Time-Invariant Spatially Coupled Low-Density Parity-Check Codes with Small Constraint Length

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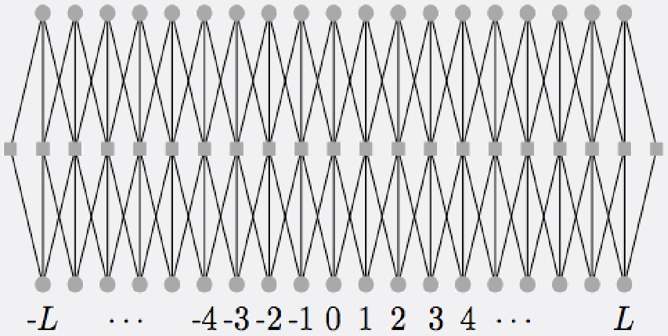
DIPARTIMENTO DI INGEGNERIA DELL'INFORMAZIONE

Spatially coupled codes

- Spatially coupled ensembles achieve capacity under Belief Propagation, thanks to the threshold saturation phenomenon[1]!

- Terminated convolutional LDPC codes belong to the class of spatially coupled codes

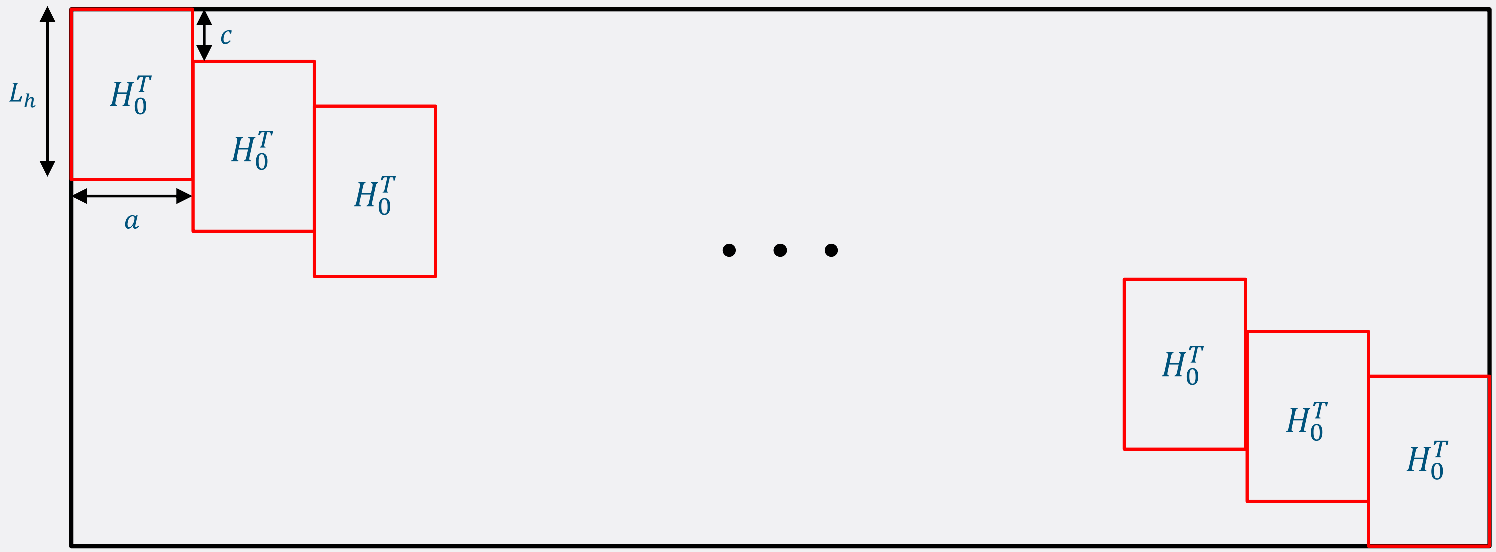
- As long as the connection is "strong", the threshold saturation effect occurs



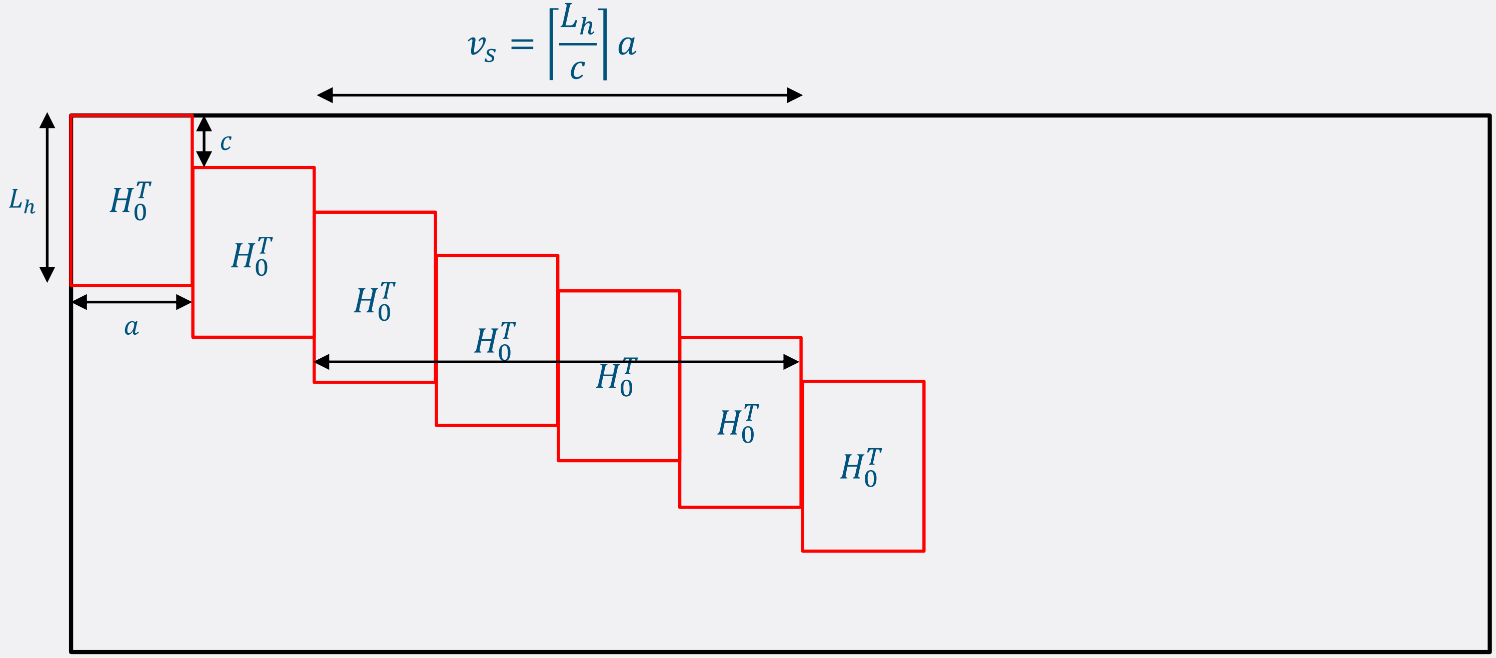
[1] K. Kudekar, T. Richardson, and R. L. Urbanke, "Spatially coupled ensembles universally achieve capacity under belief propagation," *IEEE Trans. Inform. Theory*, vol. 59, no. 12, pp. 7761–7813, Dec. 2013

LDPC codes: applications

Parity check matrix of time-invariant LDPCC codes



Parity check matrix of time-invariant LDPCC codes



Symbolic parity check matrix

$$H_0^T \rightarrow H(x) = \begin{bmatrix} h_{0,0}(x) & \cdots & h_{0,a-1}(x) \\ \vdots & \ddots & \vdots \\ h_{c-1,0}(x) & \cdots & h_{c-1,a-1}(x) \end{bmatrix}$$

$$H_0^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ \hline 1 & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad H(x) = \begin{bmatrix} 1+x & x^2 & 1 \\ 0 & 1 & x^2 \end{bmatrix}$$

$1^{st}: h_{i,j} = 0,\ j, l$

$2^{nd}: \forall\, l,\ l_d = \left\lfloor \frac{l}{c} \right\rfloor,\ l_m = l \bmod c$

$3^{rd}: h_{l_m,j}(x) \mathrel{+}= x^{l_d}$

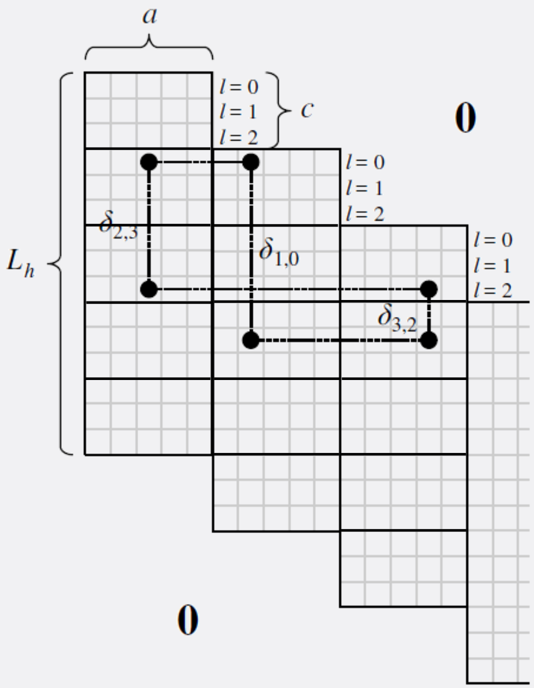
Contribution

- Complexity of encoding and decoding techniques linearly increases with the syndrome former constraint length:

1. **Design of H_0^T for rate $R = \frac{a-c}{a}$ codes and fixed minimum length g of their local cycles. The syndrome former constraint length is smaller than that obtained by unwrapping QC-LDPC codes**

2. **Theoretical lower bounds on the syndrome former constraint length**

Differences and local cycles



$$l_s = j \bmod c$$

$$\delta_{i,j}{}_{le}^{l_s}$$

$$l_e = \left(j + \delta_{i,j}\right) \bmod c$$

$$\delta_{i_1,j_1} \pm \delta_{i_2,j_2} \pm \cdots \pm \delta_{i_l,j_l} = 0$$

$$\delta_{i,j} + \delta_{x,y} \text{ iff } \quad l_{e_{i,j}} = l_{s_{x,y}}$$

$$\delta_{i,j} - \delta_{x,y} \text{ iff } \quad l_{e_{i,j}} = l_{e_{x,y}}$$

Lower bounds: absence of local cycles with length 4

- $w = 2, \quad \forall\, a, c$

$$a \leq \sum_{i=0}^{c-1} (L_h - i - 1)$$

$$L_h \geq \max\left\{c + 1, \left\lceil \frac{a + \binom{c+1}{2}}{c} \right\rceil\right\}$$

- $\forall\, w, a, c$

$$L_h \geq \max\left\{c + 1, \left\lceil \frac{a\binom{w}{2} + \binom{c+1}{2}}{c} \right\rceil\right\}$$

Lower bounds: general form for $g = 6$

- $\forall\ w_i, a, c$

$$L_h \geq \max\left\{c+1, \left\lceil \frac{\sum_{i=0}^{a-1}\binom{w_i}{2} + \binom{c+1}{2}}{c} \right\rceil\right\}$$

- Irregular codes!

Lower bounds: absence of local cycles with length 4 and 6

- $w = 2, \quad \forall\, a, c$

$$L_h \geq \max\left\{c + 1, \left\lceil \frac{2a}{c} \right\rceil\right\}$$

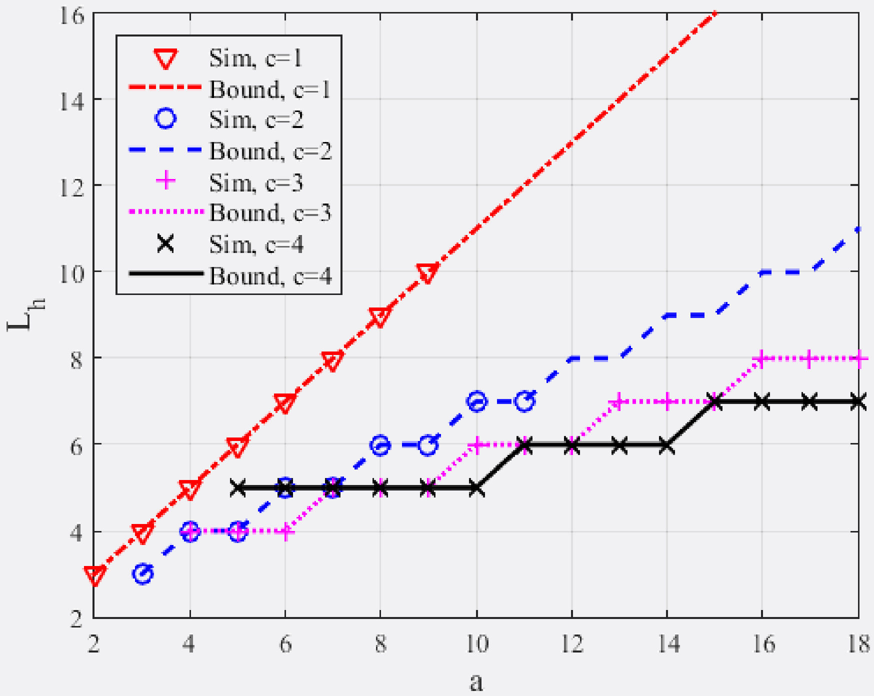
- $\forall\, w, a, c$

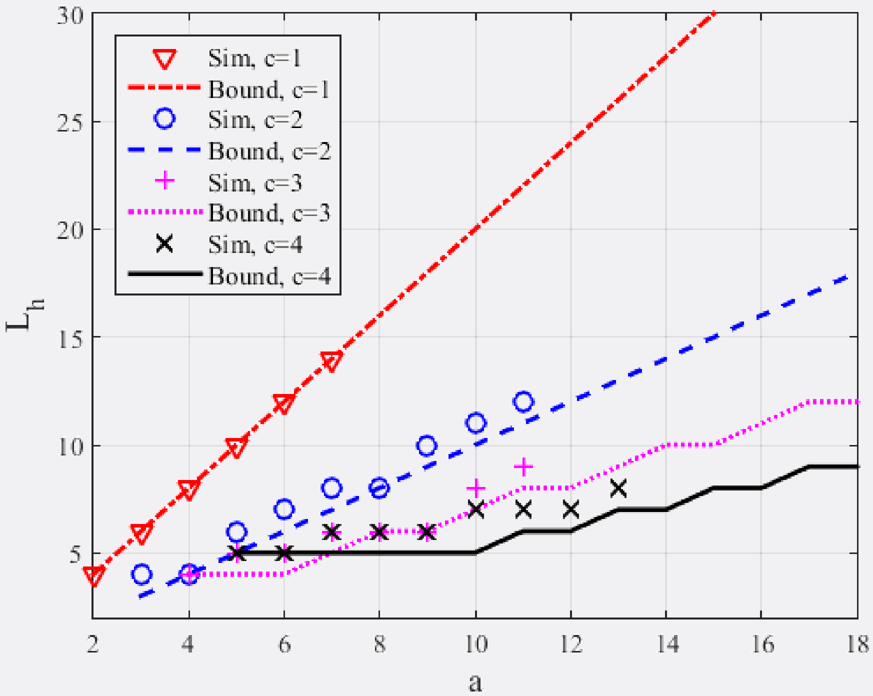
$$L_h \geq \max\left\{c + 1, \left\lceil \frac{2a\binom{w_i}{2}}{c} \right\rceil\right\}$$

- $\forall\, w_i, a, c$

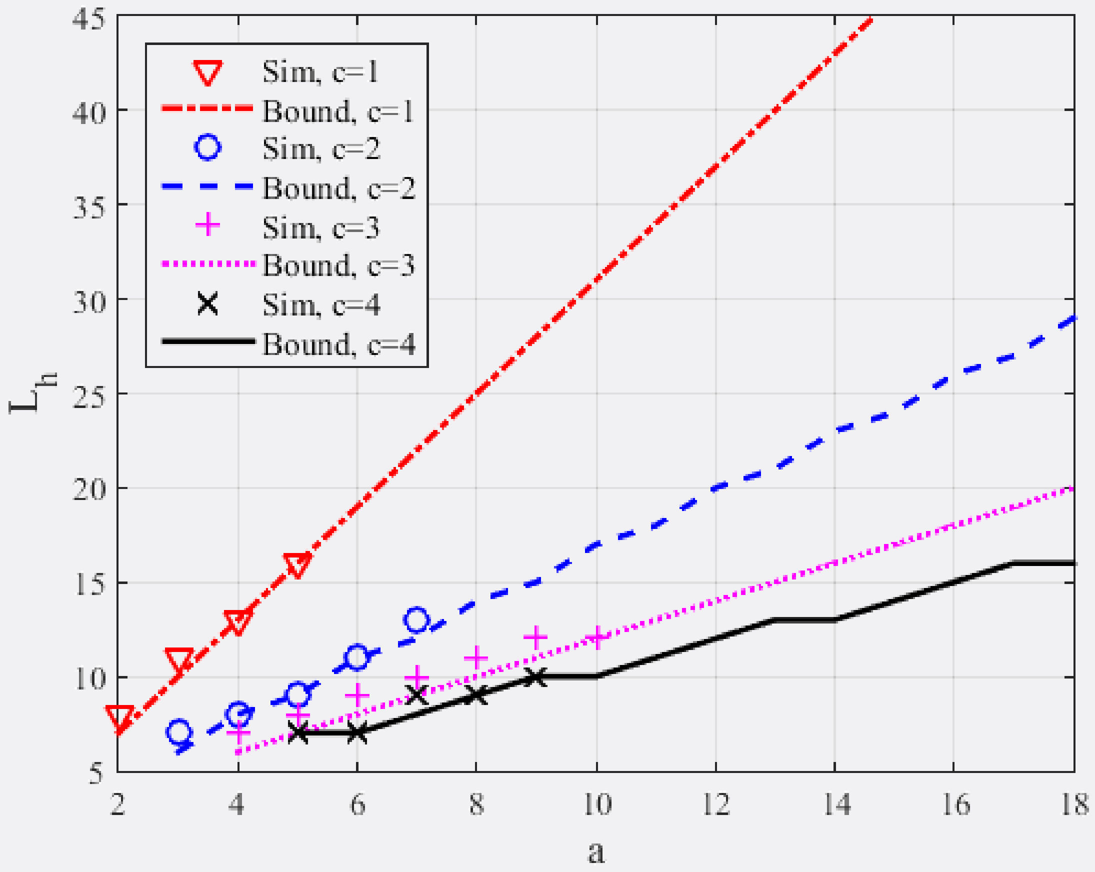
$$L_h \geq \max\left\{c + 1, \left\lceil \frac{2\sum_{i=0}^{a-1}\binom{w_i}{2}}{c} \right\rceil\right\}$$

Theoretical bound vs exhaustive search: $w = 2,3$ and $g = 6$





Theoretical bound vs exhaustive search: $w = 2$ and $g = 8$



Absence of local cycles with arbitrary length

- Montecarlo analyses
- Ex. $g = 10$

$$H(x) = \begin{bmatrix} 1 & x^{65} & 1 & 1 & x^{31} & x^{2} & x^{17} \\ x^{17} & 1 & x^{9} & x^{44} & x^{4} & x^{56} & x^{52} \\ x^{62} & x^{17} & x^{59} & x^{39} & 1 & 1 & 1 \end{bmatrix}$$

- Ex. $g = 12$

$$H(x) = \begin{bmatrix} x^{9} & x^{199} & x^{47} & x^{329} & x^{32} & 1 & 1 \\ 1 & x^{357} & x^{313} & x^{124} & 1 & x^{228} & x^{398} \\ x^{420} & 1 & 1 & 1 & x^{301} & x^{282} & x^{257} \end{bmatrix}$$

Absence of local cycles with arbitrary length

- Montecarlo analyses
- Ex. $g = 10$

x^{65}

$$H(x) = \begin{bmatrix} 1 & x^{2} & x^{14} & x^{27} & x^{67} & x^{97} & x^{130} \\ 1 & x^{21} & x^{24} & x^{1} & x^{6} & x^{75} & x^{58} \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$ [2]

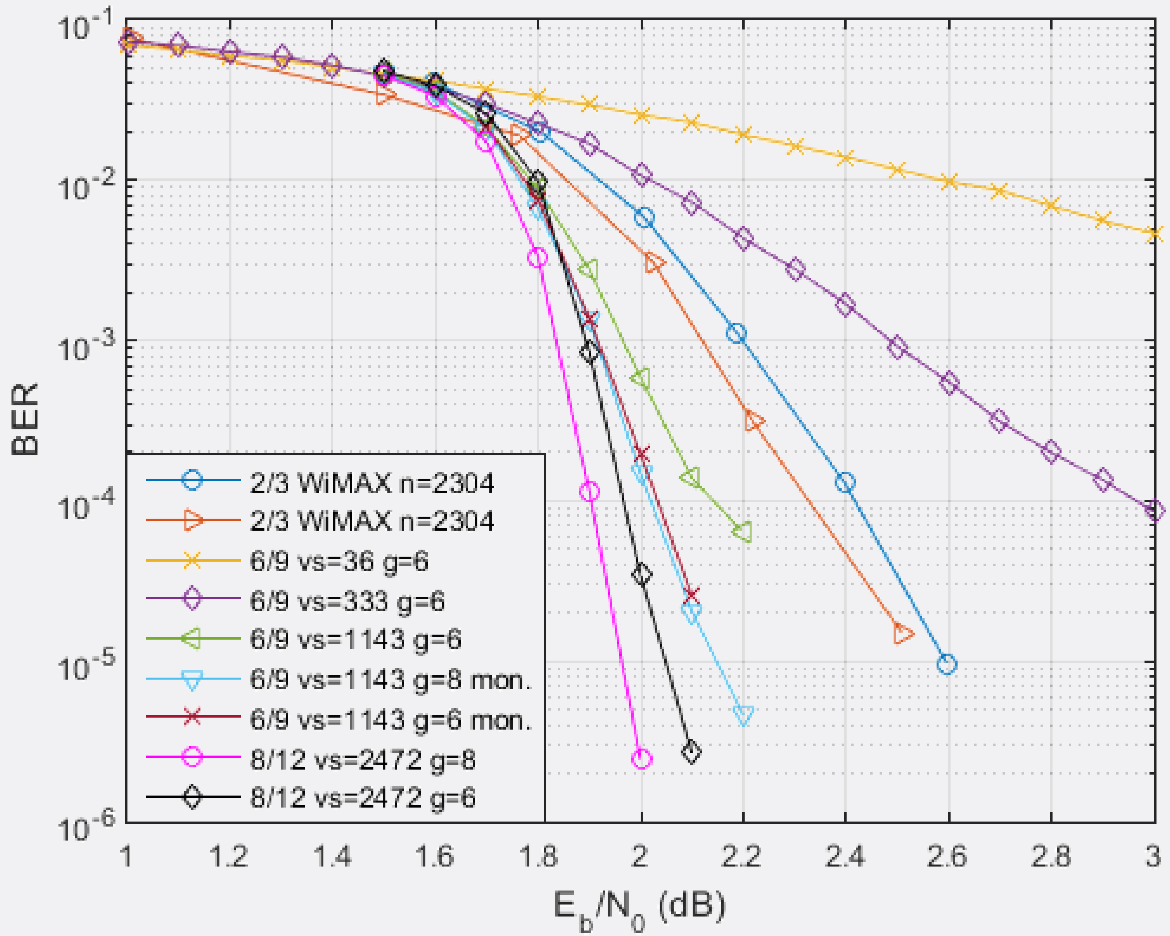
- Ex. $g = 12$

$$H(x) = \begin{bmatrix} 1 & x^{3} & x^{10} & x^{33} & x^{147} & x^{297} & x^{442} \\ 1 & x^{31} & x^{22} & x^{4} & x^{93} & x^{133} & x^{219} \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$ [2]

x^{420}

[2] I. E. Bocharova, F. Hug, R. Johannesson, B. D. Kudryashov, and R. V. Satyukov, “Searching for voltage graph-based LDPC tailbiting codes with large girth,” IEEE Trans. Inform. Theory, vol. 58, no. 4, pp. 2265–2279, Apr. 2012

Performance of the proposed codes over the AWGN channel



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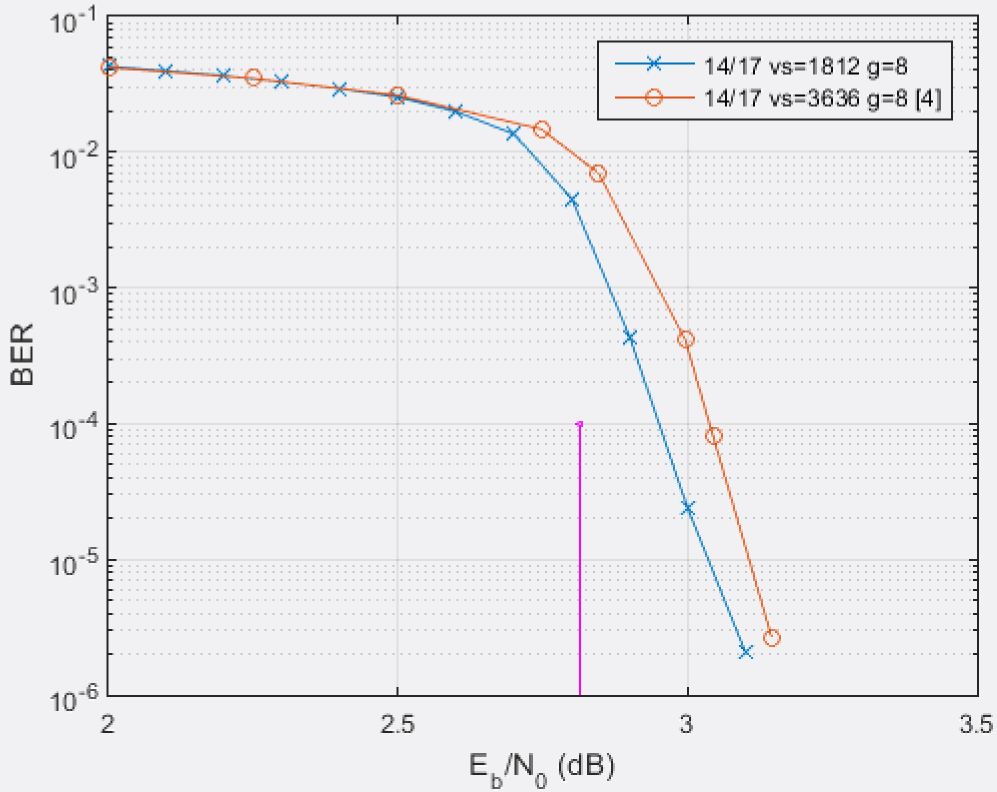
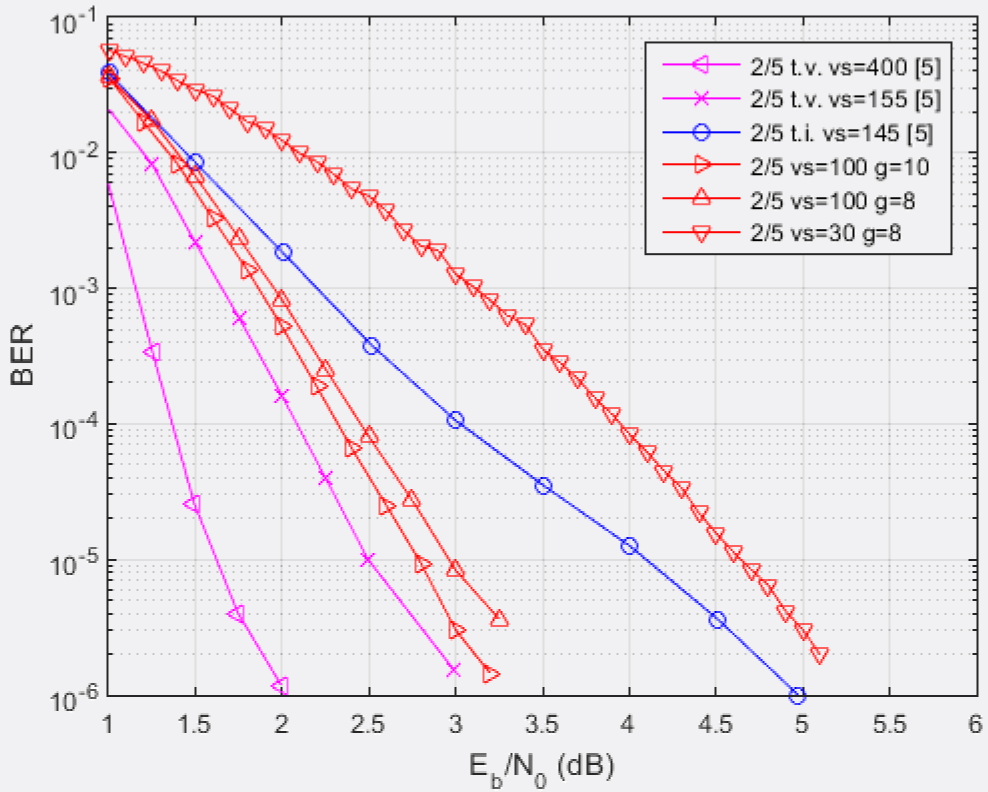


[4] R. M. Tanner, D. Sridhara, A. Sridharan, T. E. Fuja, and D. J. Costello, “LDPC block and convolutional codes based on circulant matrices,” *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 2966–2984, Dec. 2004.

Performance of the proposed codes over the AWGN channel



[5] A. E. Pusane, R. Smarandache, P. O. Vontobel, and D. J. Costello, "Deriving good LDPC convolutional codes from LDPC block codes," *IEEE Trans. Inform. Theory*, vol. 57, no. 2, pp. 835–857, Feb. 2011.

Conclusions

- Directly designing the syndrome former matrix, we have found codes with syndrome former constraint length smaller than that obtained by unwrapping QC-LDPC codes

- We have designed codes free of local cycles up to a given length

- We have found theoretical bounds on the syndrome former constraint length

Future works

- Extension of the bounds to higher values of the girth
- Study of the performance of time-varying and irregular codes
- Search of the minimum syndrome former constraint length of a code in order to reach a predetermined performance

THANK YOU!

(And use channel coding)