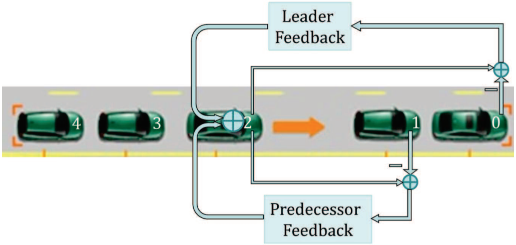
Extension Leader and Predecessor Following



- Kinematic variables are defined in terms of a **predecessor weight** $\kappa \in [0, 1]$:

$$
\begin{aligned}
p_i &\triangleq \kappa(\phi_{i-1} - \phi_i - r_i) + (1-\kappa)(\phi_0 - \phi_i - r_1 - r_2 - \ldots - r_i) - h\varphi_i \\
v_i &\triangleq \kappa(\varphi_{i-1} - \varphi_i) \qquad + (1-\kappa)(\varphi_0 - \varphi_i)
\end{aligned}
$$

- One obtains a platoon model in **identical form** (i.e. same A, B, H) with

$$d_i \triangleq \kappa\alpha_{i-1} + (1-\kappa)\alpha_0$$

- Synthesis can be performed in the same way as in predecessor following.

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String Stability of Leader and Predecessor Following

- String stability is again linked to the performance objective $\|\alpha_i\|_2 < \sigma\|d_i\|_2$.
- In this case, string stability of α_i can be guaranteed with respect to leader:

$$\sigma \leq 1 \Rightarrow \|\alpha_i\|_2 < \|\alpha_0\|_2, \ \forall i \geq 1.$$

- Spacing errors are now defined with a **smaller headway term** for $i > 2$:

$$\epsilon_i = \phi_{i-1} - \phi_i - r_i - h\varphi_i^{\mathrm{r}} \qquad (\epsilon_i = p_i - p_{i-1} + \kappa\epsilon_{i-1}, \ \epsilon_1 = p_1)$$

The reduced velocity φ_i^{r} (for $i \geq 2$) used in the headway terms is given by

$$\varphi_i^{\mathrm{r}} \triangleq \varphi_i - (1-\kappa)(\varphi_{i-1} + \kappa\varphi_{i-2} + \ldots + \kappa^{i-2}\varphi_1)$$

- By using $\hat{a}_i = \hat{\alpha}_{i-1} - \hat{\alpha}_i = \mathcal{T}(\hat{d}_{i-1} - \hat{d}_i) = \kappa\mathcal{T}\hat{a}_{i-1}$, one can show that

$$\frac{\hat{\varepsilon}_i}{\hat{\varepsilon}_{i-1}} = \kappa\mathcal{T}(s) = \kappa\frac{\hat{\alpha}_i}{\hat{d}_i}$$ (note the **difference from predecessor following**)

- String stability of ϵ_i can always be guaranteed by choosing κ small enough:

$$\|\epsilon_i\|_2 < \kappa\sigma\|\epsilon_{i-1}\|_2 \quad (\kappa \leq 1/\sigma \Rightarrow \|\epsilon_i\|_2 < \|\epsilon_{i-1}\|_2)$$

- Control with small κ would be sensitive to communication errors with leader.

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Improving Robustness against Measurement Noise

- The given LMIs might lead to large gains and hence sensitivity to noise.
- State measurement noise in the problem formulation leads to bilinear matrix inequalities, which need to be relaxed into LMIs.
- Dilated LMI conditions might be preferable for handling state noise.
- Noise sensitivity can be reduced indirectly by enforcing $\lambda_{\max}(FWF^T) < 1$. Bound on the gains can be adjusted by (typically block-diagonal) $W = W^T$.
- Sufficient LMI conditions are derived for this based on

$$FWF^T = N \underbrace{\left(Y^{-1}WY^{-1} - \lambda^{-2}W^{-1}\right)}_{\prec 0} N^T + \lambda^{-2} \underbrace{NW^{-1}N^T}_{\prec \lambda^2 I} \prec I$$

  where λ serves as an artificial variable that facilitates the decoupling.
- The conditions indicated by the braces are expressed equivalently as

$$Y \succ \lambda W \quad \text{and} \quad \begin{bmatrix} \lambda W & N^T \\ N & \lambda I \end{bmatrix} \succ 0$$

  Observe that the second condition is an LMI for fixed W.

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Robust Synthesis for Uncertain Vehicle Models

- Consider a vehicle model with uncertain $\tau \in [\tau_{\min}, \tau_{\max}]$. Platoon model with affine $\delta \triangleq \frac{1}{\tau} \in [\delta_{\min} = \frac{1}{\tau_{\max}}, \delta_{\max} = \frac{1}{\tau_{\min}}]$ dependence is obtained with

$$\underbrace{\left[\begin{array}{ccc|c} 0 & 1 & -h & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & -\frac{1}{\tau} & \frac{1}{\tau} \end{array}\right]}_{\left[\begin{array}{c|c} A(\delta) & B(\delta) \end{array}\right]} = \underbrace{\left[\begin{array}{ccc|c} 0 & 1 & -h & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]}_{\left[\begin{array}{c|c} A_0 & B_0 \end{array}\right]} + \underbrace{\left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{array}\right]}_{\left[\begin{array}{c|c} A_1 & B_1 \end{array}\right]} \delta$$

- Parameter-independent Y, N and G lead to LMIs with **affine** δ dependence:

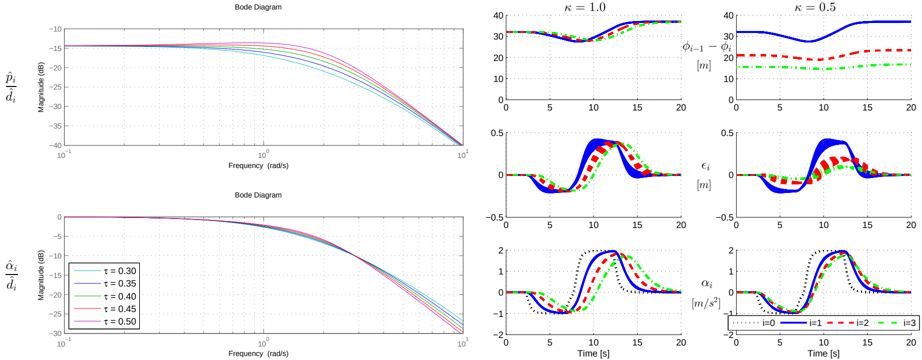
$$\mathcal{N}_0(Y,N,G) + \mathcal{N}_1(Y,N,G)\delta \prec 0, \ \forall \delta \in [\delta_{\min}, \delta_{\max}]$$

- These infinitely-many LMIs are satisfied **if and only if**

$$\mathcal{N}_0(Y,N,G) + \mathcal{N}_1(Y,N,G)\delta_{\min} \prec 0 \text{ and } \mathcal{N}_0(Y,N,G) + \mathcal{N}_1(Y,N,G)\delta_{\max} \prec 0$$

- A design based on these LMI conditions will ensure stability and robustness against arbitrary variations in $\delta = 1/\tau$ over time.
- If $|\dot{\delta}| \leq \nu_{\max}$, one can use dilated LMIs with some **δ-dependent variables**.
- Finitely many (**sufficient**) LMIs are then obtained via **relaxations**.

Example $\mathcal{H}_\infty$ State Feedback Synthesis



- Design parameters: $\tau_{\min} = 0.3$, $\tau_{\max} = 0.5$, $h = 1.1$, $c = 0.2$, $W = 0.1I$
- Optimization results: $\sigma = 1.01$, $\gamma = 0.48$ [with $\psi = 0.6$]
- Feedback Gain: $F = \begin{bmatrix} 1.6645 & 1.5061 & -0.9745 \end{bmatrix}$
- Simulation scenario: $\varphi_i(0) = 20\,\mathrm{m/s}$, $r_i = 10\,\mathrm{m}$, zero initial spacing errors.

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Concluding Remarks

- Platoon control is a good application of LMI-based multi-objective synthesis.
- Synthesis based on parameter-dependent vehicle models can also be used for the control of heterogenous platoons.
- String stability of the spacing errors is ensured by the given LMI condition only when uncertain parameters are constant and common for all vehicles. Vehicle-specific parameter variations lead to additional disturbance effects.
- When only some states are available, one can use dilated LMI conditions for static output feedback. It is possible to perform an LMI-based synthesis for dynamic output feedback as well. Nevertheless, synthesis for uncertain systems will then be more challenging.
- It is possible to implement the leader and predecessor following scheme with a predecessor weight that changes smoothly over time.
- **Reference:** Hakan Köroğlu and Paolo Falcone, "Robust Static Output Feedback Synthesis for Platoons under Leader and Predecessor Feedback", submitted to *IEEE Transactions on Control Systems Technology* for review.