Distributed protocol for platooning in the presence of time-varying heterogeneous delays

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Lecture content

1. Modeling and notation
2. Control policy and closed-loop dynamics
3. Convergence analysis
4. Switching communication topology
5. Numerical results
6. Experimental results

Note: The following notes have been extracted from "Design, analysis and experimental validation of a distributed protocol for platooning in the presence of time-varying heterogeneous delays", Mario di Bernardo, Paolo Falcone, Alessandro Salvi and Stefania Santini. *IEEE Transactions on Control Systems Technology.* To Appear

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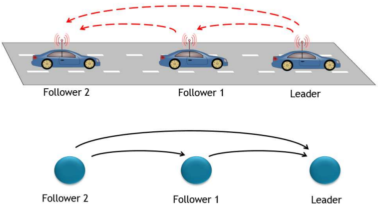
Modeling and notation

Consider the platoon in the figure. Model the i-th vehicle as $(i = 1, \ldots, N)$:

$$\dot{r}_i(t) = v_i(t),$$
$$\dot{v}_i(t) = \frac{1}{M_i} u_i(t).$$



Assumption. Constant velocity for the leader $\dot{r}_0(t) = v_0,\ \dot{v}_0 = 0.$

Problem formulation. Find the control policies $u_i(t),\ i = 1, \ldots, N$ such that the following consensus positions and velocities are reached and maintained.

$$r_i(t) \to \frac{1}{d_i} \left\{ \sum_{j=0}^{N} a_{ij} \cdot (r_j(t) + d_{ij}) \right\} \qquad \text{with } d_i = \sum_{j=0}^{N} a_{ij}$$
$$v_i(t) \to v_0.$$

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Control policy and closed-loop dynamics

Consider the following distributed, state feedback control policy

$$u_i = -\, b\left[v_i\left(t\right) - v_0\right] + \frac{1}{d_i}\sum_{j=0}^{N} k_{ij}a_{ij}\left[h_{ij}v_0 + d_{ij}^{st}\right]$$
$$-\frac{1}{d_i}\sum_{j=0}^{N} k_{ij}a_{ij}\left[r_i\left(t\right) - r_j\left(t-\tau_{ij}\left(t\right)\right) - \tau_{ij}\left(t\right)v_0\right],$$

where,

- k_{ij} and b are control gains,
- $\tau_{ij}(t)$ and $\tau_{i0}(t)$ are the *time-varying* communication delays when information is transmitted to vehicle i from its neighbor j and leader, respectively.

Assumption. The delay $\tau_{ij}(t)$ is assumed to be bounded and measurable.

Control policy and closed-loop dynamics

Define the following position and velocity errors with respect to the reference signals $r_0(t), v_0$ $(i = 1, \ldots, N)$

$$\bar{r}_i(t) = (r_i(t) - r_0(t) - h_{i0}v_0 - d_{i0}^{st}), \qquad \bar{r}(t) = \left[\bar{r}_1(t), \ldots, \bar{r}_i(t) \ldots, \bar{r}_N(t)\right]^\top,$$
$$\bar{v}_i(t) = (v_i(t) - v_0). \qquad \bar{v}(t) = \left[\bar{v}_1(t), \ldots, \bar{v}_i(t) \ldots, \bar{v}_N(t)\right]^\top.$$

Rewrite the control policy as

$$\begin{aligned} u_i\left(t\right) = & -b\bar{v}_i - \tfrac{1}{d_i}k_{i0}a_{i0}\bar{r}_i(t) + \\ & -\tfrac{1}{d_i}\sum_{j=1}^{N} k_{ij}a_{ij}\left[\bar{r}_i\left(t\right) - \bar{r}_j\left(t - \tau_{ij}\left(t\right)\right)\right]. \end{aligned}$$

For the i-th vehicle, the resulting closed-loop dynamics are

$$\begin{cases} \dot{\bar{r}}_i(t) = & \bar{v}_i(t), \\ M_i\dot{\bar{v}}_i(t) = & -\tfrac{1}{d_i}(k_{i0}a_{i0} + \sum_{j=1}^{N} k_{ij}a_{ij})\bar{r}_i(t) - b\bar{v}_i\left(t\right) \\ & +\tfrac{1}{d_i}\sum_{j=1}^{N} k_{ij}a_{ij}\left[\bar{r}_j\left(t - \tau_{ij}\left(t\right)\right)\right]. \end{cases}$$

Control policy and closed-loop dynamics

Define state vector as $\bar{x}(t) = \left[\begin{array}{cc} \bar{r}^\top(t) & \bar{v}^\top(t) \end{array} \right]^\top$ and $\tau_p(t)$, $p = 1, 2, ..., m$, with $m \leq N(N-1)$ as an element of the sequence of time-delays $\{\tau_{ij}(t) : i, j = 1, 2, ..., N, i \neq j)\}$

The closed loop dynamics of the whole vehicular network can be written as

$$\dot{\bar{x}}(t) = A_0 \bar{x}(t) + \sum_{p=1}^{m} A_p \bar{x}(t - \tau_p(t)),$$

where

$$A_0 = \left[\begin{array}{cc} 0_{N\times N} & I_{N\times N} \\ -M\widetilde{K} & -M\widetilde{B} \end{array} \right], \quad A_p = \left[\begin{array}{cc} 0_{N\times N} & 0_{N\times N} \\ M\widetilde{K}_p & 0_{N\times N} \end{array} \right],$$

$$M = \mathrm{diag}\left\{ \tfrac{1}{M_1}, \ldots, \tfrac{1}{M_N} \right\} \in \mathbb{R}^{N\times N}, \;\; \widetilde{B} = \mathrm{diag}\{b, \ldots, b\} \in \mathbb{R}^{N\times N},$$

$$\widetilde{K} = \mathrm{diag}\left\{ \tilde{k}_{11}, \ldots, \tilde{k}_{NN} \right\} \in \mathbb{R}^{N\times N}, \;\; \tilde{k}_{ii} = \frac{1}{d_i} \sum_{j=0}^{N} k_{ij} a_{ij}$$

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Convergence analysis

Let's start from the closed loop dynamics of the network

$$\dot{\bar{x}}(t) = A_0 \bar{x}(t) + \sum_{p=1}^{m} A_p \bar{x}(t - \tau_p(t)).$$

From the Leibniz-Newton formula

$$\bar{x}(t - \tau_p(t)) = \bar{x}(t) - \int_{-\tau_p(t)}^{0} \dot{\bar{x}}(t+s)\, ds.$$

Hence, the closed-loop dynamics can be written as

$$\begin{aligned} \dot{\bar{x}}(t) = A_0 \bar{x}(t) + \sum_{p=1}^{m} A_p \bar{x}(t) + \\ - \sum_{p=1}^{m} \sum_{q=0}^{m} A_p A_q \int_{-\tau_p(t)}^{0} \bar{x}(t+s-\tau_q(t+s))\, ds. \end{aligned}$$

Convergence analysis

More compactly,

$$\dot{\bar{x}}(t) = F\bar{x}(t) - \sum_{p=1}^{m} C_p \int_{-\tau_p(t)}^{0} \bar{x}(t+s)\, ds$$

where

$$C_p = A_p A_0 = \begin{bmatrix} 0_{N\times N} & 0_{N\times N} \\ 0_{N\times N} & M\widetilde{K}_p \end{bmatrix},$$

and

$$F = A_0 + \sum_{p=1}^{m} A_p = \begin{bmatrix} 0_{N\times N} & I_{N\times N} \\ -M\widehat{K} & -M\widetilde{B} \end{bmatrix},$$

with

$$\widehat{K} = -\sum_{p=1}^{m} \widetilde{K}_p + \widetilde{K}.$$

Convergence analysis

Recall the expression of

$$\widehat{K} = -\sum_{p=1}^{m} \widetilde{K}_p + \widetilde{K}.$$

and the control policy

$$u_i(t) = -b\bar{v}_i - \boxed{\frac{1}{d_i} k_{i0} a_{i0}} \bar{r}_i(t) + \\ -\frac{1}{d_i} \sum_{j=1}^{N} k_{ij} a_{ij} \left[\bar{r}_i(t) - \bar{r}_j(t - \tau_{ij}(t))\right].$$

Lemma

Supposing $k_i = \frac{k_{i0} a_{i0}}{d_i} \geq 0$ $(i = 1, \ldots, N)$, the matrix $\widehat{K}$ is positive stable if and only if node 0 is globally reachable in $\overline{\mathcal{G}}$.

What does this mean? node 0 is *globally reachable* in $\overline{\mathcal{G}}$ if there is a path in $\overline{\mathcal{G}}$ from every node i in $\mathcal{G}$ to node 0

Convergence analysis

Lemma

The matrix F in

$$F = A_0 + \sum_{p=1}^{m} A_p = \begin{bmatrix} 0_{N\times N} & I_{N\times N} \\ -M\widehat{K} & -M\widetilde{B} \end{bmatrix},$$

is Hurwitz stable if and only if $\widehat{K}_M = M\widehat{K}$ is positive stable and

$$b > \max_i \left\{ \frac{|Im(\mu_i)|}{\sqrt{Re(\mu_i)}} M_i \right\}$$

where μ_i is the i-th eigenvalue of $\widehat{K}_M$ $(i = 1, \ldots, N)$.

Convergence analysis

Theorem

The closed-loop vehicular network with time-varying delays

$$\dot{\bar{x}}\left(t\right) = A_0 \bar{x}\left(t\right) + \sum_{p=1}^{m} A_p \bar{x}\left(t - \tau_p\left(t\right)\right),$$

with the $k_{ij} > 0$ and b in

$$u_i\left(t\right) = -b\bar{v}_i - \frac{1}{d_i} k_{i0} a_{i0} \bar{r}_i(t) - \frac{1}{d_i} \sum_{j=1}^{N} k_{ij} a_{ij} \left[\bar{r}_i\left(t\right) - \bar{r}_j\left(t - \tau_{ij}\left(t\right)\right)\right].$$

such that $b > b^\star = \max_i \left\{ \frac{|Im(\mu_i)|}{\sqrt{Re(\mu_i)}} M_i \right\}$. Then, if and only if node 0 is globally reachable in $\overline{\mathcal{G}}$, there exists a constant $\tau^\star > 0$ such that, when $0 \leq \tau_p(t) \leq \tau < \tau^\star$ $(p = 1, \ldots, m)$,

$$\lim_{t \to \infty} \overline{x}(t) = 0.$$

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Switching communication topology

Introduce a switching signal

$$\sigma(t) : [0, \infty) \to \phi_\Gamma = \{1, 2, \ldots, G\},$$

with $\Gamma = \{\overline{\mathcal{G}}_1, \overline{\mathcal{G}}_2, \ldots, \overline{\mathcal{G}}_G\}$ a finite collection of graphs with a common node set $\overline{\mathcal{V}}$

Rewrite the closed-loop vehicular network as a switched time-delayed system:

$$\dot{\bar{x}}(t) = A_{0,\sigma}\bar{x}(t) + \sum_{p=1}^{m} A_{p,\sigma}\bar{x}(t - \tau_p(t)),$$

where

$$A_{0,\sigma} = \begin{bmatrix} 0_{N\times N} & I_{N\times N} \\ -M\widetilde{K}_\sigma & -M\widetilde{B} \end{bmatrix}, \; A_{p,\sigma} = \begin{bmatrix} 0_{N\times N} & 0_{N\times N} \\ M\widetilde{K}_{p,\sigma} & 0_{N\times N} \end{bmatrix}.$$

Switching communication topology

Lemma

Let $\mathcal{I} = \{i | k_{i0}a_{i0} > 0, i \in \mathcal{V}_N\}$. Assume that

$$\sum_{j=1}^{N} k_{ij}a_{ij} \geq \sum_{j=1}^{N} k_{ji}a_{ji}, \quad i \notin \mathcal{I}, i \in \mathcal{V}_N;$$

$$2k_{i0}a_{i0} + \sum_{j=1}^{N} k_{ij}a_{ij} > \sum_{j=1}^{N} k_{ji}a_{ji}, \quad i \in \mathcal{I};$$

and that vertex 0 is globally reachable in $\overline{\mathcal{G}}_{\sigma}$.
Then $H_\sigma + H_\sigma^\top = (\overline{h}_{ij})_{N\times N}$ with $H_\sigma = M\widehat{K}_\sigma$ is positive definite.

Switching communication topology

Theorem

Assume that $\overline{\mathcal{G}}_\sigma \in \Gamma$ fulfills the hypotheses of the previous Lemma. Consider the closed-loop system

$$\dot{\bar{x}}\,(t) = A_{0,\sigma}\bar{x}\,(t) + \sum_{p=1}^{m} A_{p,\sigma}\bar{x}\,(t-\tau_p\,(t))\,,$$

formed with

$$u_i\,(t) = -b\bar{v}_i - \frac{1}{d_i}k_{i0}a_{i0}\bar{r}_i(t) - \frac{1}{d_i}\sum_{j=1}^{N} k_{ij}a_{ij}\,[\bar{r}_i\,(t) - \bar{r}_j\,(t-\tau_{ij}\,(t))],$$

with $k_{ij} > 0$ and b such that $b > b_1^\star = \left\{\frac{\widehat{\mu}}{2\widehat{\lambda}} + 1\right\} M_i$, with $\widehat{\mu} = \max_\sigma\{\lambda_{\max}(H_\sigma H_\sigma^\top)\}$ and $\widehat{\lambda} = \min_\sigma\{\lambda_{\min}(H_\sigma + H_\sigma^\top)\}$.
Then, there exists a constant $\tau_1^\star > 0$ such that, when $0 \le \tau_p(t) \le \tau < \tau_1^\star$ $(p = 1, \ldots, m)$, the origin is globally asymptotically stable.

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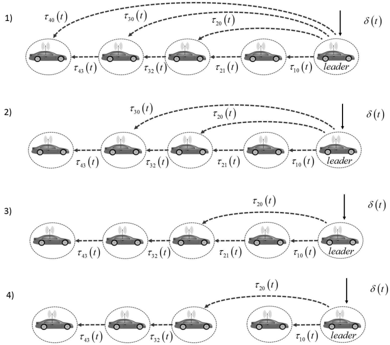
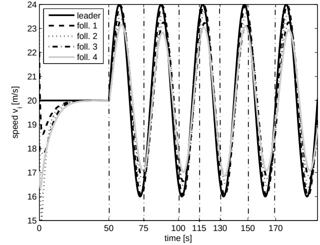
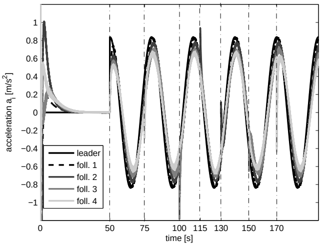
Numerical results







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