ON THE STATE RECONSTRUCTION OF 2D SYSTEMS

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ABSTRACT

The paper gives necessary and sufficient conditions for exact reconstructibility of the state of 2D systems. It includes also a technique for designing dynamic state observers.

KEY WORDS

2D system, observer, polynomial matrix description.

1. INTRODUCTION

It is well known that in 1D linear discrete time systems state reconstructibility (i.e. the possibility of computing the state using past values of inputs and outputs) is implied by observability.

In fact, a system $\Sigma = (A,B,C)$ of dimension n is reconstructible iff

$$\text{rank} \begin{bmatrix} C \\ I-Az \end{bmatrix} = n, \qquad \forall z \in \mathbb{C}$$

while it is observable iff

$$\text{rank} \begin{bmatrix} C \\ zI-A \end{bmatrix} = n, \qquad \forall z \in \mathbb{C}$$

A necessary and sufficient condition for reconstructibility is given by the existence of a matrix L that makes A+LC nilpotent. The existence of such an L allows to design a new dynamical system $\hat{\Sigma}$ which has the same internal structure as Σ, and whose output estimates exactly the state of Σ in a finite number of steps.

In this paper we introduce an extension of the reconstructibility concepts. It gives an explicit solution to the problem of constructing a 2D system $\hat{\Sigma}$ which provides in real time an exact estimate of the internal state of a 2D system $\Sigma = (A_1,A_2,B_1,B_2,C)$.

As in 1D case the 2D reconstructibility property is equivalent to a rank condition, which is here given by

$$\text{rank} \begin{bmatrix} C \\ I-A_1z_1-A_2z_2 \end{bmatrix} = n, \qquad \forall (z_1,z_2) \in \mathbb{C} \times \mathbb{C}$$

When this condition is satisfied, $\hat{\Sigma}$ exists. However internal structure of $\hat{\Sigma}$ is not necessarily similar to the structure of Σ. This depends on the fact that a 2D counterpart of the 1D nilpotency condition does not hold.

2. CAUSAL RECONSTRUCTIBILITY

The state model of a 2D system $\Sigma = (A_1,A_2,B_1,B_2,C,D)$ is given by the following equations [1]

$$\begin{aligned} x(h+1, k+1) &= A_1x(h+1, k) + A_2x(h, k+1) + B_1u(h+1, k) + \\ &\quad + B_2u(h, k+1) \\ y(h,k) &= C\, x(h,k) + D\, u(h,k) \end{aligned} \tag{1}$$

where $x(h,k) \in \mathbb{R}^n$ is the local state, $u(h,k) \in \mathbb{R}^m$ and $y(h,k) \in \mathbb{R}^p$ are the input and output vectors at $(h,k) \in \mathbb{Z} \times \mathbb{Z}$ and A_1,A_2,B_1,B_2,C,D are real matrices of suitable sizes.

Denote by

$$\mathcal{X}_0 = \sum_{i=-\infty}^{+\infty} x(i,-i)z_1^i z_2^{-i}$$

the global state on the separation set

$$\mathcal{C}_0 = \{(i,j) : i+j = 0\}$$

and by

$$X(z_1,z_2) = \sum_{i+j\geq 0} x(i,j)z_1^i z_2^j$$

$$U(z_1,z_2) = \sum_{i+j\geq 0} u(i,j)z_1^i z_2^j$$

$$Y(z_1,z_2) = \sum_{i+j\geq 0} y(i,j)z_1^i z_2^j$$

the state, input and output functions respectively.

Then, from (1) one gets

$$\begin{aligned} X(z_1,z_2) &= (I-A_1z_1-A_2z_2)^{-1}\left[\mathcal{X}_0 + (B_1z_1+B_2z_2)U(z_1,z_2)\right] \\ Y(z_1,z_2) &= C\,X(z_1,z_2) + D\,U(z_1,z_2) \end{aligned} \tag{2}$$

and, assuming zero initial conditions $\mathcal{X}_0 = 0$, the rational transfer matrix

$$W_\Sigma(z_1,z_2) \stackrel{\Delta}{=} C(I-A_1z_1-A_2z_2)^{-1}(B_1z_1+B_2z_2) + D \tag{3}$$

gives the input-output map of the system

$$Y(z_1,z_2) = W_\Sigma(z_1,z_2)U(z_1,z_2)$$

The system (1) is finite memory if the free state evolution x(h,k) vanishes for any set of initial sates and h+k sufficiently large.

The finite memory property is equivalent to assume

$$\Delta_\Sigma(z_1,z_2) = \det(I-A_1z_1-A_2z_2) = 1$$

In fact $\Delta_\Sigma(z_1,z_2) = 1$ implies that $(I-A_1z_1-A_2z_2)^{-1}$ is a polynomial matrix. Viceversa the finite memory property implies that the power series expansion of $(I-A_1z_1-A_2z_2)^{-1}$ is a polynomial matrix, so that $\Delta_\Sigma(z_1,z_2) = 1$ follows from the polynomial identity $1 = \Delta_\Sigma(z_1,z_2)\det(I-A_1z_1-A_2z_2)^{-1}$.

Any polynomial transfer matrix is realizable by a finite memory 2D system. This fact follows easily from the realization technique presented in / 1 / for the single input-single output case, which is easily extended to the multivariable case.

The structural property which is crucial in the analysis of state reconstruction is the causal reconstructibility. Intuitively, a 2D system has this property if it is possible to determine the local state $x(0,0)$ when the input and output values are known on some finite set of points in the past of $(0,0)$.

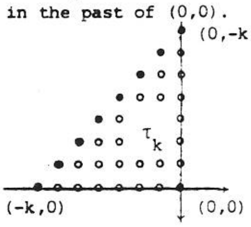
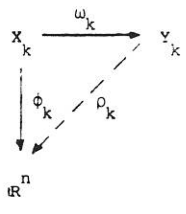


fig. 1

By the linearity assumption, a formal definition of causal reconstructibility is given referring to the free evolution of the system. Let X_k be the space of local states with support $\{(0,-k)\ (-1,-k+1),\dots,(-k,0)\}$ and $Y_k \subset \mathbb{R}^p[z_1^{-1},z_2^{-1}]$ be the space of polynomials with degree less than or equal to k, i.e. the space of output functions whose support is the triangle τ_k in fig. 1. Given Σ, we have from (1) the maps ω_k and ϕ_k that associate with each sequence of local states in X_k the output restriction to τ_k and the state $x(0,0)$. Then the intuitive notion of reconstructibility can be stated in the following form

Definition 1. Σ is causally reconstructible if there exist an integer $k \geq 0$ and a map ρ_k such that the following diagram



commutes.

Theorem 1. The following facts are equivalent

i) Σ is causally reconstructible

ii) there exist polynomial matrices $P(z_1,z_2)$ and $Q(z_1,z_2)$ which satisfy the Bézout identity

$$Q(z_1,z_2)(I-A_1z_1-A_2z_2) + P(z_1,z_2)C = I \tag{4}$$

iii) there exist a polynomial matrix $M(z_1,z_2)$ and an integer $\nu > 0$ such that

$$(A_1z_1+A_2z_2)^\nu = M(z_1,z_2)\begin{bmatrix} C \\ C(A_1z_1+A_2z_2) \\ \vdots \\ C(A_1z_1+A_2z_2)^{n-1} \end{bmatrix} \tag{5}$$

iv) the matrix

$$\begin{bmatrix} C \\ I-A_1z_1-A_2z_2 \end{bmatrix} \tag{6}$$

is full rank for any (z_1,z_2) in $\mathbb{C}\times\mathbb{C}$

proof i) → ii) ρ_k is necessarily linear on the range of ω_k and there is no restriction in assuming ρ_k linear on Y_k. Then there exist matrices F_{ij} such that

$$x(0,0) = \rho_k\Big(\sum_{(-r,-s)\in\tau_k} y(-r,-s)z_1^{-r}z_2^{-s}\Big) =$$

$$= \sum_{(-r,-s)\in\tau_k} F_{rs}\,y(-r,-s)$$

So, by shift invariance

$$x(i,j) = \sum_{(-r,-s)\in\tau_k} F_{rs}\,y(i-r,j-s)$$

Let $\{e_\mu\}$, $\mu = 1,2,\dots,n$ denote the canonical basis in $\mathbb{R}^n$ and assume the initial condition

$$\mathcal{X}_0 = \sum_{-\infty}^{+\infty}{}_h\, x(h,-h)z_1^h z_2^{-h} = e_\mu \tag{7}$$

Then, for $i+j \geq k$

$$x(i,j)z_1^i z_2^j = \sum_{(-r,-s)\in\tau_k} F_{rs}\,y(i-r,\ j-s)z_1^{i-r}z_2^{j-s}z_1^r z_2^s$$

$$= (P(z,z_2)Y(z_1,z_2),\ z_1^i z_2^j)z_1^i z_2^j$$

where $P(z_1,z_2)$ denotes the polynomial matrix

$$P(z_1,z_2) = \sum_{(-r,-s)\in\tau_k} F_{rs}z_1^r z_2^s$$

We therefore have that the difference

$$X(z_1,z_2) - P(z_1,z_2)Y(z_1,z_2) =$$

$$= (I-A_1z_1-A_2z_2)^{-1}e_\mu - P(z_1,z_2)C(I-A_1z_1-A_2z_2)^{-1}e_\mu \overset{\Delta}{=} q_\mu(z_1,z_2)$$

is a polynomial vector. Thus the polynomial matrix

$$Q(z_1,z_2) \overset{\Delta}{=} \big[q_1(z_1,z_2)\,|\,\dots\dots\,q_n(z_1,z_2)\big]$$

satisfies

$$(I-A_1z_1-A_2z_2)^{-1} - P(z_1,z_2)C(I-A_1z_1-A_2z_2)^{-1} = Q(z_1,z_2)$$

and (4) is obtained by postmultiplication of the expression above by $(I-A_1z_1-A_2z_2)$.

ii) → i) Assuming $\mathcal{X}_0$ as in (7), one gets the proof by following backward the steps of the previous point.

By linearity, the proof extends to the general case.

ii) → iii) Rewrite (4) in the following form

$$\sum_{i=0}^{\nu-1} Q_i(z_1,z_2) + \sum_{i=0}^{\nu-1} P_i(z_1,z_2)C \sum_{j=0}^{\infty} (A_1z_1+A_2z_2)^j = \sum_{j=0}^{\infty} (A_1z_1+A_2z_2)^j$$

where Q_i and P_i are the i-th degree homogeneous terms in $Q(z_1,z_2)$ and $P(z_1,z_2)$.

Hence

$$(I-P_oC)(A_1z_1+A_2z_2)^{\nu} = \sum_{i=1}^{\nu} P_iC(A_1z_1+A_2z_2)^{\nu-i} \qquad (8)$$

$$= [P_\nu P_{\nu-1}\dots P_1] \begin{bmatrix} C \\ C(A_1z_1+A_2z_2) \\ \vdots \\ C(A_1z_1+A_2z_2)^{\nu-1} \end{bmatrix}$$

If $\nu>n$, by Cayley Hamilton theorem the right hand term in (8) can be written as

$$\underbrace{[P_\nu P_{\nu-1} \dots P_1] \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \\ \hline & * & \end{bmatrix}}_{M(z_1,z_2)} \begin{bmatrix} C \\ C(A_1z_1+A_2z_2) \\ \vdots \\ C(A_1z_1+A_2z_2)^{n-1} \end{bmatrix}$$

To complete the proof, it remains to show that in (8) P_o can be assumed to be zero. If not, consider the identity

$$P_oC\,\mathrm{Adj}(I-A_1z_1-A_2z_2)(I-A_1z_1-A_2z_2) - P_o\Delta_\Sigma(z_1,z_2)C = 0 \qquad (9)$$

By adding (9) to (4) one gets a new Bézout identity

$$[Q(z_1,z_2)+P_oC\,\mathrm{Adj}(I-A_1z_1-A_2z_2)](I-A_1z_1-A_2z_2) + [P(z_1,z_2)-P_o\Delta_\Sigma(z_1,z_2)]C = I$$

where the matrix $P(z_1,z_2) - P_o\Delta_\Sigma(z_1,z_2)$ has zero constant term.

iii) → iv) For any $(z_1,z_2) \in \mathbb{C}\times\mathbb{C}$, (5) implies the reconstructibility of the 1D system $(A_1z_1+A_2z_2, C)$. Hence

$$\mathrm{rank} \begin{bmatrix} C \\ I-(A_1z_1+A_2z_2)z \end{bmatrix} = n$$

for any z in $\mathbb{C}$ and (6) follows by assuming z=1.

iv) → ii) Let $N_1(z_1,z_2)$, $N_2(z_1,z_2)$, ..., $N_s(z_1,z_2)$ be the submatrices of order n of (6) and $q_1(z_1,z_2)$, $q_2(z_1,z_2)$ .. .. $q_s(z_1,z_2)$ be their determinants.

Denote by $\mathcal{J} \subseteq \mathbb{C}[z_1,z_2]$ the ideal of polynomials p which satisfy the following equation

$$P(z_1,z_2)C + Q(z_1,z_2)(I-A_1z_1-A_2z_2) = p(z_1,z_2)I \qquad (10)$$

for some polynomial matrices P and Q.

First of all we shall prove that the algebraic variety $\mathcal{V}(\mathcal{J})$ is the variety of the polynomials $q_1,q_2,\dots,q_s$:

$$\mathcal{V}(\mathcal{J}) = \mathcal{V}(q_1,q_2,\dots,q_s)$$

From the definition of $N_k(z_1,z_2)$, $k=1,2,\dots,s$, it is clear that there exist constant matrices P_k, Q_k such that

$$N_k(z_1,z_2) = P_kC + Q_k(I-A_1z_1-A_2z_2)$$

Then premultiplying both sides by adj $N_k(z_1,z_2)$ yields

$$q_k(z_1,z_2)I = [\mathrm{adj}\, N_k(z_1,z_2)P_k]C + [\mathrm{adj}\, N_k(z_1,z_2)Q_k](I-A_1z_1-A_2z_2) \qquad (11)$$

which shows that q_k is in $\mathcal{J}$ for $k=1,2,\dots,s$ and

$$\mathcal{V}(q_1,q_2,\dots,q_s) \supseteq \mathcal{V}(\mathcal{J})$$

To prove the inverse inclusion, choose any nonzero polynomial p in $\mathcal{J}$ and polynomial matrices P and Q satisfying the equation (10). Then, for any $(z_1^\circ,z_2^\circ) \notin \mathcal{V}(p)$, we have

$$\frac{P(z_1^\circ,z_2^\circ)}{p(z_1^\circ,z_2^\circ)}C + \frac{Q(z_1^\circ,z_2^\circ)}{p(z_1^\circ,z_2^\circ)}(I-A_1z_1^\circ-A_2z_2^\circ) = I$$

Therefore the row span of

$$\begin{bmatrix} C \\ I-A_1z_1^\circ-A_2z_2^\circ \end{bmatrix}$$

is $\mathbb{C}^n$ and, consequently, (z_1°,z_2°) cannot be a common zero of $q_1,q_2,\dots,q_s$. This means that $(z_1^\circ,z_2^\circ) \notin \mathcal{V}(q_1,q_2,\dots,q_s)$ and proves the inclusion

$$\mathcal{V}(\mathcal{J}) \supseteq \mathcal{V}(q_1,q_2,\dots,q_s)$$

We complete the proof by showing that $1 \in \mathcal{J}$, which directly implies the Bézout identity. By assumption iv), $\mathcal{V}(q_1,q_2,\dots,q_s)$ is empty. Hence $\mathcal{V}(\mathcal{J})$ is empty and, by Hilbert Nullstellensatz [2], $\mathcal{J}=\mathbb{C}[z_1,z_2]=(1)$.

Remark It will be useful for the design of the observer to note that in the Bézout identity (4) it is not restrictive to assume that the matrices $P(z_1,z_2)$ and $Q(z_1,z_2)$ have real coefficients.

3. SYNTHESIS OF A 2D OBSERVER

As we shall show, the reconstructibility property of Σ is a necessary and sufficient condition for the existence of an exact observer, i.e. a new system $\hat{\Sigma}$ whose output at (h,k) is an estimate of the current state x(h,k) of Σ, that becomes exact for h+k sufficiently large.

Definition 2. Given a 2D system Σ, a 2D system $\hat{\Sigma}$ given by

$$\hat{x}(h+1,k+1) = F_1\hat{x}(h+1,k) + F_2\hat{x}(h,k+1) + G_1\begin{bmatrix} u(h+1,k) \\ y(h+1,k) \end{bmatrix} + G_2\begin{bmatrix} u(h,k+1) \\ y(h,k+1) \end{bmatrix} \qquad (12)$$

$$\hat{y}(h,k) = H\hat{x}(h,k) + J\begin{bmatrix} u(h,k) \\ y(h,k) \end{bmatrix}$$

is an exact observer if the following conditions hold:

i) $\hat{\Sigma}$ is finite memory

ii) the "estimate error"

$$e(h,k) = x(h,k) - \hat{y}(h,k)$$

vanishes for $h+k$ sufficiently large, for all initial conditions $\mathcal{X}_o$ and $\hat{\mathcal{X}}_o$ in Σ and in $\hat{\Sigma}$ respectively.

Remark Notice that for 1D observers condition i) is redundant. Actually it is implied by condition ii) since the state and the output of 1D observers coincide.

In general, this coincidence cannot be a priori assumed for 2D observers.

Theorem 2. A system Σ is causally reconstructible if and only if it admits an exact observer $\hat{\Sigma}$.

proof. Assume Σ be causally reconstructible. Then, by Theorem 1, we can express the inverse of $(I-A_1z_1-A_2z_2)$ as

$$(I-A_1z_1-A_2z_2)^{-1} = Q(z_1,z_2) + P(z_1,z_2)C(I-A_1z_1-A_2z_2)^{-1}$$

so that (2) can be written in the following form

$$X(z_1,z_2) = Q(z_1,z_2)\,\mathcal{X}_o + \hat{W}(z_1,z_2)\begin{bmatrix} U(z_1,z_2) \\ Y(z_1,z_2) \end{bmatrix} \qquad (13)$$

with

$$\hat{W}(z_1,z_2) \triangleq \left[Q(z_1,z_2)(B_1z_1+B_2z_2)-P(z_1,z_2)D \,\middle|\, P(z_1,z_2)\right] \qquad (14)$$

Let $\hat{\Sigma} = (F_1,F_2,G_1,G_2,H,J)$ be a finite memory 2D system which realizes $W(z_1,z_2)$. As we have seen in section 2, such a system exists since $W(z_1,z_2)$ is a polynomial matrix.

Then $\hat{\Sigma}$ is an exact observer of Σ. In fact the estimate error

$$E(z_1,z_2) = X(z_1,z_2) - \hat{Y}(z_1,z_2)$$
$$= Q(z_1,z_2)\,\mathcal{X}_o - H(I-F_1z_1-F_2z_2)^{-1}\hat{\mathcal{X}}_o$$

vanishes after a finite number of steps.

Vicecersa, let $\hat{\Sigma} = (F_1,F_2,G_1,G_2,H,J)$ be an exact observer of Σ, and let

$$\mathcal{X}_o = \sum_i x(i,-i)z_1^i z_2^{-i} = x(0,0)$$

$$\hat{\mathcal{X}}_o = 0\,, \quad U(z_1,z_2) = 0$$

Since $(I-F_1z_1-F_2z_2)^{-1}$ is a polynomial matrix, by (11) we obtain

$$\hat{Y}(z_1,z_2) = H(I-F_1z_1-F_2z_2)^{-1}(G_1z_1+G_2z_2)\begin{bmatrix} 0 \\ Y(z_1,z_2) \end{bmatrix}$$
$$= P(z_1,z_2)Y(z_1,z_2)\,.$$

where $P(z_1,z_2)$ is a polynomial matrix.

The estimate error given by

$$E(z_1,z_2) = X(z_1,z_2) - \hat{Y}(z_1,z_2)$$
$$= X(z_1,z_2) - P(z_1,z_2)C\,X(z_1,z_2)$$
$$= (I-P(z_1,z_2)C)(I-A_1z_1-A_2z_2)^{-1}\,x(0,0)$$

is a polynomial vector for any $x(0,0)$. Hence

$$Q(z_1,z_2) = (I-P(z_1,z_2)C)(I-A_1z_1-A_2z_2)^{-1} \qquad (15)$$

is a polynomial matrix and we obtain the Bézout identity directly from (16).

The proof of Theorem 2 above provides a constructive procedure for the design of an exact observer, based on the knowledge of the polynomial matrices $P(z_1,z_2)$ and $Q(z_1,z_2)$.

By Theorem 1, $P(z_1,z_2)$ and $Q(z_1,z_2)$ are easily computed starting from the polynomial identity

$$1 = q_1z_1 + q_2r_2 + \dots q_sr_s \qquad (16)$$

which holds for suitable polynomials $r_1,r_2,\dots,r_s$ by Hilbert Nullstellensatz. In fact, from (11) and (16) it follows

$$I = I(q_1r_1 + q_2r_2 + \dots + q_sr_s)$$
$$= \left[\sum_{k=1}^{s} \operatorname{adj} N_k(z_1,z_2)P_kr_k(z_1,z_2)\right]C +$$
$$+ \left[\sum_{k=1}^{s} \operatorname{adj} N_k(z_1,z_2)Q_kr_k(z_1,z_2)\right](I-A_1z_1-A_2z_2)$$

which gives

$$P(z_1,z_2) = \sum_{k=1}^{s} \operatorname{adj} N_k(z_1,z_2)P_kr_k(z_1,z_2)$$
$$Q(z_1,z_2) = \sum_{k=1}^{s} \operatorname{adj} N_k(z_1,z_2)Q_kr_k(z_1,z_2) \qquad (17)$$

Example 1 Consider the following system

$$A_1 = \begin{bmatrix} 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \\ 0&0&0&0 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&1&0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1&0&0&0 \end{bmatrix} \quad B_1 = B_2 = 0, \quad D = 0$$

The matrix

$$\begin{bmatrix} C \\ I-A_1z_1-A_2z_2 \end{bmatrix} = \begin{bmatrix} 1&0&0&0 \\ 1&-z_1&0&0 \\ 0&1&-z_1&0 \\ 0&0&1&-z_1 \\ 0&0&-z_2&1 \end{bmatrix}$$

is full rank in $\mathbb{C}\times\mathbb{C}$. In fact the submatrices N_1 and N_2, obtained by deleting the last and the first row respectively, have determinants

$$q_1 = -z_1^3\,, \quad q_2 = 1 - z_1z_2$$

Assuming

$$r_1 = -z_2^3\,, \quad r_2 = 1 + z_1z_2 + z_1^2z_2^2$$

$$P_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} 0&0&0&0 \\ 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{bmatrix}$$

we obtain

$$1 = q_1 r_1 + q_2 r_2$$

$$N_1(z_1,z_2) = P_1 C + Q_1(I-A_1 z_1 - A_2 z_2)$$

$$N_2(z_1,z_2) = P_2 C + Q_2(I-A_1 z_1 - A_2 z_2)$$

Matrices $P(z_1,z_2)$ and $Q(z_1,z_2)$ are easily obtained from (17). In particular $P(z_1,z_2)$ is given by:

$$P(z_1,z_2) = \begin{bmatrix} z_1^3 z_2^3 \\ z_1^2 z_2^3 \\ z_1 z_2^3 \\ z_2^3 \end{bmatrix}$$

The transfer matrix of the observer is given by (14)

$$\hat{W}(z_1,z_2) = \begin{bmatrix} z_1^3 z_2^3 & 0 \\ z_1^2 z_2^3 & 0 \\ z_1 z_2^3 & 0 \\ z_2^3 & 0 \end{bmatrix}$$

A finite memory realization of $W(z_1,z_2)$ is the following

$$F_1 \begin{bmatrix} 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \\ 1&0&0&0&0&0 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{bmatrix}, \quad G_1 = 0$$

$$G_2 = \begin{bmatrix} 0&0 \\ 0&0 \\ 0&1 \\ 0&0 \\ 0&0 \\ 0&0 \end{bmatrix}, \quad H = \begin{bmatrix} 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \\ 1&0&0&0&0&0 \end{bmatrix}, \quad J = 0$$

4. CONCLUDING REMARKS

Having in mind the structure of the observes in 1D it is natural to ask whether it is possible to design a 2D exact observer $\hat{\Sigma}$ as a copy of the original 2D system Σ, driven by static feedback laws $L_1\varepsilon(h,k)$, $L_2\varepsilon(h,k)$ on the output estimate error $\varepsilon(h,k)$.

More precisely, we want to know whether the following 2D system

$$\hat{x}(h+1, k+1) = A_1\hat{x}(h+1, k) + A_2\hat{x}(h, k+1) + B_1 u(h+1, k) + B_2 u(h, k+1) + L_1\varepsilon(h+1, k) + L_2\varepsilon(h, k+1) \quad (18)$$

$$\varepsilon(h,k) = y(h,k) - D\,u(h,k) - C\,\hat{x}(h,k)$$

is a 2D exact observer for some choice of the constant matrices L_1 and L_2, in the case when Σ is causally reconstructible.

The dynamics of the state estimate error

$$e(h,k) = x(h,k) - \hat{x}(h,k)$$

is directly obtained from (18)

$$e(h+1, k+1) = (A_1-L_1C)e(h+1, k) + (A_2-L_2C)e(h,k+1)$$

We therefore have that a 2D system admits an exact observer having structure (18) if and only if

$$\det(I-(A_1-L_1C)z_1 - (A_2-L_2C)z_2) = 1 \quad (19)$$

holds for suitable constant matrices L_1 and L_2.

It is easy to check that (19) cannot be satisfied by the system considered in Example 4. This shows that in general exact observers with structure (18) need not exist in the case when exact observers with unconstrained structure are available.

REFERENCES

[1] E. Fornasini, G. Marchesini (1978) "Doubly Indexed Dynamical Systems: State Space Models and Structural Properties", Mathematical Systems Theory, vol. 12, n. 1.

[2] O. Zariski, P. Samuel (1958) "Commutative Algebra", Van Nostrand.