On the Network Utility Optimal Allocation of Radio Resources in WCDMA Systems

Leonardo Badia, *Member, IEEE,* Adriana Telesca, and Michele Zorzi, *Senior Member, IEEE*

***Abstract*— We discuss an optimization framework for radio resource allocation in a WCDMA system supporting elastic traffic. In particular, we assume the users' preferences as driven by utility functions depending on the assigned transmission rate, and the network capacity constraints as related to interference and power limitations. In this framework, we perform the constrained instantaneous network utility maximization through a logarithmic barrier method. The obtained results are compared and discussed.**

***Index Terms*— Multi-media systems, code division multiaccess, optimization methods.**

I. INTRODUCTION

IN the recent literature, many approaches focus on microeconomics applied to Radio Resource Management (RRM) [1]–[4]. This means that a proper utility function is considered to describe the users' satisfaction as related to the perceived Quality-of-Service (QoS), and optimization policies are sought for the maximization of the overall network utility.

We adopt a utility framework similar to the one considered in [4], by considering elastic traffic, i.e., with adjustable QoS described by continuous utilities, usually assumed to be also quasi-concave. Moreover, utilities are not always increasing, since we assume that over-allocation is penalized, which is the case when, e.g., the resource consumption is not free or the network manager simply includes a virtual price to discourage the abuse of resources. After that, we regard the optimal allocation of radio resources as the constrained maximization of a proper combination of the utility functions. Usually, the radio resource constraints are modeled with oversimplifying linear relationships, as in wireline networks. To improve this aspect, we will present an approach which aims at dealing with non-linearities deriving from interference limitations of a WCDMA system. In addition, we propose a novel optimization technique for the resulting problem based on the Logarithmic Barrier Method [5], which is shown to exhibit a good trade-off between computational complexity and accuracy of the solution. Finally, we obtain numerical results which are compared and discussed.

II. OPTIMIZATION FRAMEWORK

The concept of elastic traffic [3], [6] describes multimedia services characterized by loose quality requirements, so that different QoS levels are acceptable. In particular, we focus on non real time services, thus we consider a QoS degree determined by the transmission rate. The interest toward elastic traffic is motivated by both its ability of describing reasonably well non real time best effort services, and its property of achieving a better resource usage, in the sense of allowing allocation of new calls by decreasing the quality of those already ongoing, which is particularly suitable for CDMA networks [6].

The economic instrument of utility functions seems also very suitable to study elastic traffic, since it aims at capturing the tunable QoS with a variable numerical value. The utility of an elastic service (considered to be a function of the rate r) presents many levels when r goes from 0 to a maximum technologically feasible rate R, and it is generally assumed as continuously tunable. Although many choices are possible, as a concrete example in this paper we will consider utility functions of the following form:

$$u(r) = \begin{cases} 1 - \left(\frac{r^* - r}{r^*}\right)^{\alpha_1} & 0 \le r \le r^* \\ 1 - k\left(\frac{r - r^*}{R - r^*}\right)^{\alpha_2} & r^* < r \le R \end{cases} \tag{1}$$

For numerical reference, we choose $\alpha_1 = 2$, $\alpha_2 = 4$, $k = 0.5$, $r^* = 0.4R$. For the sake of simplicity, in the following we will adopt the same utility function for all the N network users, although the presented framework can be directly applied even in the case of different utilities, i.e., when either the service or simply the service perception are differentiated among the users. In every case, the framework does not rely on this assumption, so it is directly applicable to other choices of the numerical values or shape of the utility function.

In particular, we remark that (1) describes a utility which is not always increasing, as the one proposed in [4]. In our case, $u(r)$ is increasing at first but reaching the most satisfactory value already for a fraction of the maximum available rate, i.e., r^*. Overassignments beyond this point are penalized so that an assignment equal to R only guarantees utility equal to $1 - k$. The rationale behind this choice is that the perception of the QoS related to a certain service can not indefinitely increase. This is due to the marginal decrease of the utility which is inherent in the subjective perception. Besides, in multimedia services also technological constraints determine an upper limit to the QoS. For this reason, a natural behavior of the utility would imply a saturation after the point r^* is reached. However, for this very same reason it is not suitable for the provider to supply the users with a resource amount larger than r^*. Hence, it is also likely that the provider adopts some form of over-allocation control, in order to push users' preferences toward a more efficient allocation. This control

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L. Badia and A. Telesca are with the Dept. of Engineering, University of Ferrara, Italy (e-mail: {leonardo.badia, adriana.telesca}@gmail.com).

M. Zorzi is with the Dept. of Information Engineering, University of Padova, Italy (e-mail: zorzi@ing.unife.it).

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might directly follow from an external provider intervention, or stem from a higher price, which are penalties introduced in order to avoid over-allocation. This is why we choose a utility function which is decreasing for $r > r^*$. Finally, observe that the proposed function is concave: it is common to assume concave or quasi-concave utility functions for communication services [2]–[4].

In this work, the objective of the network manager is regarded as the instantaneous network utility maximization with a constraint given by resource limitation, where the term "network utility" refers to a proper aggregate of the utilities of all users. For the sake of simplicity, and also since it is quite common in the literature, we consider the utilities to be additive, and the overall network utility to be the sum of them. This is however not restrictive, and other choices are possible.

For what concerns the allocation constraints, in wireless communication systems the resource to assign, i.e., the transmission rate, is usually considered limited by bandwidth and power. In particular, we will focus on the downlink of a single-cell WCDMA system. Thus, we account for the power constraints by connecting the rate of every connection to a power assignment through standard relationships of WCDMA networks. Hence, besides the rate r_i allocated to the ith user, we introduce other variables, called w_i, that represent the transmission power of user i. The interference that the users cause to each other can be accounted for with the Signal-to-Interference ratio (SIR) after de-spreading, i.e., E_b/N_0:

$$\left(\frac{E_b}{N_0}\right)_i = \frac{g_i w_i B / r_i}{\sum_{j=1, j\neq i}^{N} \xi_{ij} g_i w_j + \eta} \tag{2}$$

where B is the WCDMA spreading bandwidth, g_i is the link power gain for user i, $0 \leq \xi_{ij} \leq 1$ is the cross-correlation between users i and j and η represents the background noise plus the intercell interference (assumed constant, for simplicity). Note that the utility and the SIR of user i are related through the rate r_i, which appears in both (1) and (2).

The definition in (2) can be used into a relationship of the form "$(E_b/N_0)_i \geq \gamma_i$", where γ_i is a target value which defines the minimum level of acceptable QoS (depending on modulation and required error probability). We make use of the following artifice, which allows a considerable simplification of the formulae. The basic idea is to assume the SIR constraints as stringent, so that allocation of an SIR higher than the threshold is useless. In fact, the equality case is still a satisfactory choice for an SIR inequality constraint, and on the other hand requires less power. This assumption is indeed very important in order to understand the results, so it will be further discussed in Section IV.

In this way the utility maximization problem can be stated as follows. Apart from requiring that all variables be non-negative, the main constraints of the allocation are: to meet, with equality, an SIR-target requirement for every connection, and to impose that the sum of the power levels does not exceed the total power W available at the base station. For simplicity, we also put ξ_{ij} equal to 1, which does not substantially change the mathematical behavior of the problem.

Thus, if we consider an SIR target value γ_i for the ith user and the constraints about the available transmission power, the resulting optimization model is:

$$\begin{aligned}
&\max \quad \sum_{i=1}^{N} u(r_i) && \text{(a)} \\
&\text{s.t.} \quad r_i = \frac{B g_i w_i / \gamma_i}{\sum_{j=1, j\neq i}^{N} g_i w_j + \eta} \quad \forall i = 1,\dots,N && \text{(b)} \\
&\qquad \sum_{i=1}^{N} w_i \leq W && \text{(c)} \\
&\qquad 0 \leq r_i \leq R \quad \forall i = 1,\dots,N && \text{(d)} \\
&\qquad w_i \geq 0 \quad \forall i = 1,\dots,N && \text{(e)}
\end{aligned} \tag{3}$$

where the w_is and the r_is are the variables to optimize and all the terms $\gamma_i, g_i, B, \eta, R, W$ are positive constants. Note that (3b) determines a rate and power relationship by means of (2), which is used in (3a), since the objective is a function of the rate. (3c) instead states the important constraint of the power budget limit at the base-station. Problem (3) depends on the variables r_i and w_i, but through simple passages which exploit the aforementioned relationship we can reduce it into a problem depending only on the variables w_i.

We simplify the notation by letting $\tilde{\eta}_i = \frac{\eta}{g_i}$ and $\chi_i = \frac{B}{R\gamma_i}$, which yields the following problem:

$$\begin{aligned}
&\max \quad \sum_{i=1}^{N} u\left(\frac{R\chi_i w_i}{\sum_{j=1, j\neq i}^{N} w_j + \tilde{\eta}_i}\right) && \text{(a)} \\
&\text{s.t.} \quad \sum_{i=1}^{N} w_i \leq W && \text{(b)} \\
&\qquad \chi_i w_i - \sum_{j=1, j\neq i}^{N} w_j \leq \tilde{\eta}_i \quad \forall i = 1,\dots,N && \text{(c)} \\
&\qquad w_i \geq 0 \quad \forall i = 1,\dots,N && \text{(d)}
\end{aligned} \tag{4}$$

The optimal solution is found as $\mathbf{w} = (w_1, w_2, \dots w_N)$; if needed, the optimal rates can be obtained through (3b).

III. Logarithmic Barrier Method

A method of resolution based on exhaustive search, aimed at finding the exact solution by trying all possible combinations of the vector of powers, would provide the optimum at the price of an exponential increase of the complexity with the number of users. Moreover, since this method strongly depends on the values that the vector of powers can assume, a greater precision involves an increase of the number of functional evaluations and, therefore, a remarkable increment of the computational complexity.

In order to solve the problem while keeping the complexity low, we might use a procedure which replaces the bounded optimization problem with an unbounded one. In particular, we apply the Logarithmic Barrier method [5], in which the unconstrained optimization function is obtained via a linear combination of the goal function of the constrained problem and a logarithmic weighing of the constraints.

Applying this particular method to (4), the problem translates into the minimization of

$$\begin{aligned}
P(\mathbf{w}, \mu) = &- \sum_{i=1}^{N} u\left(\frac{R\chi_i w_i}{\sum_{j=1..N, j\neq i} w_j + \tilde{\eta}_i}\right) + \\
&- \mu\left(\log\left(W - \sum_{i=1}^{N} w_i\right) + \sum_{i=1}^{N} \log\left(\tilde{\eta}_i - \chi_i w_i - \sum_{j=1..N, j\neq i} w_j\right)\right)
\end{aligned} \tag{5}$$

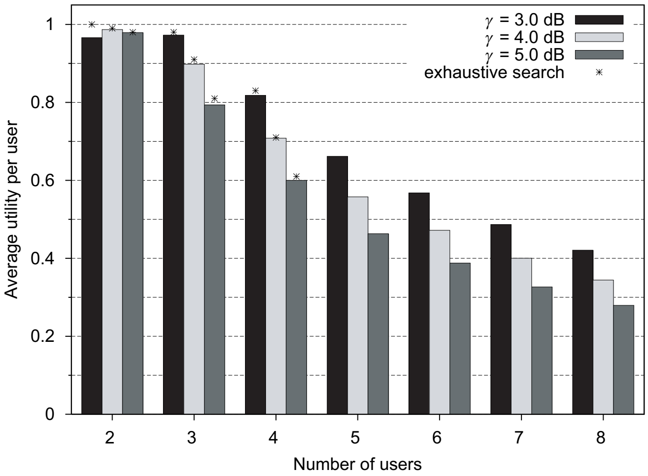

Fig. 1. Average utility per user obtained with the optimization framework for different SIR-target and number of users.

Note that the parameter μ weighs the logarithmic terms in a way similar to Lagrange multipliers.

In this way, the optimization procedure consists of finding the unconstrained minimum of $P(\mathbf{w}, \mu)$, which can be achieved by finding all roots of its gradient. For the sake of simplicity in the implementation, we adopt a local search of the roots. Thus, the efficiency of the procedure depends on the initial point chosen. Of course, other more efficient techniques could be applied within the same framework, if desired. However, this simple strategy has proven to be fast and efficient, as shown in the next section.

IV. Numerical Results and Discussion

We consider a system scenario consisting of a BS placed in the middle of a circular area, where a variable number of users are randomly placed with uniform distribution between an external radius equal to 100 m and an internal radius equal to 5 m. The link gains are obtained by considering a path loss function of the distance d, directly proportional to d^3. A log-normal shadowing term is also considered with parameter $\sigma = 6$ dB. For the optimization problem, the rate can be normalized, i.e., the highest value is set to $R = 1$. The spreading bandwidth B is taken as $1.5R$. The maximum transmission power W is 20 W. The term η, including thermal noise and intercell interference, is -37 dBm. For simplicity, γ_i is the same for all i.

The results shown refer to the search of the maximum through the minimization of (5) with a local search strategy and, for low load cases, an exhaustive search for comparison. To avoid the case in which some minima are not identified, thus missing the global maximum of $P(\mathbf{w}, \mu)$, the actual implementation of the local search tries 4 starting points. We have verified that this suffices to find a solution very close to the global maximum in all simulations where the exhaustive search has also been run. In any case, the cost of including additional starting points is almost negligible (at most it is a linear increase). About the computational complexity, we found that compared to the exhaustive search, the number of operations for the proposed technique is already four orders of magnitude lower when $N = 4$. Moreover, the complexity grows linearly in the number of users for our proposed method, whereas the exhaustive search is obviously exponential.

Fig. 1 gives an overview of the results, plotted as average utility obtained by each user in the optimal allocation found by the Logarithmic Barrier method. The overall utility saturates as the number of users increases. The allocation is satisfactory for almost all users in the case of low network load (fewer than 3 users): for the cases of SIR target equal to 3 dB, 4 dB, 5 dB, users achieves a total utility of 0.97, 0.9, 0.8, respectively. When the constraints become more stringent (more users or higher required SIR) the average utility decreases so that the total utility saturates at first, and then starts to slowly decrease.

Finally, note that for very low load the cases of higher SIR target outperform the condition $\gamma_i = 3$ dB. This is mainly due to the fact that, as introduced during the optimization discussion, the SIR constraint taken with equality is appropriate only if it is a tight bound. In the cases of low number of users or very low SIR target this might not be true. For example, when $\gamma_i < 3$ dB the optimization procedure performs even worse for low loads. However, note that in this case there is no need for constrained optimization as the system is not very much interference-limited. Anyway, the reason of this behavior has to be sought in the assumptions behind the optimization. In fact, the resource manager tries in our scenario to allocate every user by respecting the SIR condition. This is an unavoidable point, and the SIR constraints taken with equality prevent also users from being allocated with zero rate (which would be the case of users rejected by the Admission Controller). Indeed, the development of this analysis in order to obtain also efficient Admission Control techniques can be an interesting extension for future research. Up to now, the proposed technique is still promising due to its ability to achieve a good solution in a reasonable computational time.

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