A Technical and Micro-economic Analysis of Wireless LANs

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Abstract— **In this paper, we present a joint economic and technical analysis of a wireless LAN, where we model the user behavior as aimed at the choice of the transmission rate. User rate allocation requests are determined in a decentralized manner by considering the tradeoff between service requirements and willingness to pay the service price. Then, the multiple users' medium access is considered and the resulting allocation is evaluated and discussed. We provide results for what concerns the allocated resource, the number of satisfied users and the revenue coming from the assignment. All these metrics are shown to be connected, since they are strongly influenced by the service appreciation rate; however, they are also dependent on the price setting, which has to be acceptable for the users and allow efficient resource usage. Moreover, we investigate in particular the sensitivity to the number of possible system customers. These considerations are finally extended to gain general insights on the performance of the Distributed Coordination Function of IEEE 802.11b.**

I. Introduction

A very interesting and developed application of Wireless Local Area Networks (WLANs) is the creation of hot-spots, where a set of mobile terminals is connected to a central access point. It is well known that the performance in this case is heavily affected by the network operating conditions [1]–[3], especially for what concerns the number of potential users trying to access the network. In this scenario, we are interested in evaluating both technical and economic performance of the Radio Resource Management (RRM), in particular for the rate allocation issues. This means that besides the classic metric of efficiency, like the number of users admitted to the service, we are also interested, for example, in quantifying the revenue earned by the resource manager.

Modelling economic aspects of RRM is an open field for research, in which several interesting contributions have been recently presented. In particular, the concept of utility functions and ideas taken from game-theory have been employed to study the possibility to tune the Quality of Service (QoS), for example by varying the transmission data rate of the terminal [4]. It has also been shown that micro-economic concepts can be employed both to explore the effort of improving users' welfare [5], [6], and to price the resource usage in order to achieve an efficient management [7], [8]. On the other hand, we must also observe that usually in these papers the pricing issue is implemented as a *virtual* strategy, useful to avoid congestion (as in [9]).

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Our contribution here considers more directly the real price established by the operator for service tariff, which is bound to be fixed *a priori* and known in advance by the users. Also we adopt a statistical approach to regulate the tradeoff between quality and price and see how it plays in determining the fraction of users accepting the service conditions.

To this end, a user-centric model already developed by the authors [10] is used, which has been employed in combination with capacity constraints typical of cellular networks, for example Code Division Multiple Access (CDMA) based systems, where the users' multiplexing constraint is easy to express. Here, we adapt the capacity constraint of the model to a simpler approximation, so that it can be used to model the users' behavior and to dimension a WLAN hot-spot.

Secondly, we investigate the role of pricing in determining resource usage. It will be shown that an appropriate price setup is key in achieving a satisfactory revenue, but also in effectively managing the constrained bandwidth. In fact, besides causing revenue generation, pricing the system usage allows also a better coordination and a more efficient utilization. In other words, price tuning can be seen as an implicit Admission Control (AC) mechanism which improves the system performance. On the other hand, too high a price prevents users from entering the service, so that the system is under-utilized. This is another undesirable effect, and should be avoided.

Besides the price setting, we stress the impact of the load, seen as the number of users demanding allocation. In fact, both technical and economic metrics encounter severe variations when the users' set is increased or decreased. Thus, a provider of a real system needs to take this phenomenon into consideration, e.g., by accurately estimating the number of possible customers.

This work is organized as follows: in Section II we model the behavior of the WLAN users by including micro-economic considerations concerning the QoS. In Section III we present and discuss the simulation results and finally Section IV concludes the paper.

II. User-centric Allocation Model

The behavior of the wireless network users is strongly related to the achieved QoS. In this paper we assume that the users determine their service appreciation in a distributed manner and unsatisfied users refuse the service, so that they do not concur in determining overall resource allocation and hence revenue generation. This is done according to the MEDUSA model [10], where two factors are assumed as drivers of users' choices. The first factor is a tunable description of the inherent service QoS by means of a utility function, different for each user, because a generic service differentiation is assumed. The second factor is the price paid for the service, which is assumed to be a generic non-decreasing function of the obtained resource allocation. In this paper, both utility and pricing function will be related to the rate achieved on the WLAN.

In general, it is reasonable to assume that a real QoS utility function could be numerically estimated via subjective testing. Note that the proper micro-economic concept of utility function does not assign to its value any specific meaning, but it is only useful to sort users' preferences. Note also that a larger amount of resources always results in a better evaluation of the service quality, thus the QoS utility function must be non-decreasing. However, it is also realistic to assume that the QoS increase due to higher resource assignment becomes negligible when the utility is already high. For what concerns the pricing, we focus instead on usage-based tariffs. To specify the WLAN allocation case also with results, utility and pricing function will be furthermore better defined with assumptions discussed in the following, even though several different alternatives are possible as well.

To evaluate the users' choices, the MEDUSA model defines an acceptance probability $A_i(u_i, p_i)$ for each user i ($1 \leq i \leq N$, with N being the total number of users) as a function of the two parameters u_i and p_i, the former being the utility coming from the service and the latter being the price paid. If the ith user is provided with a resource assignment equal to r_i, we have $u_i = u_i(r_i)$ and $p_i = p_i(r_i)$. For the moment, r_i is still a generic resource which can be identified differently according to the kind of network under exam.

In the following, A_i will be taken as a mathematical representation of the probability that the ith user considers its service allocation fair both for quality and tariff, i.e., according to the utility/price trade-off. To have a treatable model, we will assume that the utility $u_i(\cdot)$ is the only user-specific function of the model, as it depends on many factors, such as the service enjoyed or the type of terminal. Instead, the functions $A(\cdot)$ and $p(\cdot)$ will be written without the subscript i, assuming that they are the same for all users. This is realistic if the WLAN represent a homogeneous market, where the service conditions from the economic point-of-view are uniform. However, this hypothesis is only made for the sake of simplicity and can be easily removed by considering a class-based approach.

Now, the key assumption of the following analysis is that the WLAN management can be regarded as a problem of resource allocation by identifying the resource r_i with the transmission rate of the ith user. Thus, $u_i(r_i)$ and $p(r_i)$ are the consequent utility achieved and the price paid by the ith user due to the allocation of r_i.

We assume that the transmission r_i can be regarded as a continuously tunable variable, whose values can be in the range $[0, s_i]$. This means that we introduce an upper limit s_i, which is the signalling rate. In other words, s_i corresponds to the maximum transmission rate allowed to the ith user, so that $r_i \leq s_i$. For example, in the IEEE 802.11b standard [11] the signalling rate can be equal to $1, 2, 5.5$ or 11 Mbps. The case where $r_i = 0$ mathematically represents the situation of users who do not consider the service acceptable, and therefore do not get any resource assignment.

Importantly, in the case of unconstrained resource allocation there will be no difference between what users request and what they get. However, WLAN hot-spots are in real world constrained in bandwidth, thus the allocation of a given rate vector $\mathbf{r} = \{r_i\}$ might be infeasible. For example, in IEEE 802.11b networks, the highest signalling rate of 11 Mbps equals also the total amount of available bandwidth for *all* terminals, hence either a single user requesting this allocation is the only one, or else the vector $\mathbf{r}$ can not be feasible. When the infeasibility arises, the allocation must be translated to a different feasible vector $\boldsymbol{\rho}$. In other words, we use the symbol ρ_i (in general, less than or equal to r_i) to denote the eventually allocated rate for the ith user when the requested allocation is not feasible, whereas the symbol r_i will still be kept to describe the allocation requested. The relationship between ρ_i and r_i depends on the choice operated by the network manager to meet the network capacity constraint, and also on the capacity constraint itself. In general, ρ_i can depend not only on r_i but also on the rates r_j, $j \neq i$ requested by the other users. However, the preliminary investigation performed here will consider a simplified version of this relationship, possibly not trivial and significantly dependent on the Medium Access Control (MAC) protocol used.

Let us outline the assumptions made in this paper. First of all, in the following we will assume the same signalling rate $s_i = \mathcal{M}$ for all users, which is also the total available bandwidth for all users, as in IEEE 802.11 WLANs. We consider upper-limited utilities, to respect in a simple manner the already discussed property that QoS improvements coming to larger assignments tend to saturate, as the users' perception of the service can not be indefinitely improved. We assume to have the same upper and lower bound for all utilities. Thus, all the utilities belong to a closed interval, and it is not restrictive to choose a proper normalization which translates these values into $[0, 1]$. In the sequel we model the utilities with the following formula:

$$u_i(r) = \frac{(r/\kappa\mathcal{M})^{\zeta}}{1 + (r/\kappa\mathcal{M})^{\zeta}}. \tag{1}$$

where $\kappa \in [0, 1]$ and $\zeta > 1$ are tunable parameters, whose ranges and distributions depend on the scenario.

The function $p(\cdot)$ is taken as a linear function of the allocated rate, which satisfies both the mathematical requirement of being non-decreasing and the more practical specification of conceptual simplicity, as it is likely that users do not appreciate cumbersome tariff plans. In fact, even though we do not want to discuss in detail possible pricing strategies, a linear functions has indeed the advantage of being simple [12].

To specify the acceptance probability, the following parametric expression, already proposed by the authors in [10], will be considered:

$$A(u, p) = 1 - e^{-ku^{\mu}/p^{\epsilon}}, \tag{2}$$

where the exponents μ and ϵ allow to change the shape of A by tuning the users' sensitivity to utility and price, respectively. The multiplicative constant term k depends on how utility and price are normalized. Equation (2) can be related to the Cobb-Douglas relationships, as explained in [10], where the interested readers can find more details.

We assume that the candidate users can freely choose their rate request. The MEDUSA model is then used by identifying the value of A_i, which is between $[0, 1]$, with the probability that the service is considered satisfactory, in terms of both quality and price, by the ith user. Thus, we assume that the most preferable allocation of r_i can be evaluated, for each user independently, as:

$$r_i = \arg \max_{0 \leq r \leq \mathcal{M}} A(u_i(r), p(r)) \,. \tag{3}$$

Equation (3) does not mean that user i will surely accept the service. Even though r_i is the most satisfactory request, it still has a probability of being considered acceptable equal to $A(u_i(r_i), p(r_i))$. According to the MEDUSA model, any global network metric, which is contributed to by the users, can be evaluated on average by considering the terms related to single users, weighed by their respective acceptance probability. For example, the total allocated resource can be found on average as $\sum_i r_i A(u_i(r_i), p(r_i))$.

However, the final performance is not determined by the r_i's, but by the ρ_i's, because the initial allocation vector can be infeasible. Indeed, we are also interested in evaluating the final performance by giving the requests coming from the users' side as input parameters. Thus, we assume that when the initial rate allocation vector is infeasible, it is translated to a feasible one as follows.

To keep the analysis simple, we assume to have a generic MAC and neglect for the moment the actual multiplexing mechanism, like CSMA/CA for IEEE 802.11 WLANs. For evaluation purposes, the bandwidth is regarded as an additive and perfectly elastic resource. That is, when the capacity is saturated the admitted users achieve an allocation which is an equitable share of the available bandwidth. Under not very restrictive hypotheses, this has been proven to hold, e.g., for IEEE 802.11 networks in certain cases [1]. Moreover, for the sake of simplicity, in order to keep the computational complexity low, we allocate users sequentially. Formally, at each iteration the allocation of user i must respect the condition

$$\sum_{j=1}^{i} r_j \leq \mathcal{M}. \tag{4}$$

Until Equation (4) is respected, we have $\rho_i = r_i$, i.e., the assigned rates are equal to the requests. When Equation (4) does not hold any more, all users with index j, $1 \leq j \leq i$, are re-allocated to:

$$\rho_j := \rho_j \mathcal{M} (\sum_{k=1}^{i} \rho_k)^{-1}, \tag{5}$$

and the allocations are decreased iteratively. This means that the ρ_i's in the right-hand side of Equation (5) are before the update, and at each iteration the left-hand term overwrites the old instance of ρ_i. In doing so, we neglect losses due to RTS/CTS exchange or buffer overload, thus the amount of allocated resource will tend to $\mathcal{M}$ when the number of users is sufficiently large, which is unrealistic in real network. For example, it is well known that the actual throughput reached by IEEE 802.11b WLANs is far from 11 Mbps, at most it reaches around 80% of this value [3].

TABLE I

LIST OF PARAMETERS OF SIMULATION SCENARIO

Parameter	value
maximum WLAN capacity $\mathcal{M}$	11 Mbps
utility parameter ζ	uniform in $[2, 10]$
utility parameter κ	uniform in $[0.001, 0.075]$
acceptance prob. parameter μ	2
acceptance prob. parameter ϵ	4
acceptance prob. parameter k	0.0458

Since the ρ_i's are in general lower than the r_i's, which are the values most preferred by the users, in addition to the probabilistic approach considered before to determine if the users accept to *enter* the service, we adopt also an additional evaluation to determine whether they also accept the degradation to ρ_i. This can be done by considering a conditional probability [13] defined as follows: once a user has accepted r, he accepts also $\rho < r$ with probability equal to

$$A(\rho|r) = \begin{cases} \frac{A(\rho)}{A(r)} & \text{if } A(\rho) \leq A(r) \\ 1 & \text{if } A(\rho) > A(r) \,. \end{cases} \tag{6}$$

Note that the second case is introduced only to model the fact that users will never refuse a service improvement, but since the r_i's maximize the A_i's, this situation never occurs.

Finally, note that the re-evaluation of the satisfaction status made on the $A(\rho_i)$'s implies that new amounts of bandwidth can be freed and subsequently re-allocated. However, for the sake of simplicity we will neglect this fact, which will bring to unnecessarily long computations. The results shown in the following have been obtained by following this approach. They are still consistent with more exact ones, even though small oscillations can be found due to this reason.

III. RESULTS

This section presents some results to evaluate both technical and economic metrics within the above framework. We simulate a generic WLAN hot-spot scenario, where an access point (AP) and N candidate users are present. The area where terminals are spread is assumed to be sufficiently small so that every user can have the same signalling rate $\mathcal{M}$ and the AP has full coverage of the network. This allows us to avoid advocating multi-hop capabilities in the terminals. However, preliminary results obtained with more realistic scenarios, where multi-hop connections and different signalling rates are used, exhibit completely similar behaviors. Table I shows all the choices for the economic parameters of the MEDUSA model.

The first two figures show that the value of the tariff, even with a very simple linear pricing, is key in determining the performance. In Figure 1 we show the number of the admitted

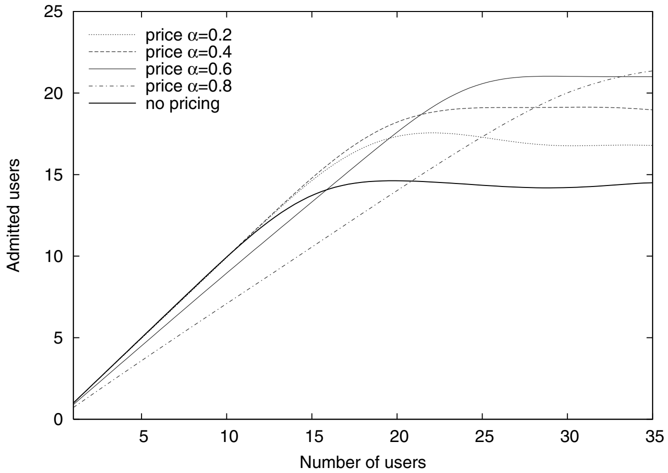

Fig. 1. Number of users accepting the service as a function of the load (candidate users)

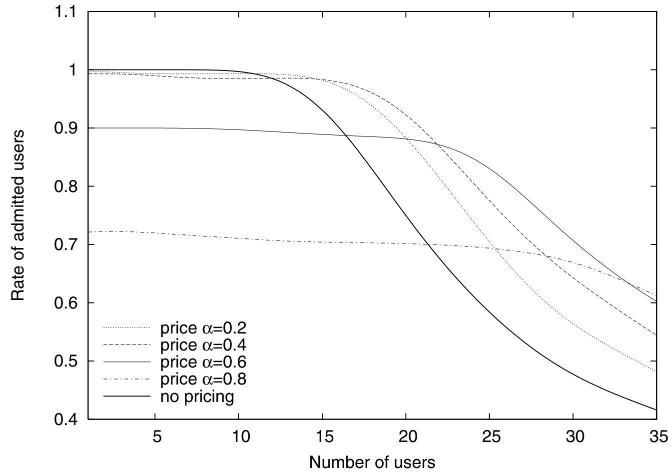

Fig. 2. Fraction of users accepting the service as a function of the load (candidate users)

users, whereas in Figure 2 the ratio between admitted and candidate users is reported. Formally speaking, we consider as *admitted* those users which achieve a rate allocation larger than 0. The *no pricing* case, shown only for the sake of comparison, considers users which are always wanting to enter the system, because $p = 0$ always implies $A = 1$. However, in this case the r_i's are determined as the ones giving 99% of the maximum possible utility (otherwise the choice of r_i as maximizing A_i's would have been indeterminate). Remember that a zero value means that the user refuses the service because either it is too expensive or the quality is poor. However, this last fact can be due to a very low rate allocation proposal, which is the case when the system is congested. This explains why the curves in Figure 1 saturate, in fact after the allocation of a certain number of users the residual capacity is very small, so that very few users can still be accommodated. Also, the smaller the price, the lower the saturation value, and the reason is that when the price is small the users' requested rate is on average larger. In fact, in our model for increasing price we have both decreasing service acceptance rate and lower average requested rate. On the other hand, considering the normalization through the number of candidate users as in Figure 1, it is emphasized that the average acceptance rate is heavily decreased even before saturation when the price is raised.

In general, the main conclusion is that it is possible to regulate the number of admitted users through an appropriate price setting. The quantitative behavior depends in general on ϵ, i.e., the users' sensitivity to the price, even though the conclusion is, qualitatively speaking, very general.

The following results, derived for this simplified model, concern the total allocated resource, from which measurements of the earned revenue can be obtained. In particular Figure 3 shows the allocated resource as a function of N. Again, the no pricing case is reported for comparison.

These results need a comment: since we are neglecting the overhead and the congestion arising from the MAC protocol, the total allocated rate is only an upperbound of the actual throughput. This can be seen from the fact that in Figure 3 the allocated resource saturates close to $\mathcal{M}$, whereas in a real WLAN the total throughput is far from this value. However, more realistic simulations considering detailed modelling of the MAC protocol can be performed and are planned to be subject of future research. We can show preliminary results in Figure 4, where the MAC protocol of the IEEE 802.11b WLAN is simulated in a more realistic, non-idealized, manner (see [14] for further details). Also, similar results, though obtained with a different simulator, have been presented in [15].

By comparing Figures 3 and 4, it becomes clearly visible that, even though the saturation value is lower, the qualitative behavior is very similar. So we infer that also the simplified model is able to capture some basic qualitative properties, although of course a more detailed MAC model is preferrable if quantitatively accurate results are needed.

For example, both in Figure 3 and 4 it can be seen that before the bottleneck of the system capacity, the throughput increases more or less linearly with the number of candidate users. The slope of this increasing behavior depends on the average acceptance rate of a single users, and in fact, the higher the price, the lower the slope (since A is decreasing in p). Then, the throughput saturates together with the saturation reached by the number of admitted users (see again Figures 1–2). However, a region can be identified, where the rate adaptation is more coordinated. In this case the allocated rate still increases even though less than linearly. Then, the effect of users leaving the service, considered too expensive, lets the throughput slightly decrease and a saturation value is found. In Figures 3–4 one can also see that the higher the price, the lower the saturation value, which means that high prices are able to admit more users but result in a lower rate allocation.

Thus, the price setting (which in our examined case reduces to the choice of α) is not trivial, as contrasting results are obtained by changing the price. In general, the provider is interested in having both a high number of admitted users, because this reflects in the long-term economic objective of gaining market shares, and a high allocation of resource, which is related to the short-term objective of high revenue.

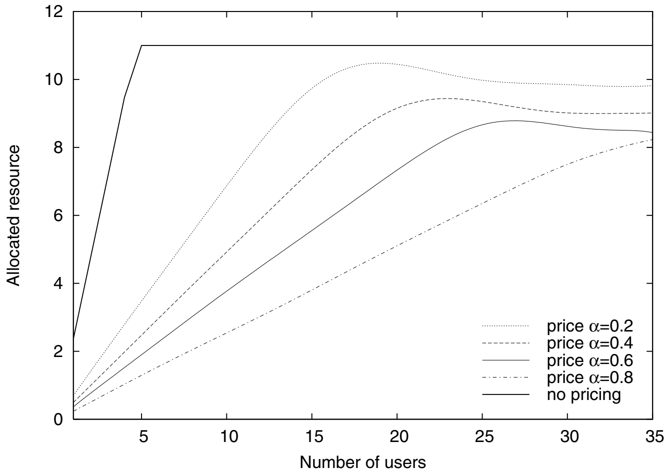

Fig. 3. Total allocated resource, including overhead, as a function of the number of candidate users

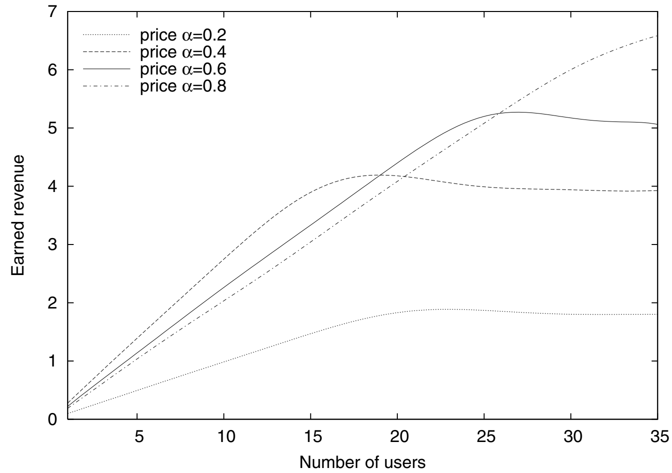

Fig. 5. Instantaneous normalized revenue as a function of the number of the candidate users

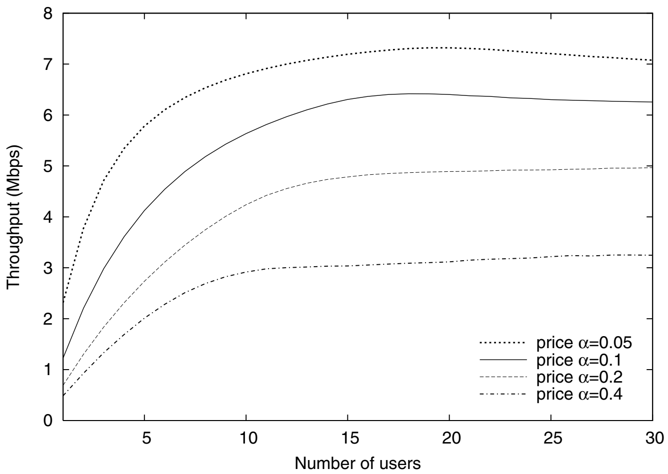

Fig. 4. Total allocated rate without overhead (throughput), as a function of the number of candidate users

To better address this topic, we can also consider more directly the revenue evaluation, as in Figure 5. This can be seen as a possible way to cut the trade-off between different provider's goals, and in fact we see that the curves overlap so that the optimal price choice in terms of earned revenue (i.e., the value of α which gives the highest curve) is strongly related to the number of candidate users. This adds another element of dependence on the number of terminals on the WLAN setup, since it is well known [3] that also the technical performance, besides the economic one, is heavily affected by the number of users.

Moreover, Figure 5 reflects many conclusions valid also for Figure 3, including the peaks of the curves which correspond to values of the load where the multiple medium access is performed in a more efficient way. These points are however dependent on the price value, which highlights a cross-relationship between the technical efficiency, the number of candidate users and the price setup. In other words, to efficiently set up the price the provider should also know the expected number of customers in the network. To this end, it is not necessary to perform accurate evaluations, but simply trivial estimations. For real WLANs, this can be achieved by preliminary investigations and may also imply a need to adjust the pricing according to the load variations. Note that the provider can still respect the hypothesis of pricing function fixed a priori, by setting the price according to an estimate of the average number of customers in a given Time-of-Day [16].

At a general level, it is possible to conclude that the tariff choice is not straightforward, as the revenue is satisfactory only for small price ranges, and also there is a strong dependence on the capacity constraint of the Wireless LAN, which generates non-trivial effects.

IV. Conclusions and Future Research

The search for efficiency in the Radio Resource Management of Wireless LANs involves many issues for the optimization and modelling, where several points of trade-off can be identified. In this work we studied the impact of the choices made by the users on the overall performance. Being a WLAN a typical distributed network, the moves of the users aimed at the local maximization of their own welfare can result in very different allocation vectors, often inefficient.

In particular, we analyzed a model for evaluating in a decentralized manner the appreciation of the service of every single user, so as to determine the global network metrics related to number of admitted users, throughput, revenue.

The goal of efficiently managing the spectrum has been shown to be characterizable from different perspectives. Since in a real network all of them should be addressed, it is very likely that a trade-off must be cut. The results we derived show that the economic aspects like price and users' utilities are key in determining the overall performance and the system might be very sensitive to variations of the parameters.

Several useful conclusions can be drawn: first of all, a careful pricing planning is able to regulate the access of the users, so that the network performance can be tuned. However, one may need to know in advance, even roughly, the estimated number of candidate users, in order to appropriately set up the price. Secondly, the network operating conditions, in particular the network dimension in terms of candidate users, affect both technical and economic results even with simple capacity constraints. We expect that this conclusion can be further generalized to more complicated cases where more details of the MAC protocol are taken into account.

More in general, our approach is able to investigate the interrelationships between number of users, tariff plan and capacity constraint of the network; this is useful both to evaluate an efficient network planning but also to investigate possible improvements of MAC protocols, in terms of enhancing the cooperation among the users' distributed access.

This work can be further developed by including more details in the analysis of the capacity, which can be taken into account through a more realistic relationship between the requested rate r_i and the allocated rate ρ_i. A deeper discussion about this topic can be found in [14].

Another possible extension accounts for different pricing strategies, not only in terms of choosing another function as $p(\cdot)$, but also by considering self-tunable pricing strategies and negotiations. This might allow a further optimization of the network management from both economic and technical standpoint.

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