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DI INGEGNERIA
DELL'INFORMAZIONE

UNIVERSITÀ
DEGLI STUDI
DI PADOVA

A 93dB 1Ms/s Sigma-Delta ADC dissipating 8.7fJ per conversion step

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SSIE 2016, Bressanone (BZ)

Topics of Discussion

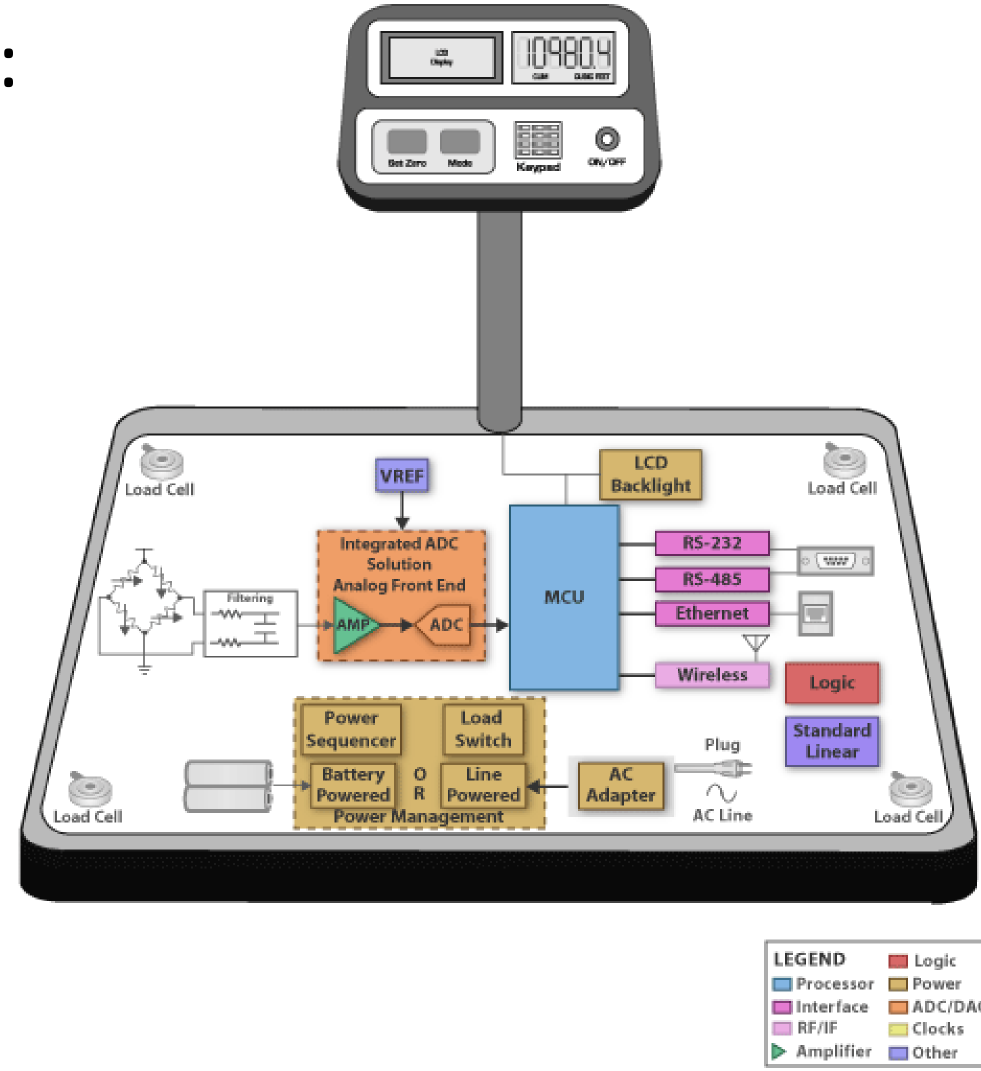
- Analog to Digital Conversion
- Sigma-Delta Modulation
- Case Study: Design and Simulations
- Measurement Issues

Analog to Digital Conversion

- Conversion of Signal from the Analog to the Digital Domain.
- Digital signals are transmitted in a more efficient way than analog signals.
- Applications:
  - Computers, Engineering Instrumentations (Oscilloscope...), Biomedical Instrumentations (Radar Imaging...), Music Reproduction...

A/D Conversion: Example

- Weigh Scale Example:
  - Capacity: 2kg
  - Sensitivity: 0.1g
  - Max V_{EXC} = 10V
  - Sensitivity = 2mV/V



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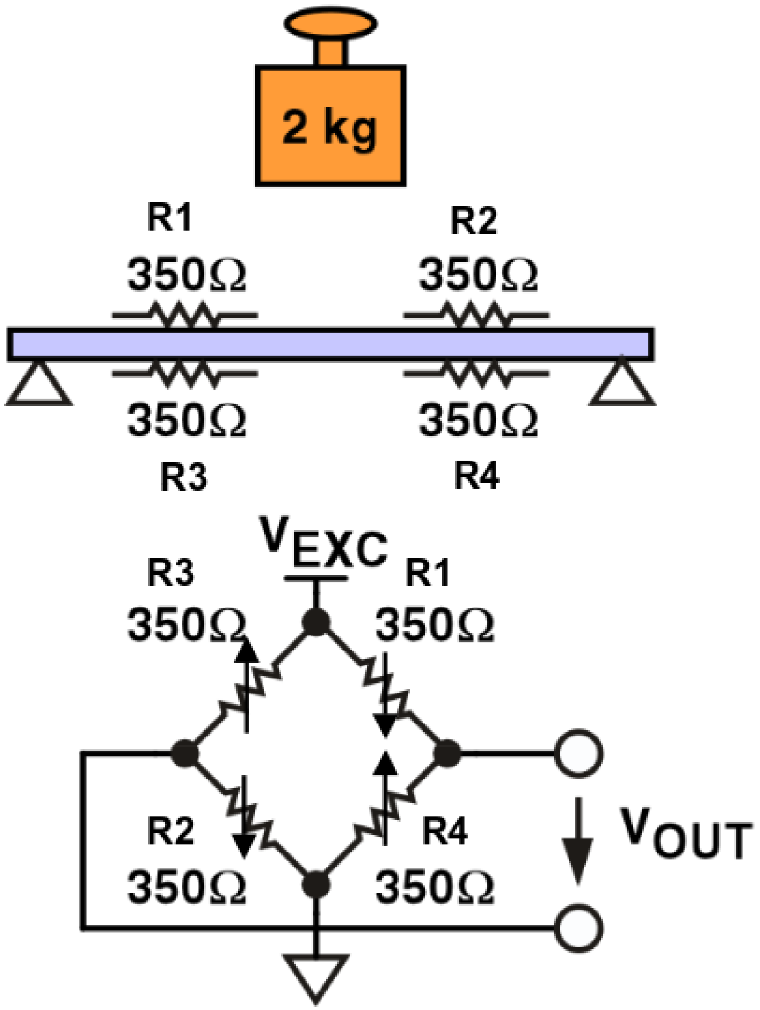
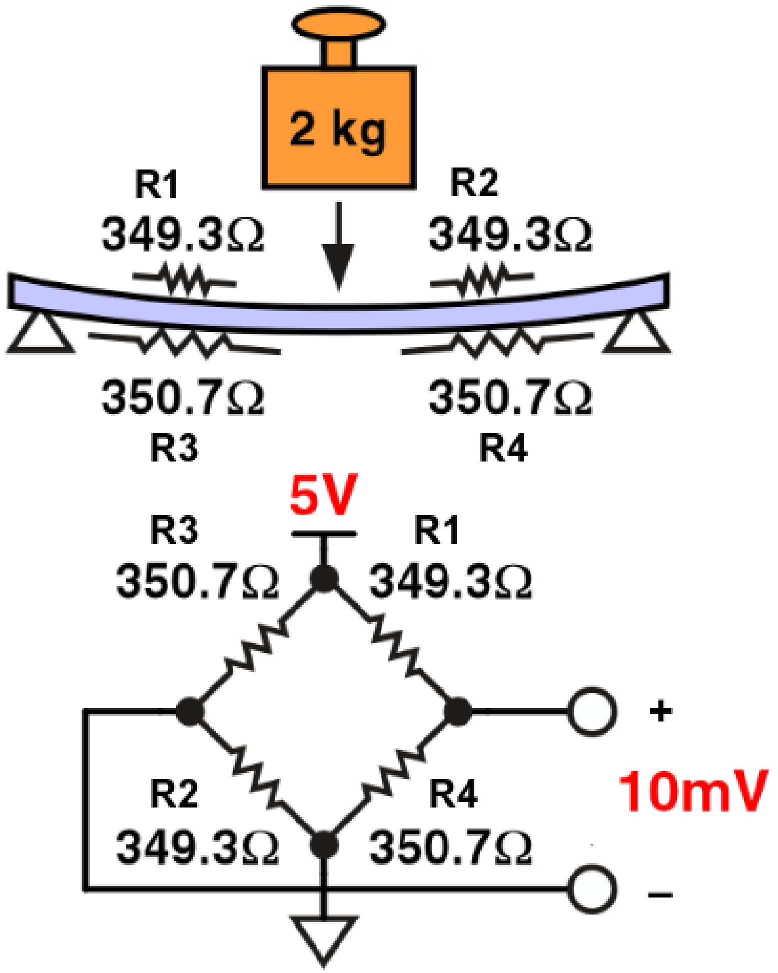
Wheat Stone Bridge

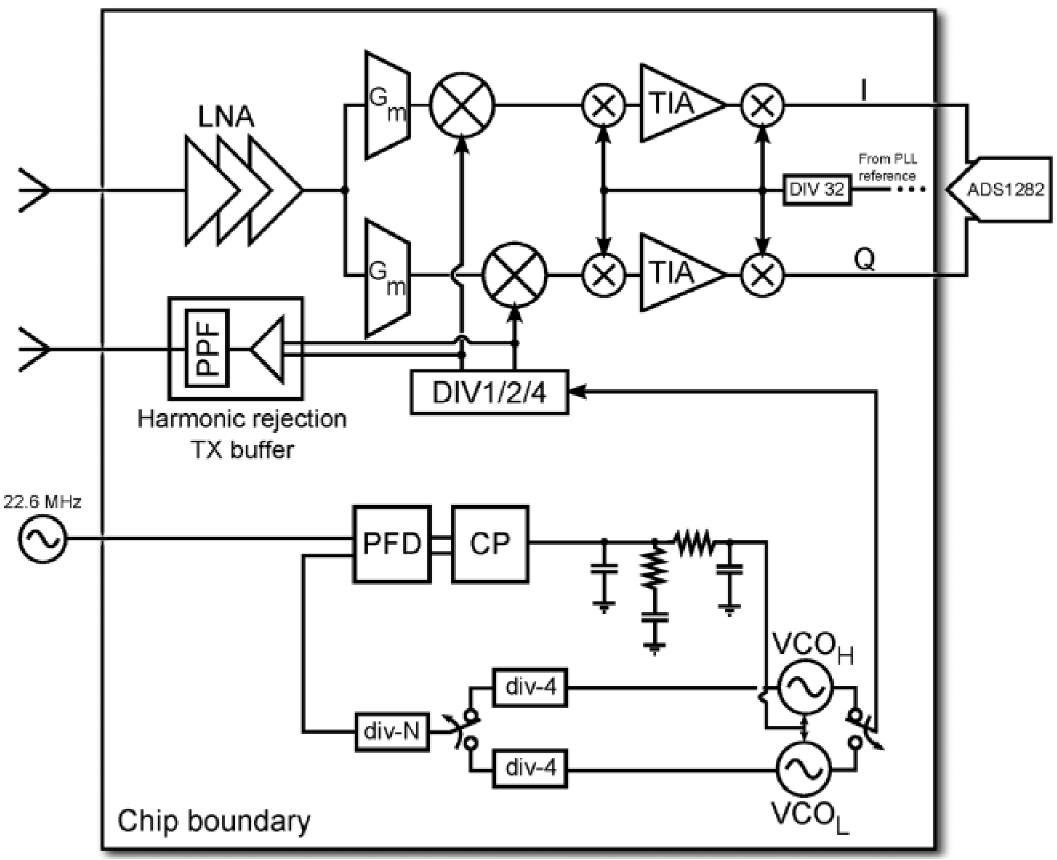




ADC Requirements

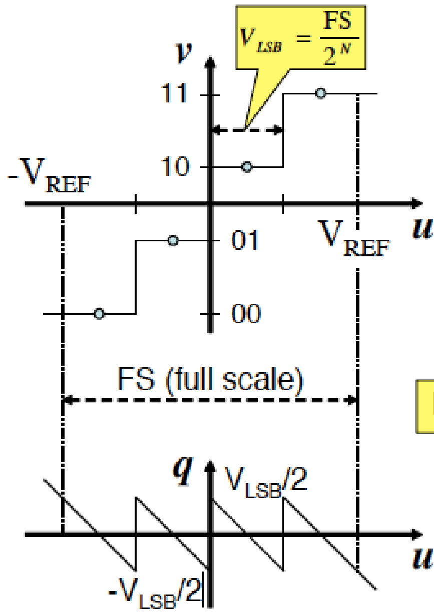
- Required 0.1g in 2kg:
  - \# counts = full-scale / resolution
  - \# counts = 2000 g / 0.1 g = 20.000
- 20.000 counts:
  - V_{FS} = 10 mV @ 5V excitation
  - $V_{PP} = V_{FS}$ / # counts = 0.5µV
- Noise-Free Code Resolution
  - $\log_{10}(V_{FS}/V_{PP}) / \log_{10}(2) = 14.3$ bits
- RMS Noise (Gaussian Noise)
  - $V_{RMS} = V_{PP}/6.6 = 75\text{nV}$ (6.6 used to convert peak-to-peak noise to rms noise for a confidence level of 99.9%)
- Noise Code Resolution
  - $\log_{10}(V_{FS}/V_{RMS}) / \log_{10}(2) = 17$ bits

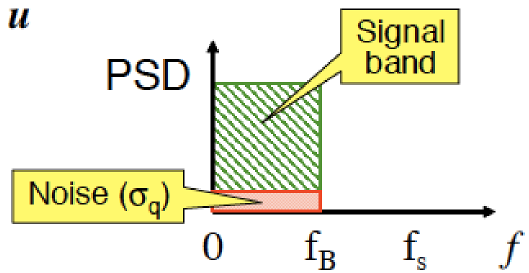
Breast Cancer Radar Imaging



- 10-30% of tumors are missed by mammography.
- UWB microwave radar technology is an attractive alternative.
- In order to recreate an high resolution map of the breast tumors an high resolution ADC is necessary.

Analog to Digital Conversion





$$SNR = \frac{P_{sig}}{\sigma_q^2}$$

- ADC: Sampling and Quantizing. In the quantization process we introduce an error.

$$\sigma_q^2 = \frac{V_{LSB}^2}{12}; \quad P_{sig} = \left(\frac{V_{LSB} \cdot 2^N}{2\sqrt{2}}\right)^2 \rightarrow SNR = 6.02 \cdot N + 1.76\text{dB}$$

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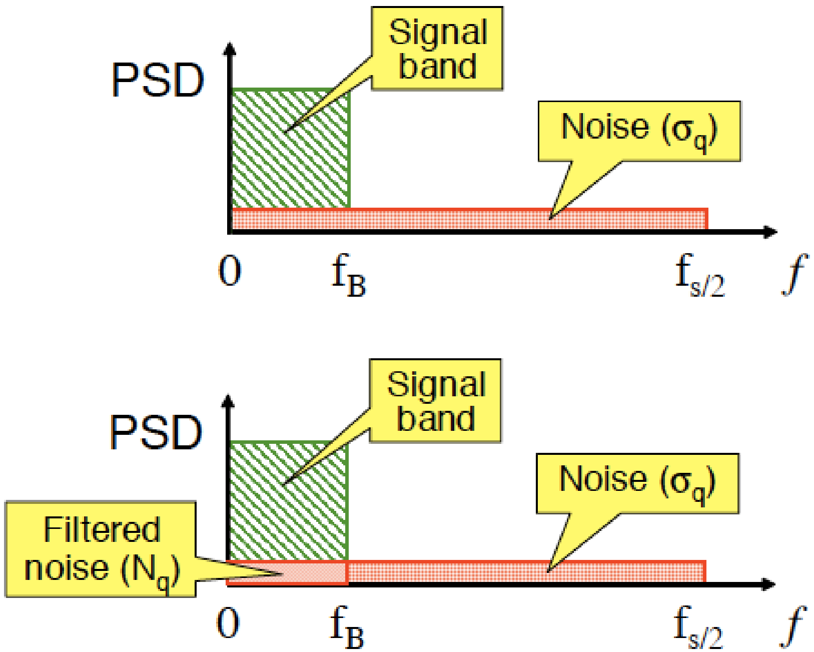
First improvement: Oversampling

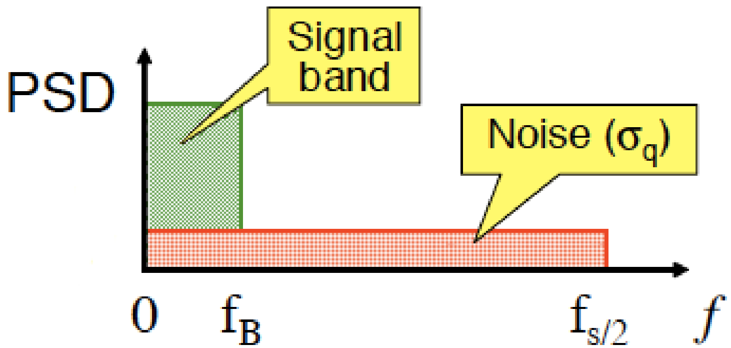


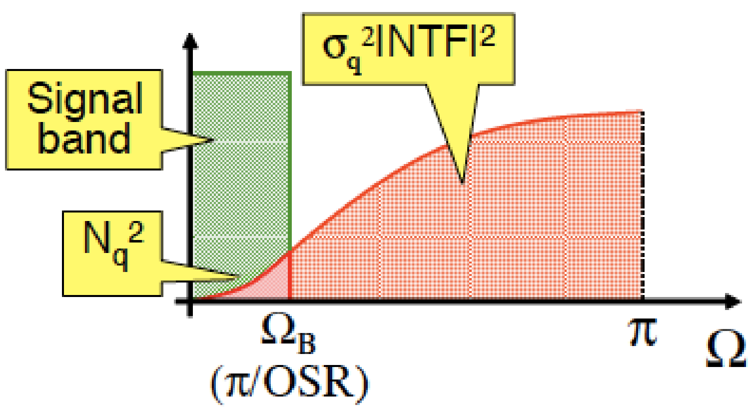
$$OSR = \frac{f_s/2}{f_B}$$

- Sampling frequency significantly higher than the Nyquist rate.

$$N_q^2 = \frac{V_{LSB}^2}{12 \cdot OSR} \rightarrow SNR = 6.02 \cdot N + 1.76\text{dB} + 10 \cdot \log OSR$$

Second Improvement: Noise Shaping (1/2)





- Pushing part of the quantization noise outside the signal bandwidth.
- High-pass filtering the quantization noise (Noise-Transfer-Function)

Second Improvement: Noise Shaping (2/2)

$$NTF(s) = \frac{s}{s+1} \leftrightarrow NTF(z) = (1 - z^{-1})$$

$$|NTF(e^{j\omega t})| = |1 - e^{-j\omega t}| = |e^{-j\omega T/2}(e^{j\omega T/2} - e^{j\omega T/2})|$$
$$= 2|sin(\omega T/2)| = 2|sin(\pi f T)|$$

$$N_q^2 = \int_0^{f_b} \frac{V_{LSB}^2}{12} \frac{2}{f_s} \left[2 \cdot sin\left(\pi \frac{f}{f_s}\right)\right]^2 \cong \int_0^{f_b} \frac{V_{LSB}^2}{12} \frac{2}{f_s} \left[2 \cdot \left(\pi \frac{f}{f_s}\right)\right]^2$$
$$= \frac{V_{LSB}^2}{12} \cdot \frac{\pi^2}{3} \cdot \frac{1}{OSR^3}$$

$$SNR = 6.02 \cdot N + 1.76\text{dB} - 5.2\text{dB} + 30 \log OSR$$

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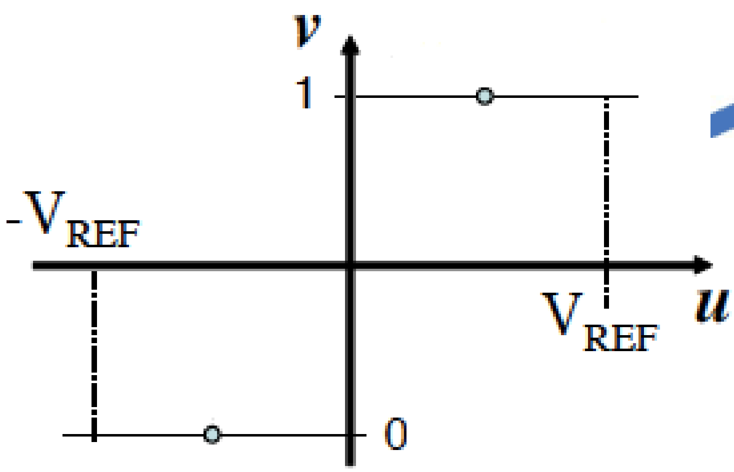
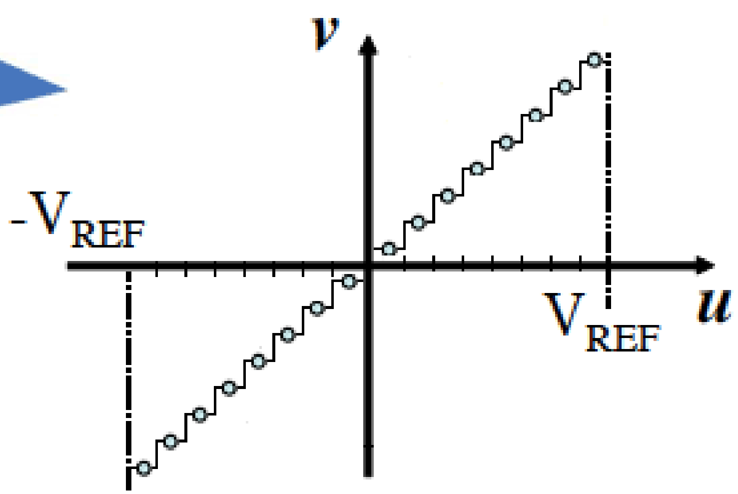
How 1-bit ADC achieves more than a 16-bit resolution

SNR in First-Order Sigma-Delta

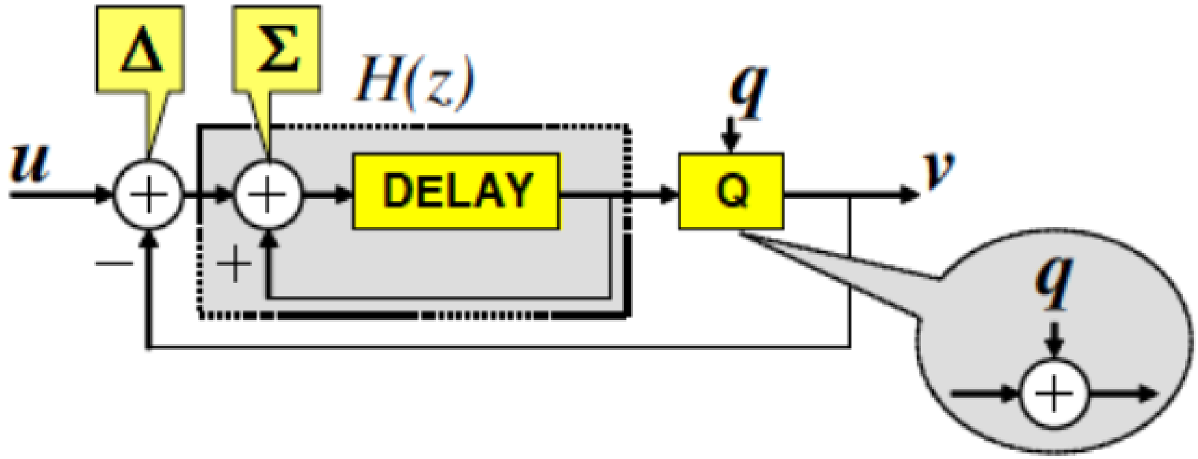
$$SNR = 6.02 \cdot N + 1.76\text{dB} - 5.2\text{dB} + 30 \log OSR$$

- N = 1, OSR = 1024 → SNR = 93 dB (16 bit)





Sigma-Delta Modulation

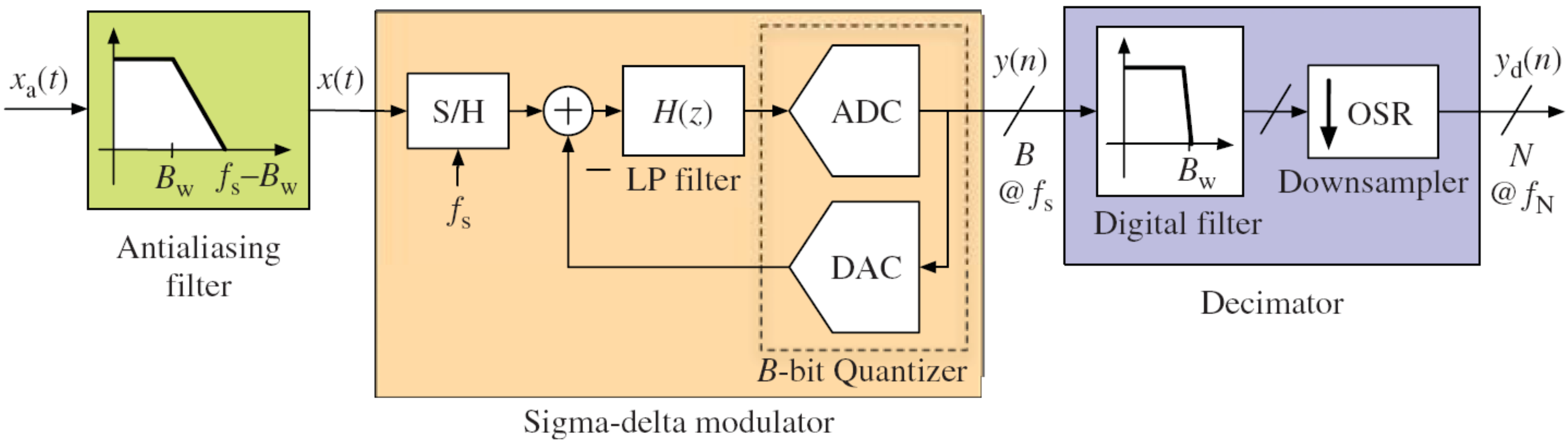


- Modeling the quantization noise as an additive quantity.

Noise Transfer Function (NTF) and Signal Transfer Function (STF)

$$\frac{v}{q} = \frac{1}{1+H(z)} = 1 - z^{-1} \quad \text{and} \quad \frac{v}{u} = \frac{H(z)}{1+H(z)} = z^{-1}$$

Sigma-Delta Architecture



- Digital Filter to remove the noise outside the signal bandwidth.
- DAC in the feedback path to return into analog domain.

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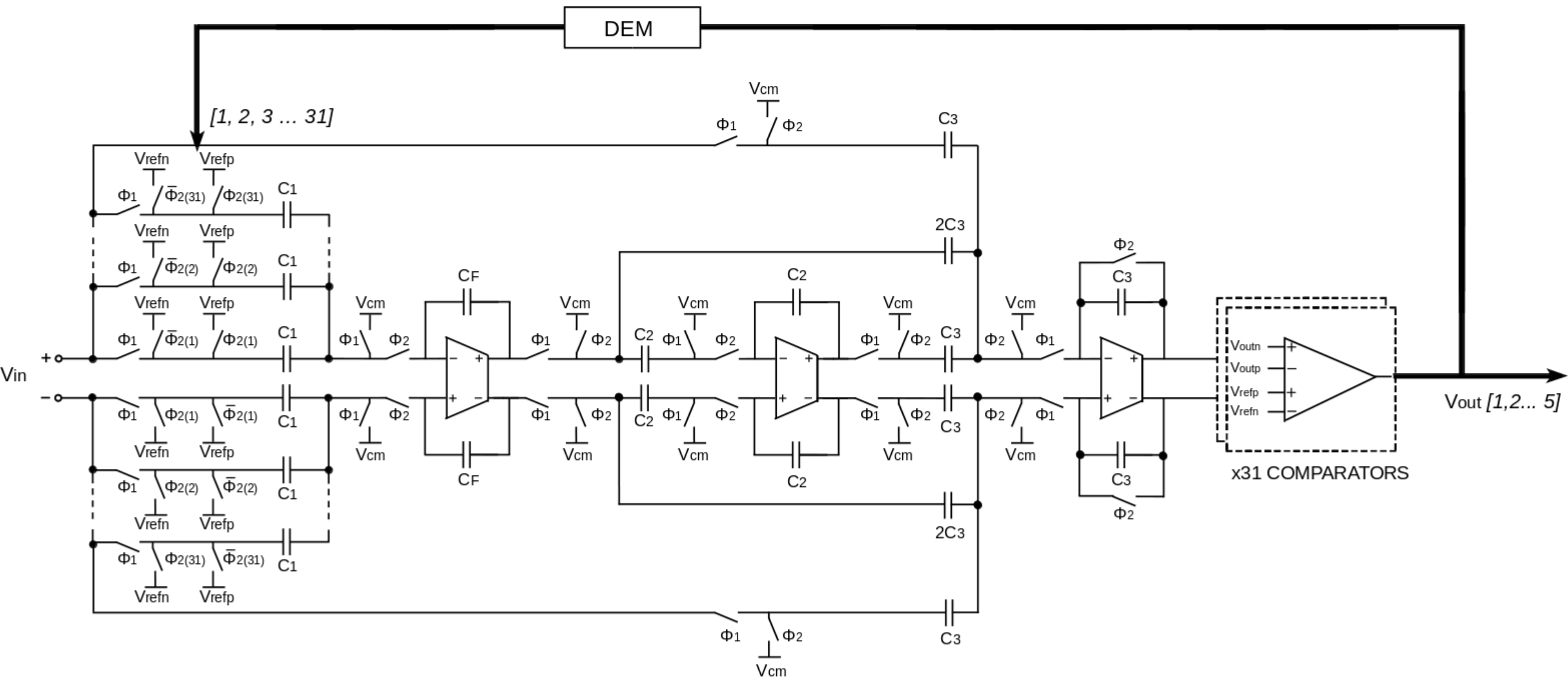
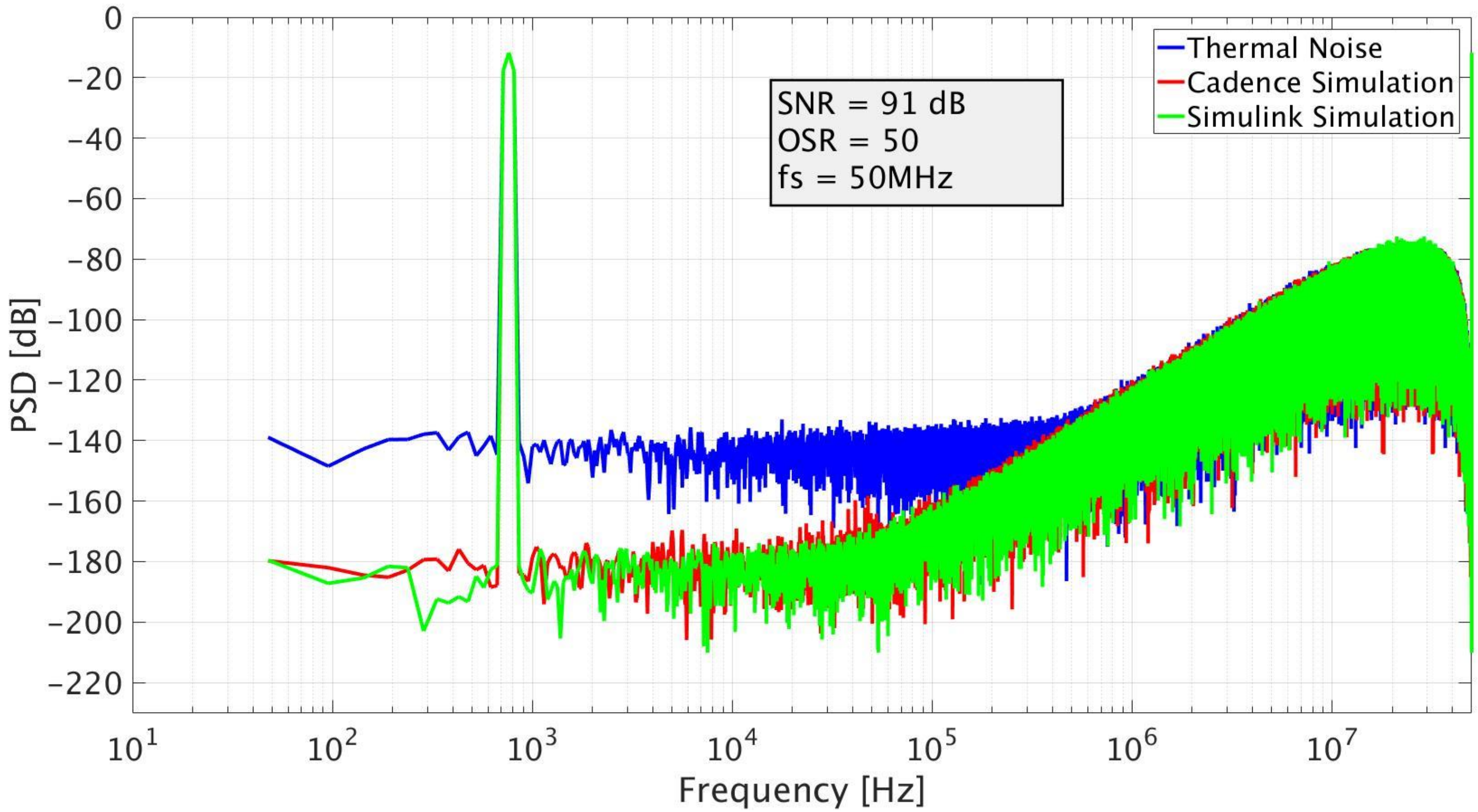
Schematic Second Order Sigma Delta

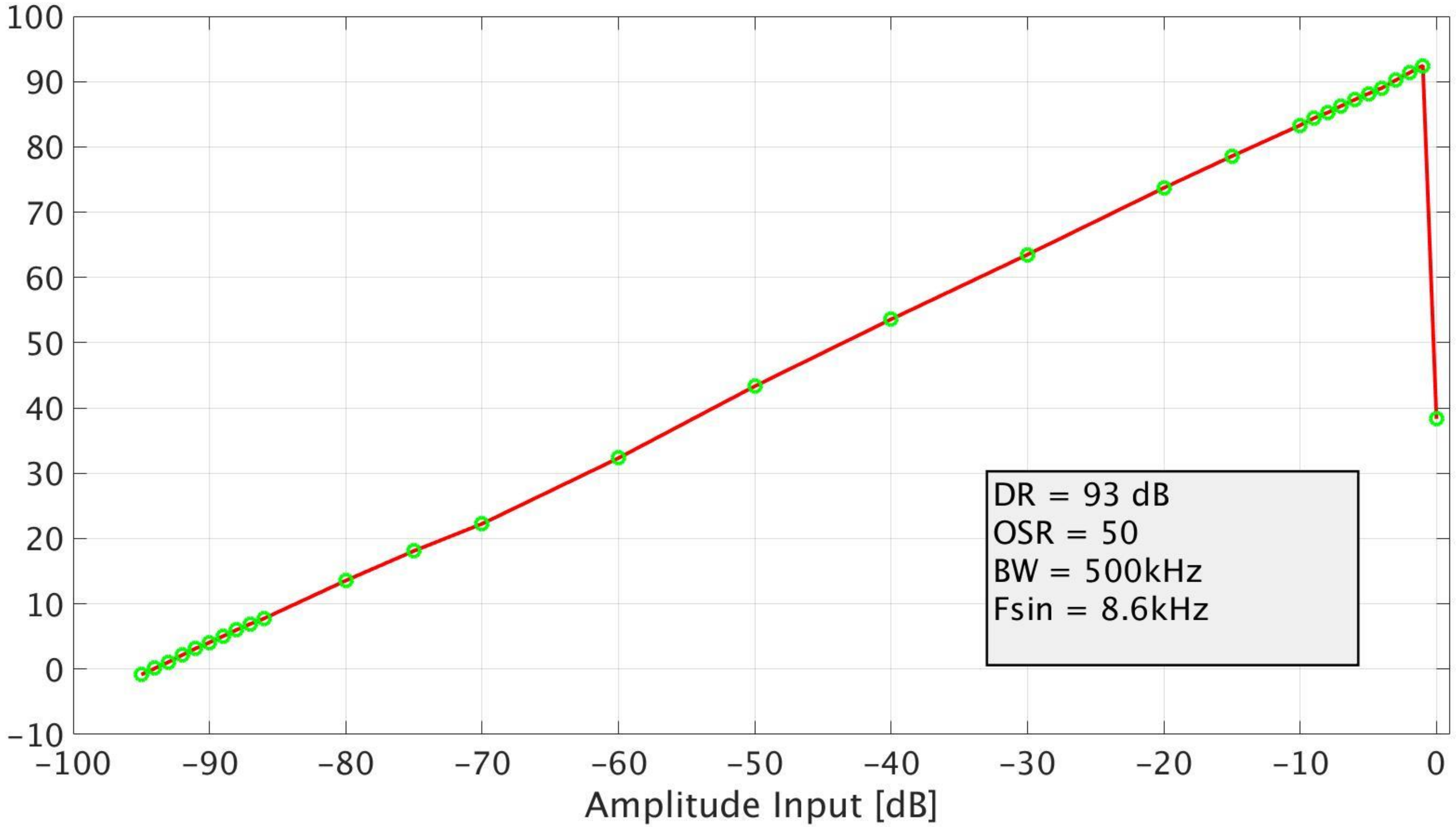


Simulations – output spectrum

- Comparison between the behavioral model and the Cadence simulation.



Simulations – dynamic range



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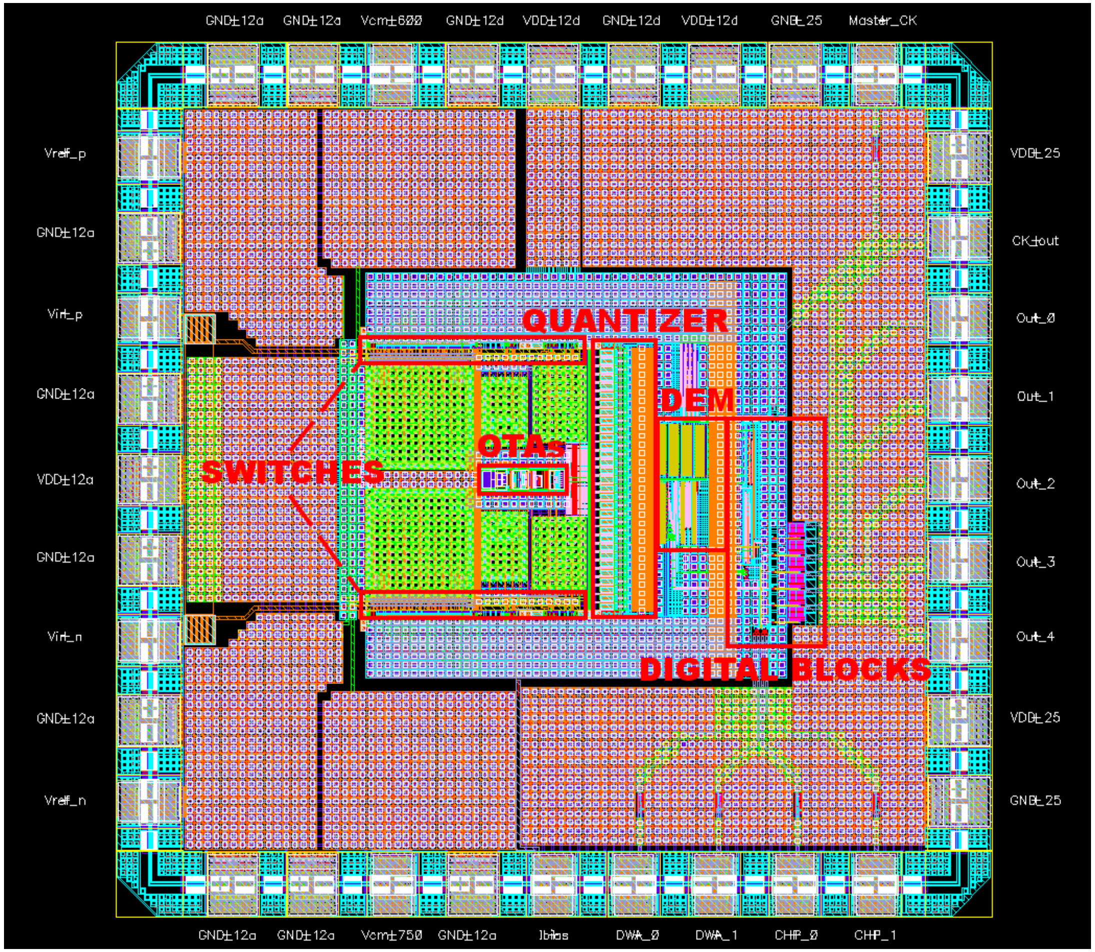
Chip layout



- Core area: 500µm x 480µm
- Area with PAD & decoupling caps: 1mm x 1mm

The chip has been fabricated thanks to ST Microelectronics in Lund (Sweden) and in particular thanks to prof. Pietro Andreani.

ADC Testing: Measurement Issues

- In order to test the linearity of the ADC a "pure" sinewave has to be used.
- Usually the instruments does not guarantee more than 12 bits of linearity.
- We have to create a low distortion and low noise oscillation to measure the effective resolution of the ADC.

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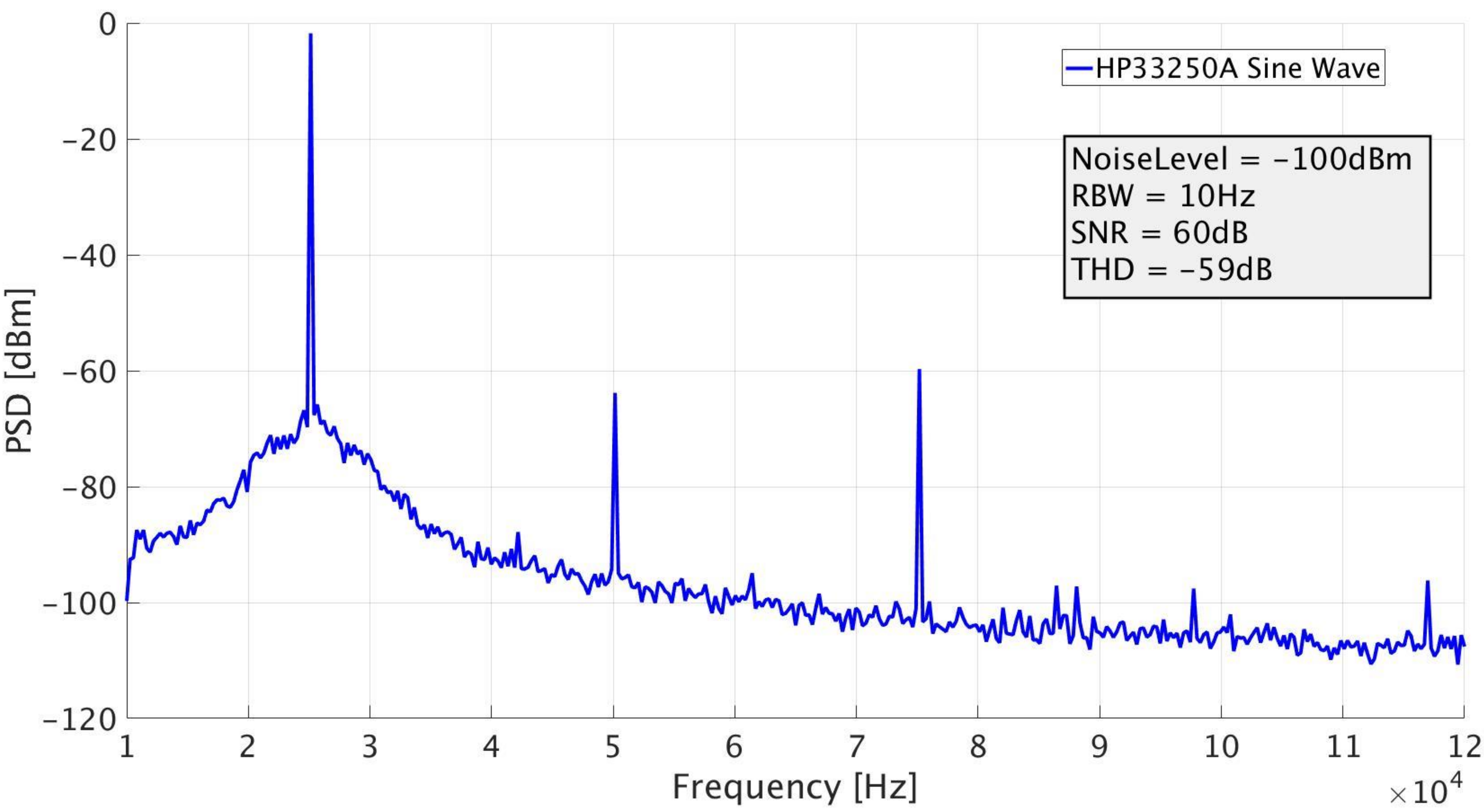
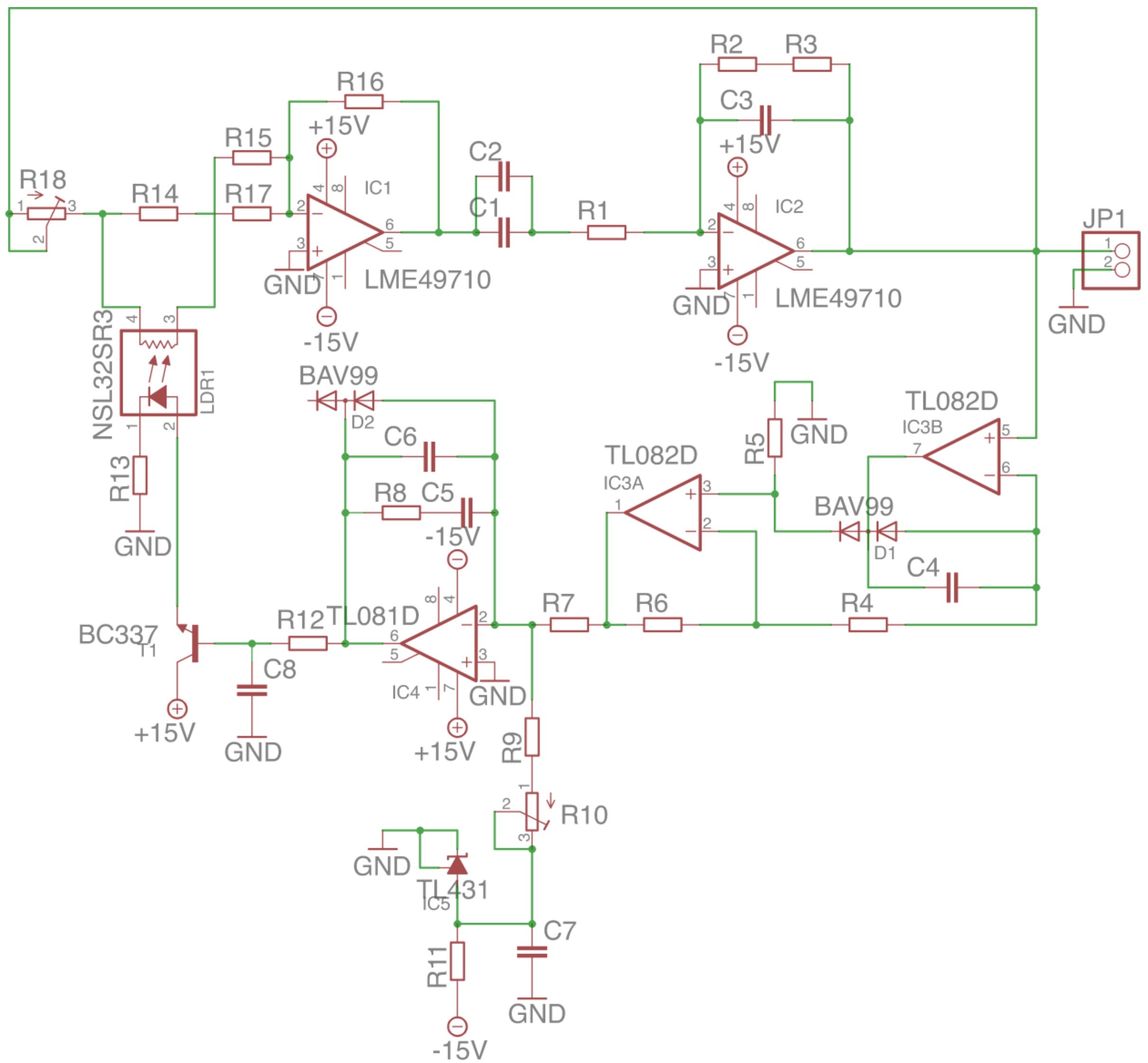
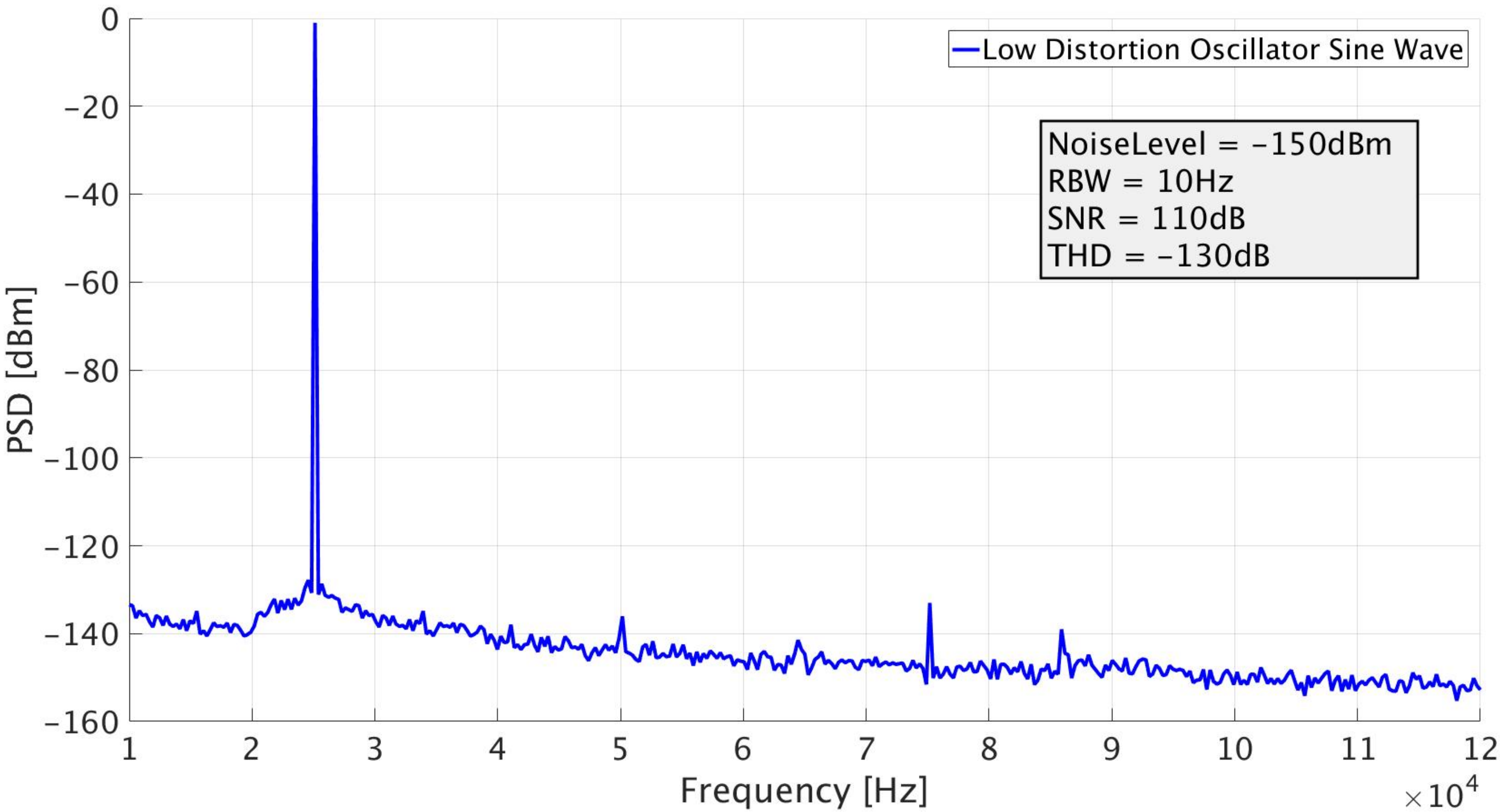
HP33250A Signal Generator Spectrum



Variable Frequency Oscillator



Measured Output Spectrum



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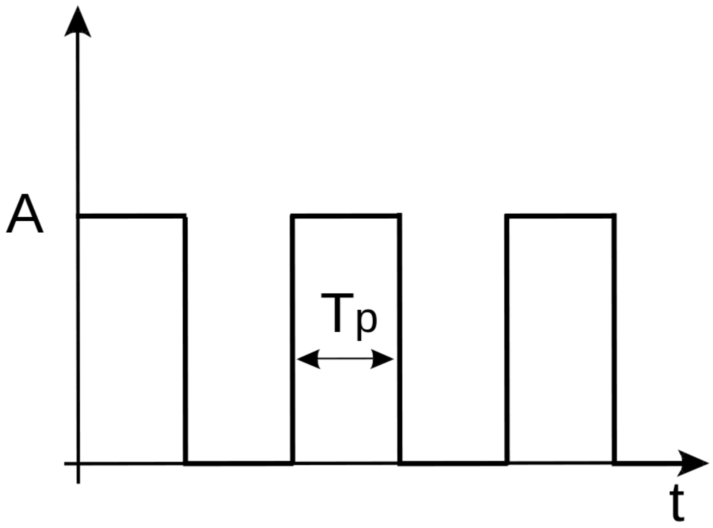
Harmonic Content of Square Wave



$$x(t) = \begin{cases} A & \text{if } |t| < Tp/2 \\ 0 & \text{if } |t| > Tp/2 \end{cases}$$

$$x(t) = \sum_{n=0}^{n=\infty} a_n \cos(n\omega_0 t)$$

$$a_n = \frac{2}{T}\int_T x(t)\cos(n\omega_0 t) = 2\frac{A}{n\pi}\sin(\frac{n\pi}{2})$$

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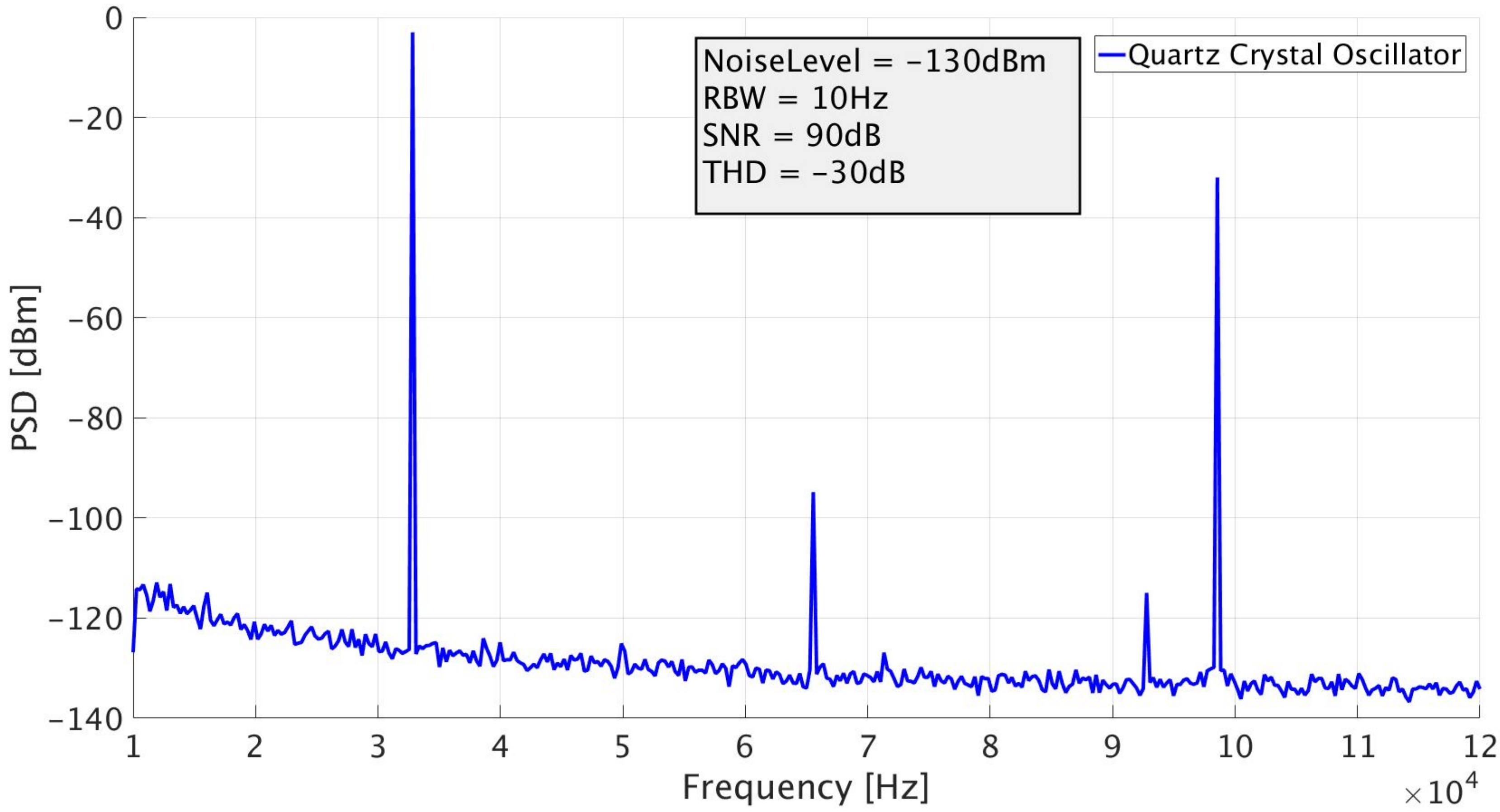
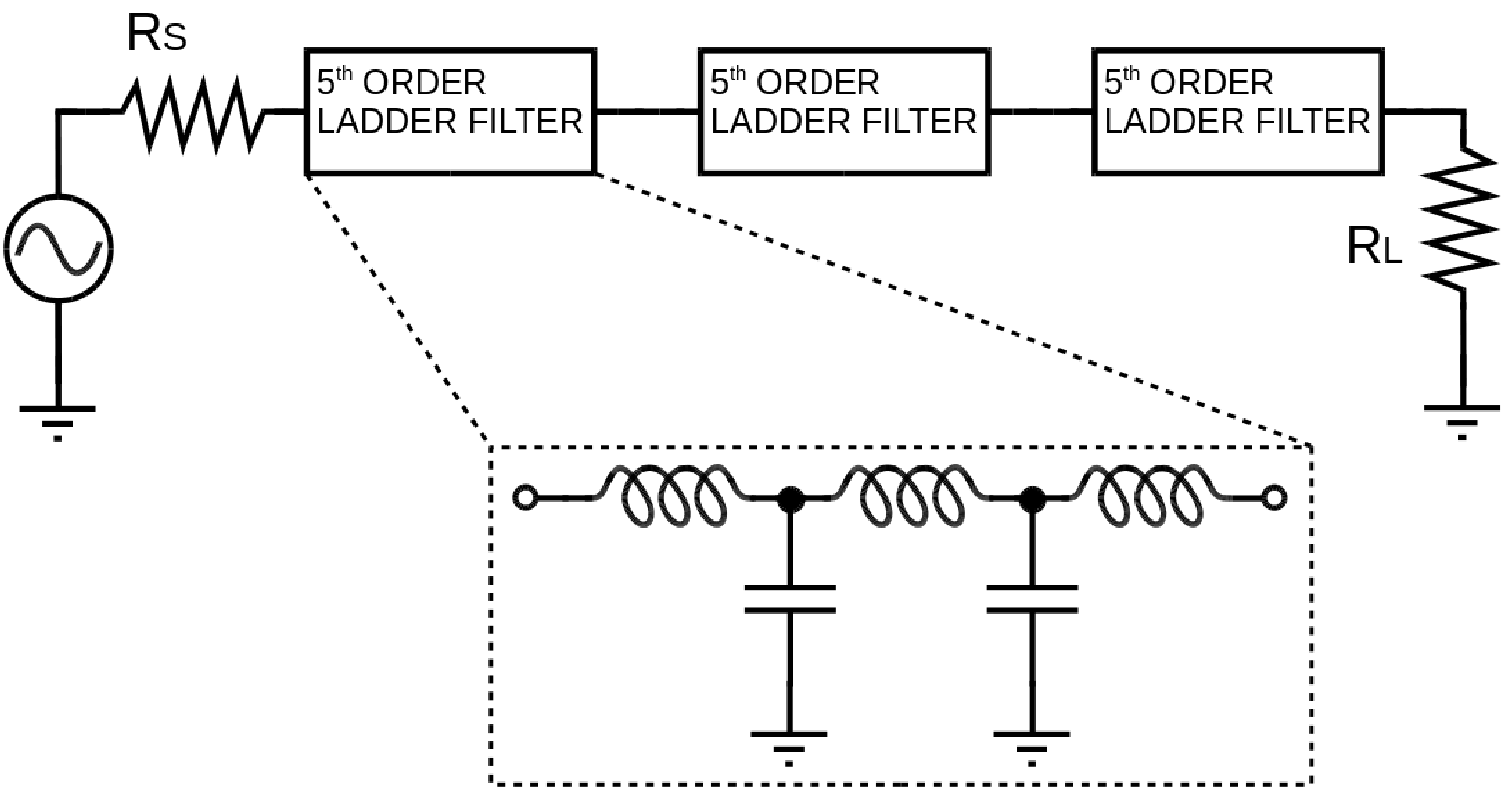
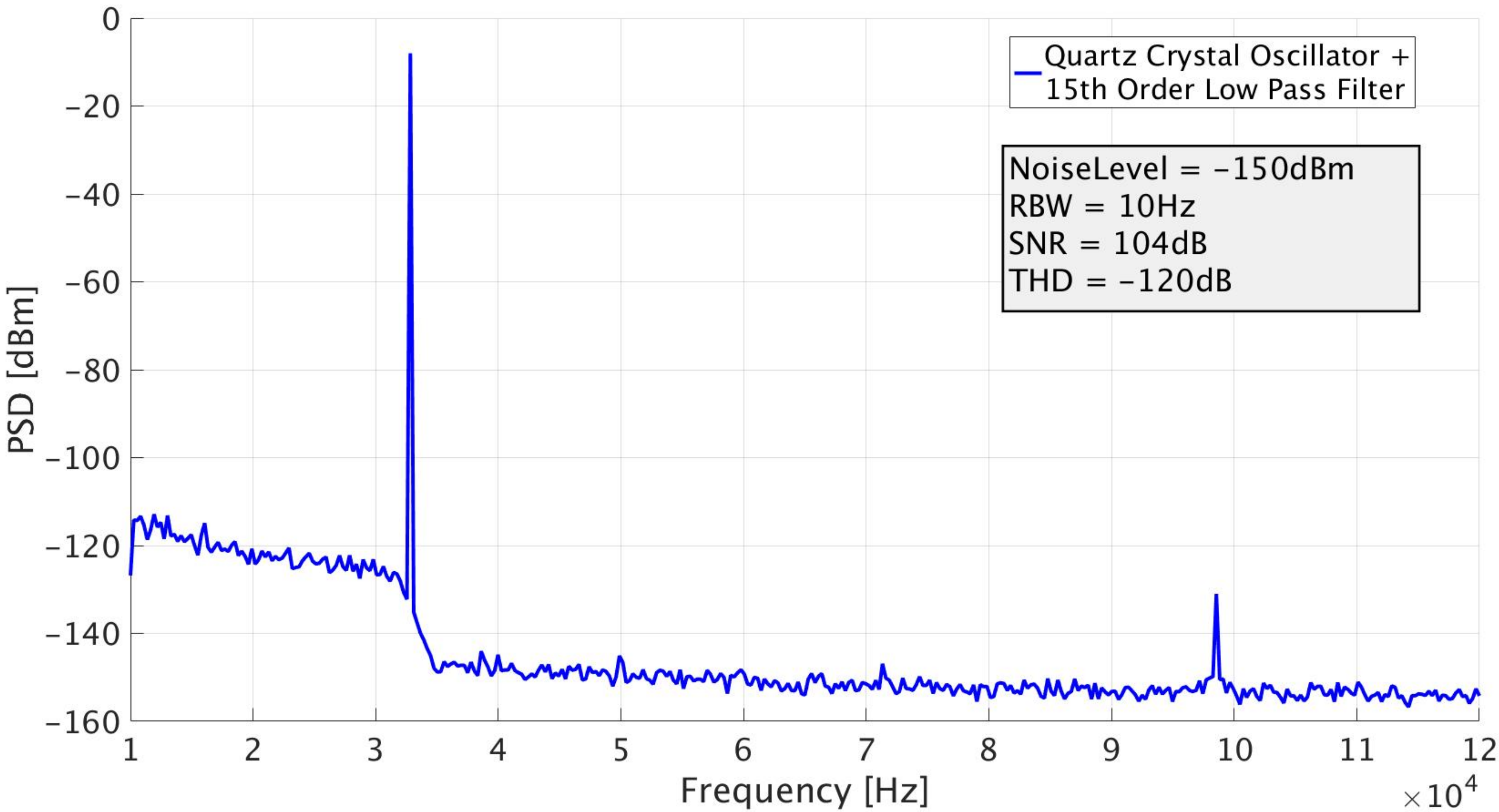
Quartz Crystal Oscillator Spectrum



Low Pass Ladder Filter



Quartz Crystal Oscillator (Filtered) Spectrum



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Conclusion

- An original design of a Sigma Delta with 93 dB DR has been proposed.
- Some of the basic characteristics of the Sigma Delta topology has been highlighted.
- Low cost / high precision oscillator solutions has been proposed in order to create an ultra pure sine wave.
- Measurements are in progress in this period.