

# Intrinsic Price of Anarchy of Age of Information for Converging Sources With Individual Costs

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**Abstract**—This paper considers a sensor network for real-time monitoring, where multiple sensors can act as equivalent information sources for a common receiver. The sensing goal is to achieve minimal freshness of status updates at the receiver's end, which is captured through the metric known as age of information (AoI). A distributed uncoordinated management is applied to the sensors, so that they send their reports independently and according to a memoryless process. Thus, nodes share the common objective of minimizing AoI but at the same time they incur an individual transmission cost when sending their updates. The goal of the analysis is to evaluate the intrinsic price of anarchy of such a distributed management, even when the nodes are in the best possible conditions, i.e., the information is converging, free from errors and collisions, and fresher updates always pre-empt older ones.

**Index Terms**—Age of Information; Queueing theory; Game theory; Remote sensing; Wireless sensor networks.

## I. INTRODUCTION

Age of information (AoI) [1] is a metric quantifying the timeliness of status updates reported by a remote information source to a receiver. It has gained popularity in the last decade especially for real time applications, which heavily relies on system controllers and actuators having up-to-date ambient information [2]–[4].

AoI is defined as the time elapsed since the generation of the most recent update, i.e., if the information source(s) send updates to a collecting point at times  $\tau_1, \tau_2, \dots, \tau_n, \dots$ , and these are received at respective time instants  $r_1, r_2, \dots, r_n, \dots$ , then the value  $\delta(t)$  of AoI at time  $t$  is

$$\delta(t) = t - \tau_j \quad \text{where: } j = \arg \max_i \{r_i < t\}. \quad (1)$$

Synthetic quantities derived from  $\delta(t)$ , such as the average AoI  $\Delta = \mathbb{E}[\delta(t)]$ , offer interesting assessments of the system performance. Sometimes, analogous values such as the Peak AoI [5] and/or the AoI violation probability [6] are used to this end as well. The purpose is to give an analytical description of the information freshness, which is ultimately impacting the system reactivity. In many scenarios, like vehicular networks [7], industrial Internet of things [8], and every time a timely alerting is required, such as surveillance or medical monitoring [9], this approach turns out to be actually more

fitting than using classic indicators such as average throughput or delay.

The literature offers evaluations of AoI-related quantities through queueing theory, framing different systems as queues with various disciplines and pre-emption rules [10]–[12]. The analytical character of AoI formalization through queueing theory makes it also particularly prone to an investigation via game theory [13]–[15]. However, as argued later, these approaches consider a game played by multiple agents, each interested in their own individual AoI value. Naturally, this leads to a non-cooperative game theoretic scenario with constrained resources, which can be further exacerbated by collisions [16] or, even in the absence of them, by buffer congestions [17]–[19]. In other words, this is a typical *tragedy of the commons* [20], where players are indirectly interested in trumping over others to push their own content.

Conversely, this paper considers a *converging source* scenario, where multiple nodes monitor and transmit equivalent content, and there is a single AoI value that all nodes are able to reset upon transmissions, even though they incur an individual cost when doing so [21]. The goal is to illustrate the *intrinsic* price of anarchy (PoA) of these distributed operations. To this end, the focus is on the most optimistic case of an M/M/1\* queue according to [22], without collisions among nodes or queueing delays, and the only inefficiency is the lack of coordination of the nodes. That is, they all want to achieve a low value of the common AoI, but at the same time minimizing their individual costs [23].

It is shown that there is indeed a PoA inherent to distributed transmission by multiple sources, which grows with the number of nodes and the transmission costs, reaching an asymptotic value of  $1 + \sqrt{c}$ , where  $c$  is the shadow price (i.e., the unit cost) of transmission, for a very large number of sources. This is still a consequence of the tragedy of the commons principle, and implies that real cooperation can be achieved only through proper incentives offered to the sensing sources for their service [24].

Beyond the analytical formalization, this result has also deep implications in practical scenarios like mobile crowdsensing or participatory federated learning [25], where the distributed contribution to the global system knowledge by individual nodes is actually not guaranteed and may require solutions to ensure fairness [26]. Indeed, if nodes are driven by individual

objectives, status reporting may be stale due to the intrinsic inefficiency of a distributed management [27], as confirmed by the analysis presented here.

The rest of this paper is organized as follows. Section II reviews the related literature. The system model is presented in Section III, also formalizing the resulting  $N$ -player complete information game, finding its Nash equilibrium, and the PoA. Numerical results are shown in Section IV. Section V concludes the paper.

## II. RELATED WORK

A multi-variable objective function, whose variables can be assumed to be controlled by different agents, calls for an immediate application of game theory [16]. As such, even seminal studies on multi-source AoI [13] already propose the evaluation of a Nash equilibrium (NE), also arguing about its Pareto inefficiency. That is, competition among greedy players naturally lead to a worse outcome than an optimal allocation requiring coordination. However, the underlying scenario in these cases is that of a competition for accessing a shared resources by multiple sources [20], each of them interested in minimizing the age of their information.

For example, [15] considers multiple sources as distributed players driven by the minimization of their AoI value, but they all pursue different individual objectives. This falls within the game theoretic narrative that competition in a scenario without any explicit trust (either coordination or a cartel, in networking or economic terms, respectively) leads to a Pareto-inefficient NE. In this same spirit, [17] proposes a game theoretic rate control mechanism that limits the competition to improve the efficiency of a distributed solution, and [7] accounts for multiple AoI values of individual nodes when optimizing UAV-aided vehicular edge computing, using game theory to shape decision-making for efficient data freshness.

Virtually, all applications of game theory to AoI in the literature explore this scenario when dealing with multiple sources, i.e., each source is a player with individual AoI minimization as the objective. Sometimes, advanced medium access techniques are considered, to see whether collisions can be resolved [16], or at least partially mitigated by improvements over the collision resolution [4] or leveraging correlation in the contents of multiple sources [18].

Simply put, all these papers can be seen as revisiting the game theoretic framework of a tragedy of the commons, which has been successfully applied to more standard metrics such as throughput in [27]–[29], but using AoI as the main objective instead. The comparative game theoretic analysis made in [3], with the explicit objective of matching these two possible objectives of throughput and AoI as network utilities, can be seen as a confirmation of this analogy.

In a number of other papers, game theoretic approaches are considered in an adversarial setup instead, that is, an AoI-related cost may be present, but some nodes try to minimize this function, while others are attackers trying to maximize it [30]–[32]. While these papers also use game theory to derive strategically optimal choices, the focus is clearly different and

mostly relates to the importance of increasing the costs for the attacker; instead, as shown later, if the objective is to improve the efficiency of a distributed management, the cost is to be kept low.

The present paper takes instead a different approach of focusing on a network where multiple flows have a single *converging* AoI, which are all able to reset upon transmission of an update. However, they all face individual costs when doing so, therefore their individual preference is that some other source does it. This is also reminiscent, but for entirely different reasons, of game theoretic analysis of distributed throughput maximization of converging flows where nodes are subject to individual costs. However, as argued in [14], the non-additive character of AoI makes it more interesting to evaluate the resulting distributed management. In other words, if the objective of throughput maximization is distributed over multiple sources, each paying their own costs, one can still expect an individual contribution that benefits the overall network utility. Conversely, if the objective is to achieve minimal AoI, all nodes are fully interchangeable and therefore prefer to leave the burden to others. The characterization can be therefore expected to be entirely different.

In this sense, it is worth mentioning that a similar analysis was proposed in [21], where a scenario with two equivalent sources with converging AoI is considered. However, that paper has remarkable differences over the analysis presented here. First of all, the number of sources is just limited to two, whereas in this paper an arbitrary number  $N$  of sources is considered. Moreover, the game that the sources play in [21] concerns a slotted time, where the per-slot probability of transmission gives the strategic choice of the agents, and instantaneous processing time. The analysis presented here considers instead a more general approach through queueing theory [10], [11], [22], which allows to account for both the generation rate of the sources and the processing time at the receiver. In particular, the best scenario is considered where newer content is always pre-emptive, but the analysis can be promptly extended to other queueing formulations with analogous results.

## III. SYSTEM MODEL

Consider a network of  $N$  nodes, transmitting to a receiver over a collision-free channel, following a memoryless process with intensity  $\lambda$ . Symmetry reasons impose to look for a solution where all nodes use the same transmission rate  $\lambda$ , which would be the consequence of a distributed decision process. It is worth mentioning that both the optimal solution and the NE will follow this criterion, even though they obtain it through different processes. That is, the optimal choice of  $\lambda$  can be seen as a centralized imposition made by the network, following an obvious symmetry requirements of the nodes. Conversely, the distributed assignment obtained at the NE computes  $\lambda$  as the best response to the other sources when they also choose  $\lambda$ , i.e., a fixed point or, as is often formalized in game theory, a symmetric allocation without any desire for unilateral deviation by any of the agents [16].

It is further assumed that transmissions incur a cost [14], [28]. This can be motivated by several considerations, the first but not exclusive one being energy reasons [19]. Yet, also from a logical perspective, a scenario without any transmission cost by the sources would simply imply that they can transmit with arbitrarily high rate with no consequence [29]. Thus, we assume that transmitters pay a cost proportional to their rate, given by  $c\lambda$ , where  $c$  is a parameter, representing the *transmission price*, i.e., the cost of transmitting with unit rate  $\lambda = 1$ .

The objective is to minimize the total system cost, defined as the sum of the average AoI at the receiver's side and the transmission cost. Due to all nodes being symmetrical, it is actually equivalent to minimize the total cost of one single node, or that of the entire network, with a factor  $N$  (which is a constant) being the only difference between the two computations.

Statistical moments of AoI can be found in several queueing systems, and in particular the present analysis considers the M/M/1\* system as defined in [22], i.e., a single-server FCFS queue with memoryless arrivals, memoryless processing and pre-emption of newer updates. This is actually the best possible scenario that avoids impairments due to buffering, packet collisions, traffic split at multiple servers and so on. However, the choice is not restrictive as the same analysis can be applied to multiple queueing systems that received an analytical characterization in the literature [10].

Without loss of generality, we take the service rate of the queue as equal to 1. This means that for stability reasons we require  $\lambda < 1/N$ , since the total rate generated by the sources, denoted by  $\Lambda$  is equal to  $N\lambda$  thanks to the superposition property of Poisson processes. This also implies that the transmission price  $c$  is the maximum value that is paid as transmission cost, since the entire network pays  $c\Lambda$ . Moreover, for the FCFS M/M/1\* queueing system, the average AoI can be written as [22]

$$\Delta = \frac{1}{\Lambda} + 1 \quad (2)$$

and therefore the individual objective of each node can be taken as the minimization of a cost  $K$  equal to

$$K = \Delta + c\lambda = \frac{1}{\Lambda} + 1 + c\lambda. \quad (3)$$

The total network utility can be taken as  $K_{\text{tot}} = NK$ . This is just written for consistency in the notation, but symmetry implies that it is equivalent to minimize  $K_{\text{tot}}$  or  $K$ .

Under this formalization, it is immediate to compute the social optimum, i.e., the transmission rate  $\lambda^*$  that minimizes

$$K_{\text{tot}} = N \left( \frac{1}{N\lambda} + 1 + c\lambda \right). \quad (4)$$

Differentiating with respect to  $\lambda$  and setting to zero, we get

$$-\frac{1}{N\lambda^2} + c = 0, \quad (5)$$

thus obtaining  $\lambda = 1/\sqrt{Nc}$ . However, this holds true if  $\lambda^*$  is an inner point of  $[0, 1/N]$ , otherwise  $K_{\text{tot}}$  always decreases

with  $\lambda$  and the minimum is attained at the extreme. Thus,  $\lambda^* = \max(N, \sqrt{Nc})^{-1}$ .

Finding the NE would preliminary verify that the equilibrium point exists and is unique. This is however immediately guaranteed by the structure of the cost function  $K$ , which is convex (i.e., concave upwards). Incidentally, this property holds for virtually all AoI expressions of different queueing systems as shown in [22], and is therefore a more general property. However, it is also possible to derive its expression explicitly, although this requires a slightly different reasoning than the direct optimization of (5), assuming instead that each node chooses  $\lambda$  to minimize its own cost, given that the other nodes play  $\lambda_0$ . Only afterwards, due to the aforementioned symmetry conditions, it is imposed that  $\lambda = \lambda_0$ . As such, the total arrival rate  $\Lambda$  in (3) is rewritten, for the time being, as  $\Lambda = (N-1)\lambda_0 + \lambda$ .

Taking the derivative of the individual cost and setting it to zero gives

$$-\frac{1}{[(N-1)\lambda_0 + \lambda]^2} + c = 0. \quad (6)$$

Substituting  $\lambda = \lambda_0$  and labelling this value as  $\lambda^{\text{NE}}$  gives

$$-\frac{1}{(N\lambda^{\text{NE}})^2} + c = 0, \quad (7)$$

which either gives an admissible solution as  $\lambda^{\text{NE}} = 1/N\sqrt{c}$ , or like before it implies that all sources select the upper extreme  $\lambda = 1/N$ . Thus,  $\lambda^{\text{NE}} = [N \max(1, \sqrt{c})]^{-1}$

It is evident that these two expressions of  $\lambda^{\text{NE}}$  and  $\lambda^*$  do not coincide, and in particular the descent of the generation rate with the number of nodes  $N$  is much higher at the NE than in an optimal management, being inversely proportional to  $N$  and only  $\sqrt{N}$ , respectively; in other words, selfish nodes are “lazier” at the NE and tend to send updates more rarely than what is optimal to do.

To better highlight this trend, one can also consider the Price of Anarchy (PoA), defined as:

$$\text{PoA} = \frac{K_{\text{tot}}^{\text{NE}}}{K_{\text{tot}}^*}, \quad (8)$$

where “NE” and “\*” denote the Nash equilibrium and the optimum. Also, since  $N$  simplifies out, we can also consider the ratio in individual terms (i.e.,  $K^{\text{NE}}/K^*$ ), and the terms can be obtained by substituting the values for  $\lambda^{\text{NE}}$  and  $\lambda^*$ , respectively, into (3). Thus,

$$K^{\text{NE}} = \begin{cases} \sqrt{c} + 1 + \frac{\sqrt{c}}{N} & \text{for } c > N \\ 2 + \frac{c}{N} & \text{otherwise} \end{cases}, \quad (9)$$

$$K^* = \begin{cases} 2\sqrt{\frac{c}{N}} + 1 & \text{for } c > 1 \\ 2 + \frac{c}{N} & \text{otherwise} \end{cases}. \quad (10)$$

This means that the NE is optimal only for low values of  $c$ . Also, for large  $N$

$$\text{PoA} \approx \sqrt{c} + 1, \quad (11)$$

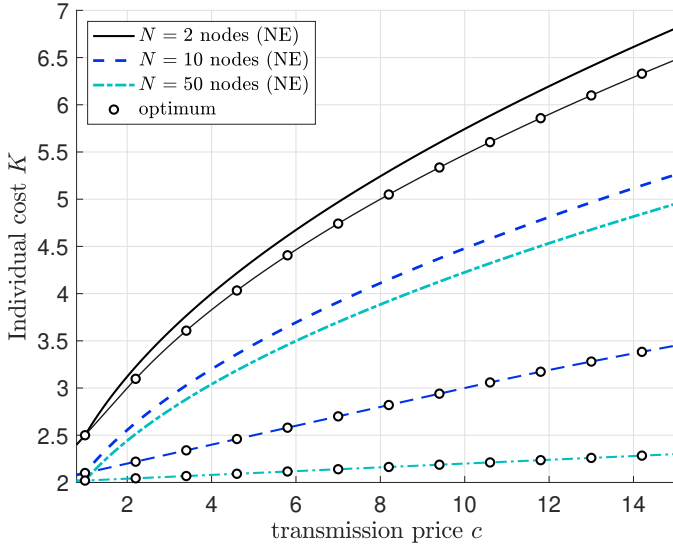


Fig. 1. Cost  $K$  vs transmission price  $c$ .

which indicates that selfish behavior leads to inefficiencies that grow with  $c$ . The usually adopted interpretation of the PoA, which must be greater than or equal to 1, is indeed that the excess value over a unit ratio is the extra cost that is suffered due to lack of coordination. That is, a PoA of, e.g., 1.25 signifies a 25% surplus cost [33]; thus, the result of (11) means that an extra cost equal to  $\sqrt{c}$  is caused by the absence of centralized control when the number of nodes is very large.

More precisely, there is an increasing trend for the PoA as a function of  $c$  and also when changing the number of sources  $N$ , albeit in this latter case the value asymptotically saturates. For growing  $c$  instead, the PoA grows unbounded, even though sub-linearly.

#### IV. NUMERICAL RESULTS

This section shows some evaluations of the formulas derived in the previous section, most notably how the transmission price  $c$  and the number of sources  $N$  impact the system management either at the optimal point or the NE. It is worth noting that the results shown correspond to intrinsic trends in the management of real time traffic, due to the characterization through the simplest possible queueing scenarios, but most considerations still applies, at least qualitatively, if different formulas for AoI are used (see [22] for a quick compendium) following from differences in the queueing systems considered and/or preemption rules.

Fig. 1 displays the individual cost versus the transmission price  $c$ , for different values of the number  $N$  of sources. It is shown that the cost suffered by nodes increases with  $c$ , but the growth is less than linear. This is a consequence of the cost being a combination between the transmission cost, which is linear in  $c$ , and the network's AoI. Thus, when  $c$  increases, the nodes trade the frequency of their more costly transmissions with the increased AoI due to more sporadic updates. Moreover, when  $N$  increases, the individual costs

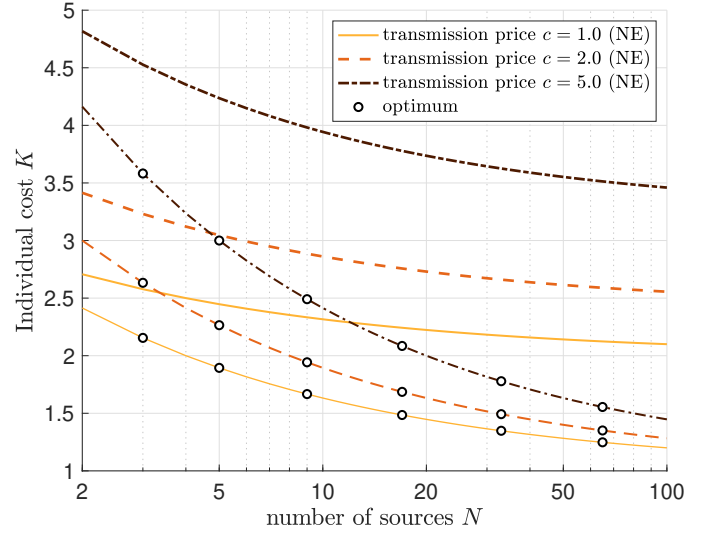


Fig. 2. Cost  $K$  vs number of sources  $N$ .

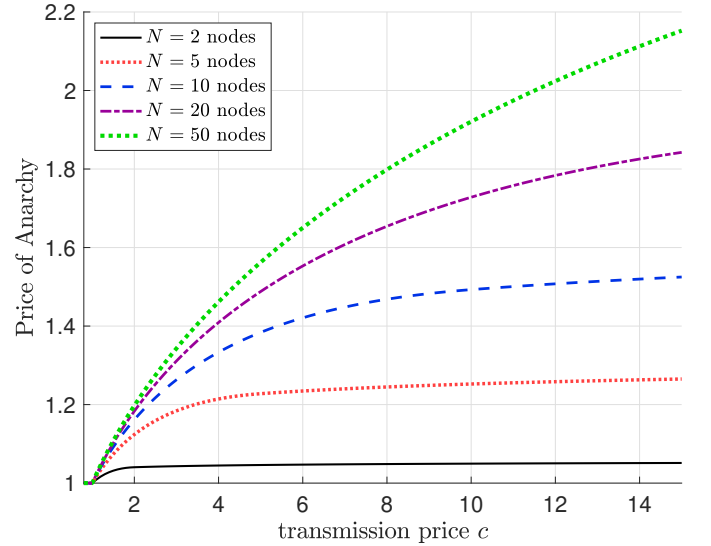


Fig. 3. PoA vs transmission price  $c$ .

become lower as the nodes can leverage their redundancy and transmit less often. While this decrease is considerable at the optimum, it is less so at the NE, due to the aforementioned principle that nodes tend to transmit considerably less often, thus a larger number of users causes higher inefficiency of AoI (i.e., the average AoI decreases but less than what it could).

This trend is analogously shown by Fig. 2, where  $N$  is instead the independent variable. A large  $N$  is virtually able to obtain an optimal network management with low cost, where all values of  $c$  converge towards the minimum value of 1, but this does not hold at the NE, whose cost descent in  $N$  is much slower. This seriously questions the ability of distributed network management without any form of coordination to obtain low AoI values.

A synthetic representation is captured by the PoA evaluation versus  $c$  displayed in Fig. 3. The asymptotic trend of the



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