# Discounted Age of Information for Networks of Constrained Devices

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***Abstract*—Remote sensing systems can exploit age of information as a performance metric to track the freshness of the data exchanged. However, sensing terminals often have a finite time span to perform their task, for example due to their limited batteries or because of mission-critical contexts. A realistic model of such scenarios ought to consider the importance of a performance metric evaluated over time, with lower weight given to future evaluations. We propose to take into account these aspects through a discounted age of information. We show how this admits some non-trivial closed form expressions, and we consider the problem of minimizing a linear combination of the discounted age of information with a transmission cost term for two different scenarios of interest, a single terminal case, and a slotted ALOHA medium access. We discuss the implications of system parameters on this metric, and its overall characterization of the resulting system performance.**

***Index Terms*—Age of Information; Internet of Things; Data acquisition; Networks; Modeling.**

## I. Introduction

A recent trend has emerged in the analysis of remote sensing that relates to the use of the *age of information* (AoI) as a performance metric for scenarios where terminals send sporadic updates to a data collection point [1]. This is the case in many applications of the Internet of Things (IoT) such as industrial, agricultural, or medical IoT [2].

In most of the related literature, we can identify three directions for the application of the general idea that AoI is worth considering as an objective for the network management, which might lead to different conclusions that a standard throughput or delay optimization. First, there are some seminal papers that illustrate the purpose of AoI and its usefulness in some specific scenarios like those already mentioned [3]–[5]. However, as a second line of investigaton, because of its advantage of allowing closed-form expressions, the AoI is also often used as a characterization metric, mostly concerning the data link layer. This ground is particularly fertile as it allows for many variations on the basic AoI subject, i.e., including energy harvesting [6]–[8], automatic repeat request [9], [10], or possible game theoretic approaches for a distributed management [11], [12]. Finally, a third and last outcome of employing AoI is its quantitative performance analysis in comparison with other terms to assess costs and benefits when acquiring new data [13], [14].

Our contribution in the present paper is to introduce a theoretical extension that may advance the state of the art under all these three aspects. The starting point of our investigation is that most of the related papers discuss AoI over an infinite horizon, often resorting to a long-term average computation. We argue that this fails to represent practical situations where the evolution of the system monitoring cannot be tracked in a constant way for an infinitely long time. Indeed, the operation of sensing devices is often limited, for several possible reasons, like a restricted lifetime due to a finite battery [15] or deterioration of components [16], or a time span between recharges [17]. Also, when mission-critical operations are considered, future evaluations can be considerably less important than present-time ones [18], [19]. To keep this into account, we investigate the introduction of a *discount factor* in the AoI computation and we investigate the analytical consequence of this extension in the formulas.

As a result, our model can be considered as an augmentation suitable for many scenarios where infinite duration of the sensing operation is not appropriate. We show that this can still be framed as the usual formulas for AoI evaluations, albeit with some mathematical intricacies, which nevertheless admit closed-form derivations. Finally, we present the comparison of this extended metric with cost parameters [9], to investigate how some results of AoI-based network management extend in this case.

In particular, we will show some analytical results for two standard settings, i.e., a single terminal sending updates, and a network of terminals under a slotted ALOHA medium access constraints. These are just sample references, yet they can be regarded as representative of other realistic cases in the Internet of Things, with similar rationale. For example, even when modern random-based access control protocols are considered, the results are still based on those for classic ALOHA-like scenarios [20], [21].

More in general, we will discuss how our proposed introduction of a discount factor in the AoI can lead to more challenging problems related to a better physical characterization of practical scenarios, also possibly considering dynamic multi-agent or multi-objective optimizations [1], [19], [22], [23].

## II. Discounted AoI – Analysis

Consider a discrete (slotted) time where information updates can be sent from a general terminal to a sink. The AoI is defined as the difference between the current time index and that of the last slot where a successful update occurred. This also means that the AoI is equal to 0 in all the time slots where a successful update occurs.[1]

[1]Throughout this paper, we consider both a discrete time and also, as a result, a discrete-valued AoI [13], [17]. Thus, the formulas are slightly different, but substantially equivalent, to papers such as [3], [4] where the AoI is taken as a continuous value.

The success of an update is a stationary random process, which means that, whenever an update is attempted, it is successful with conditional probability $\mathsf{P}_{\text{succ}}$, which is assumed to be independent and identically distributed (i.i.d.) for all the attempts. Thus, if we denote with $p$ the probability that an update is attempted (which, from a sensor's perspective, is nothing but the transmission probability) in a given slot, we can denote the probability of a successful update as $\rho = p\mathsf{P}_{\text{succ}}$. For the sake of simplicity, we focus on the case of sensors acting without any coordination or information on the current AoI, so they set their probability $p$ and attempt an update with i.i.d. probability $p$ for all the slots, which is a quite standard setup in the literature [5], [15], [24].

This discrete-time representation prompts a direct representation through renewal processes, since the AoI simply increases by one at each slot, unless a successful update occurs, in which case it is reset to 0. Thus, the average AoI can be computed as the average of the reward $\delta$ of a renewal process over the average duration of a renewal cycle, where $\delta$ is increasing by 1 at each step and is set to 0 at every renewal (successful update, happening with probability $\rho$). The duration of a renewal cycle has geometric distribution, i.e., an update occurs after $\kappa + 1$ slots with probability $\rho(1-\rho)^{\kappa}$, with $\kappa = 0, 1, \ldots$.

We can introduce a *discount factor* $a$, whose meaning is to devalue the future over the present. Discount factors are used in economics, but also in game theory, infinite-horizon optimizations, and reinforcement learning [9], [25], [26], to represent a decrease in value over time, since it is commonly assumed that present-day benefits are more important for an individual than future ones. This is also relevant in a scenario where AoI is to be minimized, since AoI indefinitely increases if no update is performed [3], but this is only attained asymptotically, and it may be less relevant if the horizon of interest is limited to few interactions. On the other hand, the only way to decrease the AoI is to perform an update, which can be assumed to have a cost. In this situation, a present expense is borne to obtain a benefit (lower AoI) in the future. We will discuss how, when the discount factor has a strong impact on the future evaluations, it may be more convenient to limit the updates.

The discount factor $a$ is chosen such that $0 < a < 1$, and geometrically multiplies all future values, so that any metric computed at discrete time slot $t = 0, 1, 2, \ldots$ is weighted as $a^t$. If we consider a *discounted* AoI with such a discount factor $a$, we can apply the same reasoning and compute the average discounted AoI as the average *discounted* reward of a renewal process divided by the average *discounted* duration of the cycle [26]. In other words, we are looking for a value, denoted as $D_a$, and computed as

$$D_a = \frac{\displaystyle\sum_{\kappa=0}^{\infty}\left(\sum_{j=0}^{\kappa} j a^j\right)\rho(1-\rho)^{\kappa}}{\displaystyle\sum_{\kappa=0}^{\infty}\left(\sum_{j=0}^{\kappa} a^j\right)\rho(1-\rho)^{\kappa}} \tag{1}$$

where one can exploit the notable sums

$$\sum_{j=0}^{\kappa} j a^j = a\frac{1-(\kappa+1)a^{\kappa}+\kappa a^{\kappa+1}}{(1-a)^2}, \quad \sum_{j=0}^{\kappa} a^j = \frac{1-a^{\kappa+1}}{1-a}$$

and after some algebra obtain

$$D_a = \frac{a}{(1-a)^2}\rho(1-a+a\rho)\left(\frac{1}{\rho} - \frac{1}{1-a+a\rho} - \frac{a(1+a)(1-\rho)}{(1-a+a\rho)^2}\right) \tag{2}$$

Despite its more complicated expression than the regular AoI for a slotted time, we remark that this value $D_a$ obeys the following properties, which are immediate to verify.

*Property 1:* The limit value of $D_a$ for $a \to 1^-$ is $\rho^{-1} - 1$, which corresponds to the case without any discounting [13].

*Property 2:* The limit value of $D_a$ for $\rho \to 0^+$ is $a/(1-a)$. This implies that when the information is never updated, the value of the discounted AoI is still kept limited, which is different from the AoI without discount, that goes unlimited.

*Property 3:* $D_a$ is decreasing in $\rho$. This follows the same trend of the AoI without any discounting, as is pretty intuitive.

According to the last property, to minimize the discounted AoI, the best sensing policy will be to maximize the probability of a successful update, which does not make any difference from the non-discounted case. However, similar to that case, a limitation to an aggressive updating policy can be given by the combination of $D_a$ with a transmission cost, and its differences with the standard analysis will be discussed next. To do so, we consider two special cases, namely, that of a single terminal (or access without collisions) and a slotted ALOHA medium access, and we discuss them in relation to the implications of employing the *discounted* AoI instead.

### A. Single terminal

We consider a scenario where a terminal is sending updates that are always successful. This corresponds to a single terminal case, or a setting where sporadic updates take place at the application level, over a collision-free medium. The only factor that prevents such a device from constantly updating the information in every slot is the transmission cost.

Thus, we can set $\rho = p$, where $p$ is the transmission probability of the single terminal. In other words, we assume $\mathsf{P}_{\text{succ}} = 1$. In such a case, the minimum possible value of the AoI is obtained for $p = 1$, and this is also true for $D_a$, which is decreasing in $\rho$ according to (2). However, a way to regulate the transmission probability of the terminal might be to consider a cost for sending updates, proportional to a coefficient $c$. This implies that the terminal is trying to minimize a weighted sum of the expected AoI and cost. If we consider the expression of the discounted expected AoI for this computation, we are looking for the solution of

$$\min\left(D_a + cp\right). \tag{3}$$

It can be noted that $D_a$ is decreasing in $p \in [0, 1]$, thus when $c = 0$, one can minimize $D_a$ by trivially setting $p = 1$, but if a higher cost is introduced, (3) is solved when

$$-\mathrm{d}D_a/\mathrm{d}p = c. \tag{4}$$

In other words, $c$ imposes a specific transmission probability $p^* < 1$ to the terminal, which is the solution of (4).

The interpretation of this result is two-fold. On the one hand, $c$ can be seen as an exogenous parameter that depends, for example, on the energy expenditure to perform a transmission. In that case, $p^*$ is simply determined as a side effect [9]. On the other hand, one can also consider $c$ to be a tunable parameter (especially if it is introduced as a

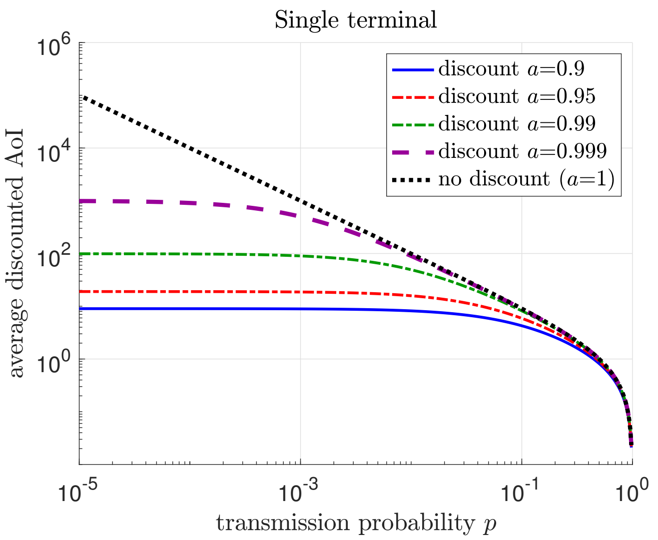


Fig. 1. Average discounted AoI $D_a$ vs. the transmission probability $p$, single terminal case.

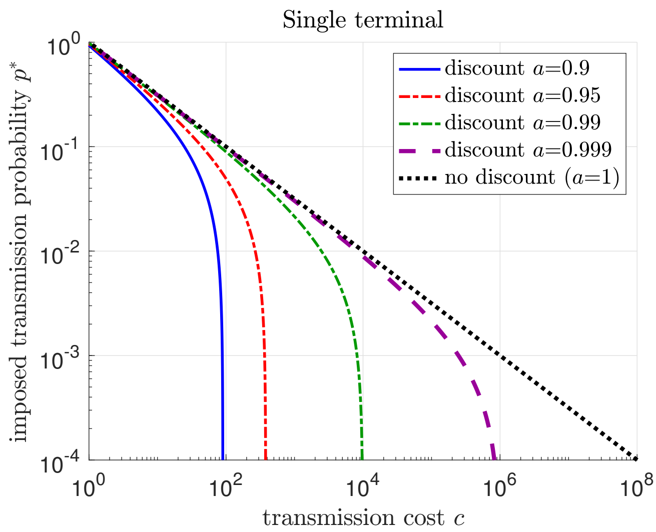


Fig. 2. Imposed transmission probability $p^*$ vs. the transmission cost $c$, single terminal case.

*virtual* cost, unrelated to physical consumption) and use it to regulate the frequency of updates sent by the terminal [13].

To gain some numerical insight, Fig. 1 plots the average discounted AoI $D_a$ for different values of the discount factor $a$, as a function of the transmission probability $p$. The case of non-discounted AoI is plotted for comparison. The curves overlap for $p \to 1$, while the asymptotic behavior for $p \to 0$ is different, as per Property 2.

Fig. 2 presents the value of $p^*$ as a function of the cost parameter $c$, according to (4). For the non-discounted AoI, a higher cost implies imposing an ever decreasing transmission probability $p^*$ that never reaches 0, though, whereas this trend is much more rapid for the discounted case, and in particular, there is a threshold value of $c$ that gives $p^* = 0$. Therefore, if the cost turns out to be higher than this threshold, the device never sends an update since the cost for doing so is not worth the ultimate benefit of a lower discounted AoI.

### B. Slotted ALOHA

These results can be extended by considering a slotted ALOHA medium access. This is a common reference for AoI investigations, and there can be expansions also considering more sophisticated medium access protocols, still based on the rationale of ALOHA [4], [5], [20], [21].. In this case, we assume $N$ identical terminals to be perfectly syncrhonized over the slotted time axis, where they all randomly send updates in each slot with probability $p$. The probability that a transmission is successful depends on the medium being collision-free, which means that $\mathsf{P}_{\text{succ}} = (1-p)^{N-1}$. Thus, $\rho = p(1-p)^{N-1}$.

Hence, not all updates are successful, and an aggressive transmission policy $p = 1$ leads to $\rho = 0$. In the classic case without discount, it is immediate and well-known to derive that the best choice is $p = 1/N$.[2] However, this is true when no cost is considered for the transmission. We can keep the same approach of the previous subsection and consider a transmission cost $cp$, i.e., proportional to the frequency of transmission, so that the single terminal is trying to solve (3).

Note that the terminals are assumed to be identical in terms of their objectives and also the discount factor, so they all use the same optimal value $p^*$. Also, it is still true that $D_a$ has a dependence on $p$, albeit more complicated than before, and therefore the result of $p^*$ is obtained through (4).

To assess the resulting performance, we focus on a sample case with $N$=10 and we consider once again the average discounted AoI $D_a$ for different values of the discount factor $a$, as a function of the transmission probability $p$, which is reported in Fig. 3. It is visible that the minimum AoI is obtained for $p = 0.1$, which is $1/N$, even for the discounted case, which is in accordance to Property 1 of (2).

However, when a cost term $c$ is introduced, the choice of $p^*$ is strongly impacted by the discount factor. In fact, setting the transmission probability under a non-zero transmission cost would correspond to moving away from the minimum of the curves in $p = 0.1$, towards the left. The lower $a$, the less steep the slope of the curve, which implies a lower resistance towards decreasing $p$ once a higher transmission cost is set. This is visible also in Fig. 4 where the required $c$ to obtain a certain $p^*$ is plotted versus the value of $p^*$ itself. It is evident that for lower discounts, the required cost becomes lower; in other words, a terminal seeking to minimize the discounted AoI is more inclined to decreasing the transmission probability when the cost increases.

## III. Future Developments

We showed how to compute the discounted AoI for a remote sensing system, and remarked how this implies interesting extensions of the analysis. The idea of applying a discount to the AoI can be inserted in any existing analysis. Although this admits closed-form expressions, the equations become more complicated. Still, a numerical analysis is easy to perform, and offers the advantage of realistically representing a scenario where the terminals are more concerned of the present expenditure for transmitting an update than its benefits of decreasing the AoI in the long run.

[2]More specifically, it is intuitive that the non-discounted case, where the AoI is equal to $\rho^{-1} - 1$, has a minimum when $\rho$ is maximized, which is immediately found to happen when $p = 1/N$.

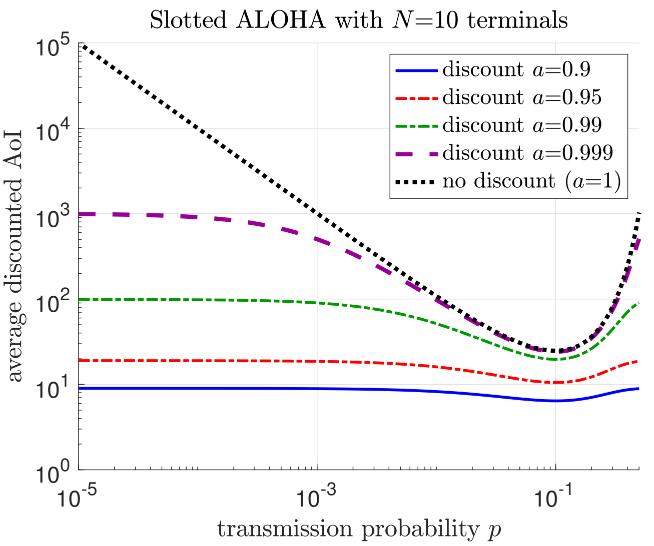


Fig. 3. Average discounted AoI vs. the transmission probability, slotted ALOHA with $N = 10$ terminals.

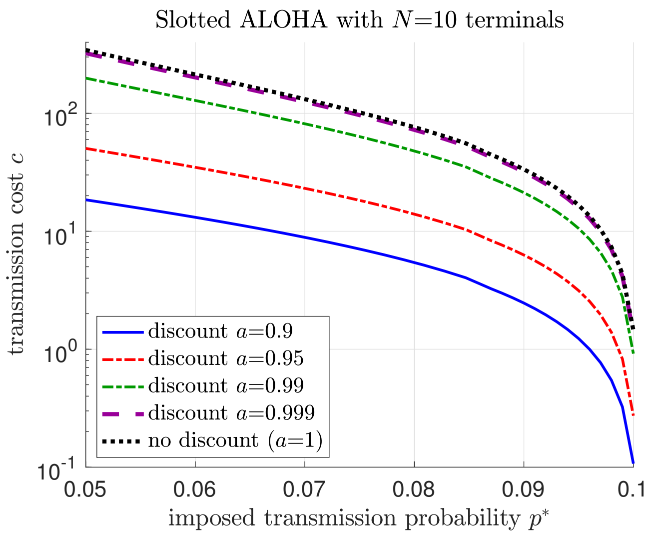


Fig. 4. Transmission cost $c$ vs. imposed transmssion probability $t^*$, slotted ALOHA with $N = 10$ terminals.

This contribution is not limited to the extension of the mathematical relationships, but can also open the door to further investigations. It may be interesting to relate the duration of the time horizon to the lifespan of the terminal's battery, which in turns depends on the rate at which updates are sent. In other words, instead of being unrelated parameters, the discount factor $a$ and the transmission cost $c$ can be put in relationship depending on the battery consumption [9]. This looks like an interesting extension for future work.

Another aspect that may be worth considering is the attitude towards an aggressive updating approach represented by the discount factor, whose lower value ultimately represents an intention to be less persistent when the update is costly. In a multi-agent scenario, this can be seen as a Bayesian type of the terminal [23], which the other terminals may want to discover or keep into account.

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