Using Game Theory and Bayesian Networks to Optimize Cooperation in Ad Hoc Wireless Networks

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***Abstract*—Infrastructure sharing has been recently investigated as a viable solution to increase the performance of coexisting wireless networks. In this paper, we analyze a scenario where two wireless networks are willing to share some of their nodes to gain benefits in terms of lower packet delivery delay and reduced loss probability. Bayesian Network analysis is exploited to compute the correlation between local parameters and overall performance, whereas the selection of the nodes to share is made by means of a game theoretic approach. Our results are then validated through use of a system level simulator, which shows that an accurate selection of the shared nodes can significantly increase the performance gain with respect to a random selection scheme.**

I. Introduction and Related Work

Cooperation is one of the most promising enabling technologies to meet the increasing rate demands and quality of service requirements in wireless networks, especially since nowadays many techniques to share the spectrum resources among different networks are envisioned. Beyond spectrum sharing, also infrastructure sharing is possible, namely, when a network decides to cooperate, it shares some or all of its nodes, that become relays for another network. In such a scenario, cooperation can leverage the benefits of diversity, obtaining a considerable gain in the efficiency of shared resources. Usually, sharing the whole set of nodes can grant the biggest advantage to both networks. However, this clearly comes at the cost of additional traffic that should be handled by some of the shared nodes. In addition, in a realistic environment, an operator may not be willing to share too many nodes to improve the traffic of another operator, e.g., for security or privacy reasons. Therefore, the operator may decide to share only a limited number of nodes, receiving the same treatment from the operator of the other network. If this is the case, an optimal choice of the shared nodes, according to certain criteria, is needed. Indeed, some nodes may be deployed in crucial positions, and they may be particularly suited for helping the other network; on the contrary, nodes placed close to the network border are likely to be less useful or even useless. Furthermore, sharing a node implies that a higher amount of traffic will be routed through it, which may result in a higher latency for the traffic of its own network.

In this paper we consider two wireless networks deployed in the same region but operated by different entities. In the first scenario, the two coexisting networks perform their operations separately: each network only uses its own resources to deliver the data packets generated by its nodes. Clearly, since they are assumed to share the same spectrum resources, cross–network interference may limit the overall performance. For such a scenario, we select a set of local parameters: some of them are directly observable and depend only on the topology of the network, like the number of neighbors at a given node, and some others are not observable and depend on the link characteristics and on the traffic load. We exploit Bayesian Network (BN) [1] analysis to estimate the joint probability distribution of this set of parameters, and we use BN also to predict, given the evaluation of the observable parameters, the values of the other parameters that will be used to calculate a performance metric. The use of this probabilistic tool is very promising for wireless network optimization, and it has been recently exploited, e.g., for predicting the occurrence of congestion in a multi-hop wireless network [2].

Such an approach can also be used to improve the performance of both networks in our scenario. The key idea is that a higher node density may help both networks to augment the available diversity, and thus to find shorter routes for multi-hop communications. It is straightforward that this may be obtained if each network can take advantage of some of the nodes of the other one. We model the interaction between the two networks through Game Theory. In spite of the considerable theoretical gain that a cooperative transmission allows, modeling the involved agents as smart selfish decision-makers usually leads to inefficient non-cooperative equilibria. For example, [3] shows that the IEEE 802.11 distributed Medium Access Control (MAC) protocol leads to an inefficient Nash Equilibrium (NE) and in [4] a situation similar to the Prisoner's Dilemma occurs in a slotted Aloha MAC protocol. To improve the performance of the network, cooperation among the players is often desirable. In the present paper, we achieve this goal formulating the problem as a repeated game, which consists in a number of repetitions of a base game. It captures the idea that a player has to take into account the consequences of his current action on the future actions of other players. Cooperation is in fact obtained by punishing deviating users in subsequent stages. Similar approaches have been used for example in [5]–[7].

In brief, the main contributions of this paper are:

- the definition of the cooperation problem of two networks sharing the same spectrum resources as a strategic game;
- the use of BN theory to learn the probabilistic relationships among a set of parameters of interest in the network, in order to infer the performance metric parameters from some observable topological parameters;

- the proposal of a game theoretic algorithm to choose the best nodes to share;
- the implementation of the strategic game and the BN predictor in an actual wireless network simulator, that simulates the network behavior at the Physical, the MAC and the network layers of the protocol stack;
- to numerically show the effectiveness of our algorithm in improving the performance of the wireless networks by accurately selecting the nodes to be shared.

The rest of the paper is divided as follows. In Section II we briefly review the Bayesian Network and Game Theory approaches, then in Section III we introduce our network scenario. In Section IV we describe the cooperation strategy adopted, while in Section V we detail the simulation setup and show the main results. Section VI concludes the paper.

II. Mathematical Preliminaries

In this section we briefly describe the mathematical tools, i.e., Bayesian Networks and Game Theory, that we adopt to identify techniques for the selection of the best cooperating nodes in the network. The former is a method to learn an approximate joint probability distribution among a set of random variables from a set of instances of such variables. The latter is a branch of mathematics studying interactions between decision makers.

A. Bayesian Networks (BNs)

A Bayesian Network (BN) is a probabilistic graphical model [1] describing conditional independence relations among a set of M random variables through a Directed Acyclic Graph (DAG). This graph is used to efficiently compute the marginal and conditional probabilities of the M variables. A node i in the DAG represents a random variable x_i, while an arrow that connects two nodes i and j represents a direct probabilistic relation between the corresponding variables x_i and x_j. The absence of a direct arrow between two variables implies that the variables are independent, given certain conditions on the other variables of the graph. The orientation of the arrow is also relevant to describe the relationship between the two variables. If the arrow is directed from node i to node j, we say that i is a parent of j, and we write $x_i \in \mathrm{pa}(x_j)$. To clarify this concept, consider the following example. If nodes h, i, and j are represented in a BN such that h is a parent of i and i is a parent of j, the joint probability of the corresponding variables is

$$P(x_h, x_i, x_j) = P(x_h)P(x_i|x_h)P(x_j|x_i) \ , \tag{1}$$

that is simpler than a general joint probability among three variables. See [1] for further details on the BN properties.

a) Learning the structure: The technique to learn the approximate joint probability distribution through a BN is divided into two phases, structure learning and parameter learning. The former is a procedure to define the DAG that represents the qualitative relationships between the random variables, i.e., the presence of a direct connection between a couple of variables, not conditioned by other variables. According to the score based method, e.g., see [8], we do not assume any a priori knowledge on the data, but we just analyze the realizations of the variables and we score each possible DAG with the Bayesian Information Criterion (BIC) [9], that we have chosen as a score function. The BIC is easy to compute and is based on the Maximum Likelihood (ML) criterion, i.e., how well the data suits a given structure, and penalizes DAGs with a higher number of edges. If each variable is distributed according to a multinomial distribution, i.e., it has a finite number of possible outcomes, then the BIC becomes very simple to compute, involving only summations for all possible outcomes of the variables and all possible outcomes of the parents of each variable, see [8].

b) Learning the parameters: The parameter learning phase consists in estimating the parameters of the simplified joint distribution according to the probability structure defined by the DAG chosen in the structure learning phase. In order to have the joint distribution, it suffices to estimate the probability of each variable conditioned by the variables that correspond to its parent nodes in the graph. Coherently with the choice of the BIC as a scoring function, we use the ML estimation technique also to determine all the conditioned probabilities for each variable considered.

B. Game Theory

Game Theory [10] is a branch of mathematics that studies *games*, i.e., strategic situations where many players interact together. The aim of game theory is to provide interaction models and predict the outcomes of a game. In game theory, utility functions are used to represent the players' appreciation of the outcomes. A utility function is associated to each player and its value depends on the actions taken by all the players. It is common to assume that the players are selfish and rational. The former implies that each of them only wants to maximize its own utility, independently on the utilities of the others. The latter means that they choose their strategies on the basis of all the information they have about the game.

We describe a game in the normal form as a triplet:

$$\mathcal{G} = (P, \mathbf{S}, \mathbf{U}) \ , \tag{2}$$

where $P = \{1, \dots, N\}$ is the set of the N players, $\mathbf{S} = (S_1, \dots, S_N)$ is the N-tuple of strategy sets, where S_i is the set of all actions that i can take, and $\mathbf{U} = (u_1, \dots, u_N)$ is the N-tuple of utility functions, where $u_i : \mathbf{S} \to \mathbb{R}$ is the utility function of player i. A vector $\mathbf{s} = (s_1, \dots, s_N)$, where $s_j \in S_j$ for all $j = 1, \dots, N$, is called a strategy profile.

An important concept in Game theory is the Nash Equilibrium (NE), defined as the strategy profile $\mathbf{s^{NE}} = (s_1^{NE}, \dots, s_N^{NE})$ where each player obtains its maximum utility given the strategies of the other players, i.e., such that

$$u_j\left(\mathbf{s^{NE}}\right) \geq u_j\left(s_1^{NE}, \dots, s_{j-1}^{NE}, s_j, s_{j+1}^{NE}, \dots, s_n^{NE}\right) \ , \tag{3}$$

for all $j = 1, \dots, N$, for all $s_j \in S_j$. In other words, it is an equilibrium against unilateral deviations. However, the existence and uniqueness of such an equilibrium point in $\mathbf{S}$ are not guaranteed. Further, NEs can be inefficient from a social point of view, leading to the so called tragedy of the commons [11], an inefficient situation occurring when individuals share a common resource in a selfish manner.

A possible way to increase the efficiency of an NE is to formulate the problem as a repeated game, i.e., a base game repeated in time. In this case a rational player is forced to take into account how his current action can influence the future actions of other players. A cooperative behavior is induced by punishing deviating users in subsequent stages.

III. System Model

In this section, we describe the network scenario under investigation from the physical up to the routing layer. In our scenario, two ad hoc wireless networks coexist and share the common spectrum resource. Each network consists of N terminals randomly deployed, and each node is a source of traffic, which generates packets according to a Poisson process with intensity λ. The end destination of each packet is another node in the network, chosen at random. Furthermore, time is divided in slots and slot synchronization is assumed across the whole network.

A. Physical Layer

At the physical layer, CDMA with fixed spreading factor is employed to separate simultaneous transmissions, since both networks share the same spectrum resources, and a training sequence is transmitted at the beginning of each transmission to help channel estimation. The receiving node, $D^{(0)}$, uses a simple iterative interference cancellation scheme to retrieve the desired packet when M simultaneous communications, namely $T^{(1)}, \ldots, T^{(M)}$, are heard. In order to describe this scheme, we need to define the Signal to Interference plus Noise Ratio (SINR) at $D^{(0)}$ for the incoming transmission $T^{(i)}$ from node $D^{(i)}$, i.e.,

$$\Gamma^{(i)} = \frac{N_s P^{(i)}}{N_0 + \sum_{j \neq i} P^{(j)}} , \tag{4}$$

where N_0 is the noise power and N_s is the spreading factor. $P^{(j)}$ indicates the incoming power due to $T^{(j)}$, i.e., for all $j = 1, \ldots, M$:

$$P^{(j)} = \frac{P_T |h_{D^{(j)},D^{(0)}}|^2 d_j^{-\alpha}}{A} , \tag{5}$$

where P_T is the transmission power, which is considered to be the same for all the nodes in the network, A is a fixed path-loss term, d_j is the distance between the receiving node and the source of $T^{(j)}$, α is the path loss exponent, and $h_{D^{(j)},D^{(0)}}$ is a complex zero mean and unit variance Gaussian random variable, which represents the effect of multi-path fading. More precisely, in our scenario, we consider a time correlated block fading. Therefore, for the channel between nodes $D^{(j)}$ and $D^{(0)}$, the multi-path fading coefficient in time slot t is

$$h_{D^{(j)},D^{(0)}}(t) = \rho \; h_{D^{(j)},D^{(0)}}(t-1) + \sqrt{1-\rho^2}\, \xi , \tag{6}$$

where ρ is the time-correlation factor and ξ is an independent complex Gaussian random variable with zero mean and unit variance. Now we can describe the iterative interference cancellation scheme as follows:

- the destination node $D^{(0)}$ sorts the M incoming transmissions according to the received SINR, in decreasing order (for simplicity, we assume $\Gamma^{(1)} \geq \cdots \geq \Gamma^{(M)}$);
- starting from transmission $T^{(1)}$, $D^{(0)}$ tries to decode the corresponding packet, with a decoding probability that is a function of $\Gamma^{(1)}$;
- if the packet is correctly received, its contribution is subtracted from the total incoming signal;
- $D^{(0)}$ attempts to decode the transmission with the next highest SINR, $T^{(2)}$, and goes on until it can try to decode the packet of interest.

B. MAC Layer

At the MAC layer, we implement a simple transmission protocol based on a Request-To-Send/Clear-To-Send (RTS/CTS) handshake. Every time node $D^{(i)}$ wants to send a packet to node $D^{(j)}$, it checks the destination availability by sending a RTS packet; if $D^{(j)}$ is not busy, it replies with a CTS so that $D^{(i)}$ can start transmitting the packet. Correct reception is acknowledged by means of an ACK packet. In the case of decoding failure, after a random backoff time, node $D^{(i)}$ schedules a new transmission attempt, or discards the packet, if the maximum number of retransmissions has been reached. The signaling packets are very short, i.e., they are transmitted within a single time slot, and are protected by a simple repetition code of rate $1/2$. Instead, data packets may span several time slots, so error detection coding is used to verify their correct reception, i.e., redundancy bits are added at the end of each packet.

C. Network Layer

The source node and the destination node are not necessarily within coverage range of each other, so we consider multi-hop transmissions. Two nodes are neighbors, i.e., they can communicate directly, if their distance is lower than a threshold value ℓ. In order to transmit to a node that is not within coverage, the nodes use a static routing table, which is built using Optimized Link State Routing (OLSR) [12], a traditional routing protocol, and is available at every node of the network. Each time a node generates a new packet, or receives a packet to be forwarded, the packet is put in the node queue, with FIFO policy. The maximum queue length is fixed and equal for all nodes. If a new packet arrives when the queue is full, it is simply discarded.

IV. Cooperation Strategy

In this section we describe how the two networks that coexist in our scenario can share efficiently the spectrum resources by means of cooperation.

A. Performance metric

Given the path from $D^{(i)}$ to $D^{(j)}$, we define the delivery delay $\zeta^{(i,j)}$ as the average end-to-end delay of a packet sent along the path, given that the packet is received; and the packet loss probability $p_c^{(i,j)}$ as the probability that a packet is lost along the path. The former depends on the channel and interference conditions, which may require one or more retransmissions, and on the overall traffic level. Indeed, for multi-hop routes, a packet has to wait at each relay node until all the packets it finds in the FIFO queue have been sent. Regarding the latter, the packet loss, there are two main events to be accounted for. One is a high interference level, that may lead to a packet drop due to an excessive number of retransmissions; the other is buffer overflow, i.e., the packet is discarded if the next relay has no room for it in its queue.

We consider a metric to measure the gain offered by the various cooperation strategies, which takes into account the average end-to-end delay of a packet sent along the path from $D^{(i)}$ to $D^{(j)}$. Since no end-to-end packet retransmission mechanism is implemented in our network, the effect of lost packets must also be considered. Ignoring lost packets (i.e., computing the delay statistics only on correctly delivered packets) may lead to an optimistic evaluation of the network performance under heavy traffic, where few packets actually reach the destination. In this case, a high-loss path might end up being considered better than a more reliable path with a slightly higher delivery delay. The other extreme, i.e., defining the delay contribution of a lost packet as infinite, makes the delay evaluation meaningless since the average delay would also be infinite for any positive loss probability. Clearly, neither option is desirable in our case.

Therefore, we propose another definition that gives a finite bias to the average delay in case of a packet loss. In particular, when a packet is lost when going from $D^{(i)}$ to $D^{(j)}$, we increase the delay of the following packet in the same path by the interarrival time between packets routed on that path. [1] This additional delay is given by $(N-1)/\lambda$, i.e., the inverse of the per-path average traffic intensity (recall that each packet generated at $D^{(i)}$ has a randomly chosen destination among the remaining nodes of the network, so that the per-node traffic λ needs to be divided by the number of possible destinations, $N-1$).

According to this reasoning, we recursively define the *weighted delivery delay* of a data packet sent via multi-hop transmission by node $D^{(i)}$ to node $D^{(j)}$ as:

$$\mathcal{W}^{(i,j)} = \left(1 - p_c^{(i,j)}\right)\zeta^{(i,j)} + p_c^{(i,j)}\left(\frac{N-1}{\lambda} + \mathcal{W}^{(i,j)}\right) . \tag{7}$$

In this calculation, the channel and interference conditions, and thus the loss probability, are assumed to be independent for different packets. This is due to the fact that the destination for each packet is chosen at random, and the time between two subsequent packet transmissions over the same path is deemed to be long enough.

From Eq. (7) we obtain:

$$\mathcal{W}^{(i,j)} = \frac{N-1}{\lambda}\frac{p_c^{(i,j)}}{1-p_c^{(i,j)}} + \zeta^{(i,j)} . \tag{8}$$

The delivery delay $\zeta^{(i,j)}$ and the loss probability $p_c^{(i,j)}$ depend on the nodes that the routing protocol selects as relays. In a static network, it is possible to estimate these values during a training period. Instead, if the network is dynamic (mobile nodes or time-varying traffic statistics), this is not possible. We propose a different way of estimating the delay and the loss probability, based only on instantaneous geographic and routing information. Since a packet sent over a multi-hop path has to traverse a number of nodes before reaching the destination, we make the assumption that both the overall path delivery delay and the overall path loss probability can be decomposed into contributions given by the various traversed nodes. More precisely, the overall delivery delay is given by the sum of the average delays required to traverse every single node (time in queue plus transmission time), whereas the overall loss probability is obtained from the loss probabilities at every node (probability of transmission failure and probability of buffer overflow). If $\mathcal{R}^{(i,j)}$ is the set of nodes belonging to the path between $D^{(i)}$ and $D^{(j)}$ (excluding $D^{(i)}$ and $D^{(j)}$), we have:

$$\zeta^{(i,j)} = \zeta_q^{(i)} + \sum_{h\in\mathcal{R}^{(i,j)}} \zeta_q^{(h)} , \tag{9}$$

where $\zeta_q^{(h)}$ is the average time between the arrival of a packet at node $D^{(h)}$ and its reception at the next hop. This delay depends on the next relay; indeed, while the time needed for traversing the queue is the same for all packets, the time required for a successful transmission depends on the channel condition, and hence on the next hop chosen. We estimate $\zeta_q^{(h)}$ averaging over all the possible next-hop relays, thus over all the neighbors of node $D^{(h)}$.

The packet loss in the multi-hop path is calculated in a similar way, i.e.,

$$p_c^{(i,j)} = 1 - (1-p_t^{(i)})(1-p_q^{(j)}) \prod_{h\in\mathcal{R}^{(i,j)}} (1-p_t^{(h)})(1-p_q^{(h)}) , \tag{10}$$

where $p_t^{(h)}$ is the probability that a transmission from node h to the next hop fails because the maximum number of retransmissions is reached, and $p_q^{(h)}$ is the probability that a packet correctly received at node $D^{(h)}$ is discarded due to buffer overflow. Furthermore, we notice that $p_q^{(h)}$ depends on the queue of the receiving node $D^{(h)}$, while $p_t^{(h)}$ depends also on which node is used as next hop. For this reason, similarly to what we have done for $\zeta_q^{(h)}$, we consider a value averaged over all the neighbors of $D^{(h)}$.

With (9) and (10) we can calculate the weighted delivery delay $\mathcal{W}^{(i,j)}$, defined in (8). This parameter should be estimated for each couple of nodes, with a sufficiently long training period. From (8), we define $\overline{\mathcal{W}}$ as the average over all the couples of nodes belonging to the network. This will be used in the following as the performance metric of the whole network.

B. Stochastic estimation of local parameters

In a real network, the values of the parameters $\zeta_q^{(i)}$, $p_t^{(i)}$, and $p_q^{(i)}$ should be estimated based on local information. Our idea is to use some parameters that can be easily calculated at each node $D^{(i)}$. We consider in particular the number of

[1] Equivalently, we assign to lost packets a delay contribution equal to the interarrival time, to received packets the actual delay incurred, and then divide the sum of all contributions by the number of correctly received packets only.

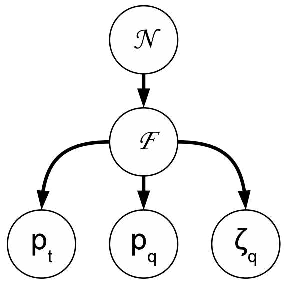

Fig. 1. Bayesian Network showing the probabilistic relationships among the 5 parameters of interest: ζ_q, p_t, p_q $\mathcal{F}$, and $\mathcal{N}$.

flows $\mathcal{F}^{(i)}$, that can be easily calculated from the routing table, and the number of neighbors, $\mathcal{N}^{(i)}$. We have estimated the probabilistic relationships among ζ_q, p_t, p_q, $\mathcal{F}$, and $\mathcal{N}$. Notice that we removed the dependence on the specific node. In fact, the Bayesian Network approach exploits the collected data, which are specific for each node, to find out the correlation between the local parameters and the values of $\mathcal{N}$ and $\mathcal{F}$. The result is a set of general conditional distributions (one per each local parameter) which can be therefore applied to any node of the network. It follows that once the number of flows or neighbors of a given node is known, the distributions of $\zeta_q^{(i)}$, $p_t^{(i)}$, and $p_q^{(i)}$ for that node are also known.

We first collected the measures of these parameters in our scenario as a function of the traffic load λ, for different topologies. Then we calculated the structure of the Bayesian Network (BN). We should notice that this procedure is different from using a training period to directly derive the local parameters. In fact, in this case a training period would be needed every time the topology changes, so as to evaluate their value for each specific node or path. On the contrary, with our procedure we can estimate the general joint probability among these parameters, that does not depend on the specific topology.

The structure of the BN is reported in Fig. 1. The structure of this BN is the same for all the values of λ, while quantitatively the probabilistic relationships change with λ. We notice that $\mathcal{N}$ does not influence, to a first approximation, the values of the three performance parameters, once the value of $\mathcal{F}$ is observed. In other words, once we calculate from the routing table the value of $\mathcal{F}$, we can have an estimate of the probability distribution of the three performance parameters. From these estimated parameters, we can calculate also the overall network performance $\overline{\mathcal{W}}$.

C. Cooperation

When cooperation is exploited, some nodes are shared between the two networks, and the routing tables calculated via OLSR change accordingly. By using the framework introduced above, we can estimate the overall performance of the two networks with and without cooperation. We denote with $\overline{\mathcal{W}}_k(\mathcal{D}_1, \mathcal{D}_2)$ the weighted delivery delay of network k, with $k = 1, 2$, when the two networks share the set of nodes $\mathcal{D}_1$ and $\mathcal{D}_2$, respectively. In particular, $\overline{\mathcal{W}}_k(\emptyset, \emptyset)$ is the performance metric of network k when no nodes are shared. Thus, for any choice of the nodes shared we can calculate

TABLE I
SIMULATION PARAMETERS

Number of nodes per network	10
Transmission power [dBm]	24
Noise floor [dBm]	-103
Modulation used	BPSK
Time slot duration [ms]	1
Packet length [bit]	4096
λ [pkt/s/node]	1 to 5
Spreading factor	16
Fading correlation factor ρ	0.9

the variation in $\overline{\mathcal{W}}_k$ for the two networks. Then, we can model the cooperation strategy by means of Game Theory, by considering each network as a selfish agent whose utility function can be any decreasing function of $\overline{\mathcal{W}}_k$. To sum up, the following steps are followed in our framework:

- we learn the network behavior by measuring the parameters of interest over several random topologies with fixed setup;
- we use the BN method to infer the joint distribution among ζ_q, p_t, p_q, $\mathcal{F}$, and $\mathcal{N}$;
- we evaluate the utility functions $\overline{\mathcal{W}}_k(\mathcal{D}_1, \mathcal{D}_2)$, for the two networks $k \in \{1, 2\}$, for all the possible choices of the sets $\mathcal{D}_1$ and $\mathcal{D}_2$.
- we select the two subsets $\mathcal{D}_1$ and $\mathcal{D}_2$ to be shared, based on the game theoretic approach described in Section IV-D.

D. Game theoretic approach

The problem is formulated as a repeated 2-player game, where the players are the two networks. We name the nodes of the networks from 1 to $2N$, where the nodes in the sets $\mathcal{Q}_1 = \{1, ..., N\}$ and $\mathcal{Q}_2 = \{N+1, ..., 2N\}$ belong to network 1 and 2, respectively. The strategy of each network is represented by the set of nodes $\mathcal{D}_1$ and $\mathcal{D}_2$ they decide to share, therefore in the most general formulation the strategy sets are the power sets $\mathcal{S}_1 = 2^{\mathcal{Q}_1}$ and $\mathcal{S}_2 = 2^{\mathcal{Q}_2}$. The utility function of each network, $u_k : 2^{\mathcal{Q}_1} \times 2^{\mathcal{Q}_2} \to \mathbb{R},\ k = 1, 2$, is the reciprocal of the average weighted delay per path for that network, that is, $\overline{\mathcal{W}}_k^{-1}(\mathcal{D}_1, \mathcal{D}_2)$. Each of these metrics jointly depends on the strategies of both players: if a network decides to share a given node, that node is loaded by the traffic of the other network that passes through it. On the other hand, an additional shared node decreases the overall amount of traffic that passes through the other nodes.

In this paper, we assume for simplicity that the networks do not have the freedom to choose the number of nodes to share. They can share either no nodes or exactly 2 nodes, therefore the cardinality of each strategy space is $\binom{N}{2}+1$. Although our approach can be extended to a larger number of cooperating nodes, our numerical results show that a large fraction of the available cooperation gain is already achieved with this simple choice.

If we consider a single stage of this game it is immediate to see that the unique NE is the strategy profile $s = (\emptyset, \emptyset)$, i.e., no network cooperates. In fact, given the strategy of the

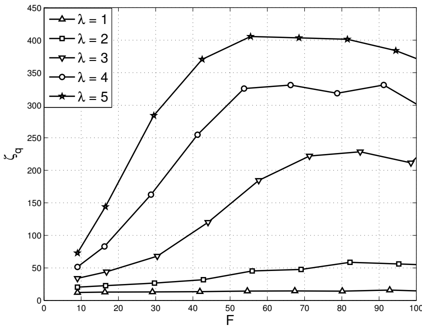

Fig. 2. BN estimation of the average delivery delay ζ_q as a function of the number of flows $\mathcal{F}$ passing through the node.

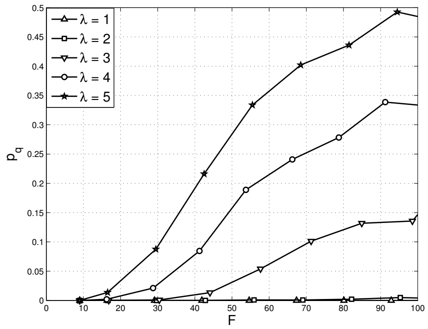

Fig. 3. BN estimation of the probability of buffer overflow p_q as a function of the number of flows $\mathcal{F}$ passing through the node.

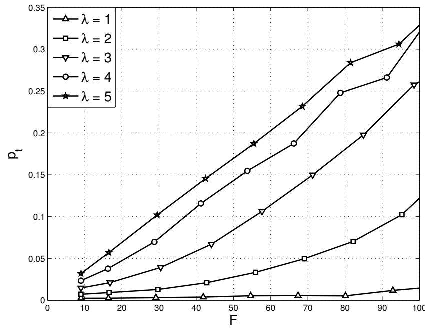

Fig. 4. BN estimation of the probability of transmission failure p_t as a function of the number of flows $\mathcal{F}$ passing through the node.

other, each network prefers to share no nodes in order not to increase the total traffic through its nodes. However, in the repeated formulation it can be shown that each strategy profile that allows to reach a better utility for both players is a NE. A player deviating from that strategy profile can be punished by the other player during subsequent stages. The duration of this punishment can be set so that the gain obtained during the deviating stage does not compensate the loss during the subsequent stages. Punishment strategies in repeated games allow multiple equilibria with varying utilities for each player.

Inspired by the Nash bargaining solution [10], we decide to maximize the product $\left(u_1 - u_1^{NC}\right)\left(u_2 - u_2^{NC}\right)$, where u_1^{NC} and u_2^{NC} are the status quo utilities, i.e., the utilities $\overline{\mathcal{W}}_1^{-1}(\emptyset,\emptyset)$ and $\overline{\mathcal{W}}_2^{-1}(\emptyset,\emptyset)$ obtained when networks do not cooperate. We additionally impose the mathematical constraint $u_k - u_k^{NC} \geq 0$, $k = 1, 2$, to avoid the situation where the maximum corresponds to a decrease in the utilities of both networks. The solution found results in increased utilities for both networks compared to the non cooperative case, therefore it is a NE for the repeated game formulation.

V. Results

In this section we present the simulation setup and the main results of our approach for cooperation.

A. Simulation Setup

In order to prove the effectiveness of our cooperation strategy, we developed a network simulator which encompasses the layers from physical to routing, as described in Section III. The system parameters are reported in Table I. Each simulation run is performed with randomly generated connected networks, and lasts for 10000 time slots, including an initial transient phase. Different values of the traffic generation intensity λ were considered, from 1 packet/s, corresponding to a lightly loaded network, up to 5 packet/s, which is instead the case of an overloaded network. In each scenario, 500 simulation runs were performed to collect the data required for the BN inference. Based on this information, the empirical distributions and the average values of ζ_q, p_t and p_q, conditioned on $\mathcal{F}$, were derived.

In the subsequent steps, a new set of 500 simulation runs was performed for each value of λ. In each run, two networks are again randomly deployed; the overall system performance is theoretically evaluated by computing the values of $\overline{\mathcal{W}}_k$, based on the routing tables, and the values of $\overline{\mathcal{W}_k}(\mathcal{D}_1, \mathcal{D}_2)$, with $k \in \{1, 2\}$, where $\mathcal{D}_1$ and $\mathcal{D}_2$ are the optimal sets of nodes to be shared, according to the game theoretic framework proposed in Section IV.

The aim is to verify how much gain is achievable with our approach with respect to a random selection of the nodes shared and a fully cooperative strategy. Therefore, the network performance obtained by using our Game Theoretic node selection strategy is compared to those achieved by the following strategies: 1) no cooperation, 2) two nodes shared, randomly chosen by each network, and 3) all nodes shared.

B. Bayesian Network estimation

Exploiting the stochastic estimation of local parameters through the BN approach proposed in Section II-A, we can evaluate the expected value of the three parameters of interest, namely the average delivery delay ζ_q, the probability of buffer overflow p_q and the probability of transmission failure p_t, as a function of the number of flows $\mathcal{F}$ passing through the node and of the traffic intensity λ. The expected values of ζ_q, p_q, and p_t are shown in Figs. 2, 3 and 4, respectively. We notice that the highest number of flows through a single node is reached when that node becomes the only connection among three separate clusters of nodes. If these groups have similar cardinalities, and the number of nodes in each network is N, we can rise up to a maximum of about $4(N-1)^2/3$ flows through a single node, that is close to the maximum value of $\mathcal{F}$ represented in the figures. We also observe in Fig. 2 that for very high values of $\mathcal{F}$ and λ, the average delivery delay decreases. We conjecture that this happens for two reasons: (1) the queue of these nodes are always almost full, so that the time to traverse them cannot grow much further, whereas (2) a node traversed by a high number of flows is often chosen as receiver by most of his neighbors. For these reasons, when it transmits, a lower number of communications can interfere, thus leading to a lower time needed to deliver a packet to the next hop.

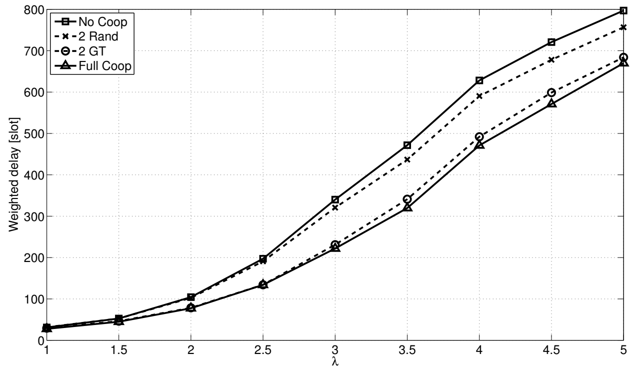

Fig. 5. Weighted delay as a function of the packet generation intensity λ, for the four compared scenarios: with no nodes shared (No Coop); with two nodes shared, randomly chosen (2 Rand); with two nodes shared, chosen via Game Theory (2 GT); and with all the nodes shared (Full Coop).

C. Cooperation performance

In Fig. 5, we present the actual gain, in terms of delay reduction, offered by the considered cooperation strategy. The curves are obtained by averaging over 500 random topologies, each consisting of two networks of $N = 10$ nodes. The other system parameters are reported in Tab. I. We plot the average weighted delay of each network (due to the symmetry of the scenario, it is not necessary to distinguish between the networks) in four different cases, that is: (1) when no nodes are shared, namely No Coop; (2) when 2 nodes randomly chosen are shared, namely 2 Rand; (3) when 2 nodes, selected through the proposed Game-theoretic approach, are shared, namely 2 GT; (4) when all nodes are shared, namely Full Coop.

It can be observed that, as intuition suggests, full cooperation grants the highest benefits, due to the higher diversity. Hence, this is the maximum achievable gain for the scenario investigated. This gain is more pronounced when the networks are heavily loaded, since congested paths are more frequent, and spatial diversity becomes more advantageous.

When only two nodes can be shared, the choice of the shared nodes makes the difference. In fact, Fig. 5 shows that a careful selection of the resources to be shared can significantly increase the achievable gain when compared to a blind random selection. A random selection can not offer a significant gain for lightly loaded networks, while, for heavily loaded networks, it can offer only one third of the gain granted by full cooperation. On the contrary, if the same number of nodes are shared, but chosen by means of our game-theoretic approach, the maximum achievable gain is fully obtained for lightly loaded networks and closely approached for heavily loaded networks.

VI. Conclusions

In this paper, we developed a framework which can be used to select the best cooperation strategy between two coexisting wireless networks sharing some of their nodes. Bayesian Network theory was used to derive the statistical correlation between local parameters and global system performance. Based on this information, a game theoretic selection of the nodes which can guarantee the highest benefit was made. Even when a small fraction of nodes is shared, we obtained a significant gain. In particular, both for lightly and heavily loaded scenarios, the selection scheme based on Game Theory can achieve almost the same performance as a full cooperation scheme.

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