Proceedings Intern. Conference Signals & Systems, Al-Ain (U.A.E.), Jan. 29-31, 1990
AMSE Press, Vol. 1, pp. 213-224

FEEDBACK DECOUPLING OF 2D SYSTEMS

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Abstract Necessary and sufficient conditions for the existence of a decoupling bicausal precompensator for multivariable 2D systems are derived in state space and frequency domains.

In general, the decoupling problem for 2D systems can be solved by feedback compensators if suitable injectivity assumptions are introduced on the input-state matrices.

The structure of dynamic compensators is derived for this case and the 2D decoupling problem with stability is solved.

1. INTRODUCTION

Since many years the decoupling problem constitutes one of the most attracting research topics in multivariable 1D systems theory. Besides several appealing consequences in the applications, the interest in this field relies on the analytical tools that have been introduced in developing the underlying theory. The decoupling schemes considered in the literature have different characteristics. These include the topology of the interconnections (based on the use of of precompensators, feedback compensators or compound strategies), the dynamical characteristics of the subsystems that enter in the interconnections, the use of state-space or input/output models and, finally, the algebraic structures (fields, rings) which provide the framework where the systems are defined [1-5]. In most applications we are required to solve at the same time the decoupling and the stabilization problems. In these cases state or output feedbacks have to be considered and only those schemes that include dynamic compensators become relevant to the solution.

2D systems provide input/output and state-space models representing physical processes which depend on two independent variables. In some cases one of these variables is time and the other represents a spatial dimension - as in the study of some classes of distributed parameter systems and delay differential systems, while for other problems - such as image processing - none of the independent variables can be sought of as time. Typically they apply to two dimensional data processing in several fields, as seismology, X-ray image enhancement, image deblurring, digital picture processing, etc. Also, 2D systems constitute a natural framework for modelling multivariable networks, large scale systems obtained by interconnecting many subsystems and,in general, physical processes where both space and time have to be taken into account [6,7].

Recently the feedback control theory of 2D systems attracted the interest of research people and a great deal of attention has been deserved to problems related to stabilization and characterization of closed loop characteristic polynomials [8–11]. Moreover the systematic application of 2D polynomial matrices techniques allowed to extend the original single-input/single-output analysis up to include multivariable 2D systems.

In this paper we aim to analyze how 2D compensators apply to noninteracting control of multivariable 2D systems and to find necessary and sufficient conditions for the existence of a feedback law that makes diagonal and nonsingular the closed loop transfer matrix. We shall tackle this problem using MFD's in two variables, applied to input/output and state space models. It is worthwhile to remark that several equivalent strategies, based on bicausal precompensators, static precompensators and compensators, static precompensators and dynamic compensators, can be implemented in generating noninteracting controls for 1D systems. In the case of 2D systems these strategies are not equivalent [14], since they allow decoupling of different classes of systems.

The state equation of a multivariable 2D system $\Sigma = (A_1 , A_2 , B_1 , B_2 , C , D)$ having m inputs and m outputs are given by

$$x(h+1,k+1) = A_1x(h+1,k) + A_2x(h,k+1) + B_1u(h+1,k) + B_2u(h,k+1) \qquad (1.1)$$
$$y(h,k) = Cx(h,k) + Du(h,k)$$

where u and y are the m-dimensional vectors of input and output values, x is an n-dimensional local state vector and A_1 , A_2 , B_1 , B_2 , C, D are matrices of appropriate dimensions. In the following we shall adopt the standard convention that a scalar sequence $\{s(h,k)\}$ with nonnegative indices h,k is associated with a formal power series $\Sigma s(h,k)z_1{}^h z_2{}^k$ having nonnegative powers in z_1 and z_2. According to this convention, a proper (strictly proper) rational function can be represented as a quotient $p(z_1,z_2)/q(z_1,z_2)$ of coprime polynomials with $q(0,0)\neq 0$ ($q(0,0)\neq 0$ and $p(0,0)=0$).

Therefore, the transfer matrix of Σ is the mxm rational matrix

$$W(z_1,z_2) = C(I-A_1z_1-A_2z_2)^{-1}(B_1z_1+B_2z_2)+D \qquad (1.2)$$

whose entries are proper rational functions in two variables. The system (1.1) is called *strictly proper* if D=0 and *bicausal* if D is an invertible matrix. It is immediate to see that Σ is strictly proper if $W(0,0) = 0$ and bicausal if $W(0,0)$ is an invertible matrix.

Because of the structure of 2D systems a number of different state feedback schemes are allowed. The simplest of these is represented by the static control law

$$u(h,k) = Kx(h,k) \quad , \quad K \in R^{m\times n} \qquad (1.3)$$

Comparing with static state feedback in 1D theory, the possibilities of modifying the dynamical behaviour by applying (1.3) are much poorer [12].

If we consider 2D systems as 1D systems defined over a suitable polynomial ring, we are lead to introduce feedback control laws of the following form

$$u(h,k) = \sum_{i=-N}^{N} K_i x(h-i,k+i) \quad , \quad K_i \in R^{m \times n} \tag{1.4}$$

In particular, the structure (1.4) is obtained when we interpret in a 2D framework the decoupling techniques presented in [4]. An obvious consequence of (1.4) is that the typical 2D quarter plane causality is not preserved and in general the resulting closed loop system is *weakly causal* [10,13].

In this contribution we are interested in defining control laws which can be generated by 2D systems located in the feedback chain (2D compensators) and that give rise to systems which still exhibit the original quarter plane causality. Consequently the control laws we shall take into account are represented by the following recursive equation

$$u(h,k) = \sum_{\substack{i,j=0 \\ (i,j)\neq(0,0)}}^{N} H_{ij} u(h-i,k-j) + \sum_{i,j=0}^{N} K_{ij} x(h-i,k-j) \quad , \quad H_{ij} \in R^{m \times m} \; , \quad K_{ij} \in R^{m \times n} \tag{1.5}$$

It is interesting to notice that if we try to solve separately the decoupling and the stabilization problems for 2D systems, the dynamical feedback law (1.5) works successfully even in cases where the static law (1.3) fails. This makes a significant difference with respect to the 1D case, where dynamic and static state feedback compensators have essentially the same potentiality [3], when the solutions of these two problems are separately considered. Since in the sequel we will always use feedback control laws which are "dynamic" and "causal", we shall omit these attributes.

2. DECOUPLING BICAUSAL PRECOMPENSATORS: STATE VARIABLE APPROACH

In this section we are concerned with the existence of a decoupling bicausal precompensator for a strictly proper 2D system $\Sigma = (A_1, A_2, B_1, B_2, C)$ represented by the state updating equations (1.1). As we shall see, the conditions that will be derived are only partially reminiscent of those obtained in [1] for 1D state-space systems. In fact the 1D decouplability condition can be expressed as a rank condition on a constant matrix, which allows to construct a decoupling static feedback law, while the decoupling compensators for 2D systems are dynamical systems and the decouplability condition is expressed in terms of algebraic properties of a polynomial matrix in two indeterminates.

To shorten our notations, we write $\mathbf{A} = A_1z_1 + A_2z_2$ and $\mathbf{B} = B_1z_1 + B_2z_2$

Let

$$d_i := \min\{j : C_i\mathbf{A}^j\mathbf{B} \neq 0\ ,\ j = 0, 1, \dots n-1\} \quad \nu_i := d_i + 1$$

Clearly, the existence of $d_1, d_2, \dots d_m$ is guaranteed if and only if the system transfer matrix $W(z_1,z_2)$ is nonsingular. Actually the nonsingularity of W is necessary to solve the decoupling problem and in the sequel this condition will be always assumed.

Proposition 1 *Let* M_0 *be the* mxm *2D polynomial matrix given by*

$$M_0 = \begin{bmatrix} C_1\mathbf{A}^{d_1}\mathbf{B} \\ C_2\mathbf{A}^{d_2}\mathbf{B} \\ \vdots \\ C_m\mathbf{A}^{d_m}\mathbf{B} \end{bmatrix} \tag{2.1}$$

Then the system can be decoupled by a decoupling bicausal precompensator if and only if: i) there exists a constant nonsingular matrix Q_0 *such that* $M_0Q_0 = \operatorname{diag}(\varepsilon_1\ \varepsilon_2 \dots \varepsilon_m)$, *where* ε_i , $i = 1, 2, \dots m$ *are homogeneous 2D polynomials of degree* d_i+1; ii) $M_0^{-1}C(I-\mathbf{A})^{-1}\mathbf{B}$ *is proper rational.*

proof Assume that i) and ii) hold. It is immediate to see that $p := \det M_0$ is an homogeneous polynomial of degree $m + \Sigma_i d_i$ and that the i-th column of $\operatorname{adj} M_0$ is an homogeneous polynomial vector of degree $m - 1 + \Sigma_{j\neq i} d_j$, $i = 1, 2, \dots m$.

Consider the following series expansion of the transfer matrix

$$C(I-\mathbf{A})^{-1}\mathbf{B} = \begin{bmatrix} C_1\mathbf{A}^{d_1}\mathbf{B} \\ C_2\mathbf{A}^{d_2}\mathbf{B} \\ \vdots \\ C_m\mathbf{A}^{d_m}\mathbf{B} \end{bmatrix} + \begin{bmatrix} C_1\mathbf{A}^{d_1+1}\mathbf{B} \\ C_2\mathbf{A}^{d_2+1}\mathbf{B} \\ \vdots \\ C_m\mathbf{A}^{d_m+1}\mathbf{B} \end{bmatrix} + \dots \qquad = M_0 + M_1 + \dots \tag{2.2}$$

and premultiply both sides by M_0^{-1}. We obtain

$$M_0^{-1}C(I-\mathbf{A})^{-1}\mathbf{B} = p^{-1}[(\operatorname{adj} M_0)M_0 + (\operatorname{adj} M_0)M_1 + \dots] \tag{2.3}$$

The degrees of the nonzero polynomials in the matrices $(\operatorname{adj} M_0)M_r := [p_{ij}^{(r)}]$, $r = 0, 1, \dots$ are given by $\deg p_{ij}^{(r)} = m + r + \Sigma_i d_i$

By assumption ii), the left hand side of (2.3) admits a power series expansion

$$M_0^{-1}C(I-\mathbf{A})^{-1}\mathbf{B} = \mathbf{p}_0 + \mathbf{p}_1 + \ldots \tag{2.4}$$

where $\mathbf{p}_i$ are homogeneous matrices of degree i. Comparing (2.3) and (2.4) and equating the homogeneous terms of the same degree, we have $(\mathrm{adj}\,M_0)M_0 = p\mathbf{p}_0$ and hence $\mathbf{p}_0 = p^{-1}(\mathrm{adj}\,M_0)M_0 = I$. This implies that the matrix

$$M_0^{-1}C(I-\mathbf{A})^{-1}\mathbf{B} = I + \mathbf{p}_1 + \mathbf{p}_2 + \ldots \tag{2.5}$$

is a bicausal transfer matrix. Recalling assumption i), by (2.5) we have

$$C(I-\mathbf{A})^{-1}\mathbf{B}[I + \mathbf{p}_1 + \mathbf{p}_2 + \ldots]^{-1}Q_0 = M_0Q_0 = \mathrm{diag}\{\varepsilon_1, \varepsilon_2, \ldots \varepsilon_m\}$$

which shows that $[I + \mathbf{p}_1 + \mathbf{p}_2 + \ldots]^{-1}Q_0$ is a decoupling bicausal compensator.

Conversely, suppose that there exists a decoupling bicausal precompensator with transfer matrix $U = U_0 + U_S$, where U_0 is a nonsingular constant matrix and U_S is a strictly proper rational matrix. Then

$$C(I-\mathbf{A})^{-1}\mathbf{B}(U_0+U_S) = \mathrm{diag}\{\delta_1, \delta_2, \ldots \delta_m\} \tag{2.6}$$

where δ_i are proper rational functions.

Denote by ε_i the homogeneous polynomial of minimum degree in the series expansion of δ_i and equate the minimum degree homogeneous rows on both sides of (2.6). We obtain

$$\begin{bmatrix} C_1\mathbf{A}^{d_1}\mathbf{B} \\ C_2\mathbf{A}^{d_2}\mathbf{B} \\ \\ C_m\mathbf{A}^{d_m}\mathbf{B} \end{bmatrix} U_0 = \mathrm{diag}\{\varepsilon_1\ \varepsilon_2 \ldots \varepsilon_m\}$$

Therefore the property i) holds with $Q_0 = U_0$.

It remains to prove property ii). Consider a MFD of the transfer matrix given by

$$C(I-\mathbf{A})^{-1}\mathbf{B} = \overline{N}(I+D_S)^{-1} \tag{2.7}$$

where $\overline{N} = NQ_0$ satisfies the condition ii) of Proposition 1 and $D_S(0,0) = 0$. Let $\Delta := \mathrm{diag}\{\bar{n}_{11}\ \bar{n}_{22} \ldots \bar{n}_{mm}\}$ and rewrite $\overline{N}$ in the form $\overline{N} = \Delta + P$. Then the above condition implies that in $\overline{N} = \Delta(I+\Delta^{-1}P)$ the matrix fraction $\Delta^{-1}P$ is strictly proper. By (2.2) and (2.7) we have

$$
\begin{aligned}
&M_0 + M_1 + M_2 + \ldots \\
&= \bar{N}Q_0^{-1}(I+D_S)^{-1} = \bar{N}(I+Q_0^{-1}D_SQ_0)^{-1}Q_0^{-1} = \Delta(I+\Delta^{-1}P)(I+Q_0^{-1}D_SQ_0)^{-1}Q_0^{-1}
\end{aligned} \tag{2.8}
$$

and hence $M_0 = \Delta Q_0^{-1}$. Thus, premultiplying (2.8) by M_0^{-1} we see that the matrix $M_0^{-1}C(I-\mathbf{A})^{-1}\mathbf{B} = Q_0(I+\Delta^{-1}P)(I+Q_0^{-1}D_SQ_0)^{-1}Q_0^{-1}$
is proper rational.

3. DYNAMIC FEEDBACK DECOUPLING

To carry through the analysis of the feedback decoupling scheme for a strictly proper 2D system $\Sigma = (A_1, A_2, B_1, B_2, C)$, an important remark is that the application of dynamic state feedback together with static precompensation produces transfer matrices that can be obtained also using suitable bicausal precompensators.

In fact, let $K(z_1, z_2)$ and Q_0 be the transfer matrices of the compensator and the precompensator respectively. Then the transfer matrix of the closed loop system is given by

$$
W(z_1, z_2)\ [(I-K(z_1, z_2)(I-\mathbf{A})^{-1}\mathbf{B})^{-1}Q_0] \tag{3.1}
$$

and the term in square brackets can be viewed as the transfer matrix of a bicausal precompensator.

A significant difference with respect to 1D systems is that, given a bicausal precompensator $U(z_1, z_2)$, the 2D transfer matrix WU needs not be implementable using a dynamic state feedback compensator and a static precompensator.

This is illustrated by the following example. Consider the system $\Sigma = (A_1, A_2, B_1, B_2, C)$ given by

$$
A_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}
$$

$$
C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
$$

Since the transfer matrix of Σ

$$
W(z_1, z_2) = \begin{bmatrix} z_1 & 0 \\ 0 & z_1 \end{bmatrix} \begin{bmatrix} 1 & z_2 \\ z_2 & 1 \end{bmatrix}^{-1}
$$

satisfies condition ii) of Proposition 1, there exists a decoupling bicausal precompensator.

Its transfer matrix is easily computed and is given by

$$U(z_1,z_2) = \begin{bmatrix} 1 & z_2 \\ z_2 & 1 \end{bmatrix}$$

However we cannot decouple the system adopting state feedback and static precompensation, since in this case it is not possible to find a constant nonsingular Q_0 and a proper rational $K(z_1,z_2)$ that make the matrix (3.1) diagonal.

Our aim is now to find structural conditions on the matrices of the state model (1.1) which guarantee that a decoupling bicausal compensator $U(z_1,z_2)$ can be replaced by a feedback compensator $K(z_1,z_2)$ and, possibly, a static precompensator Q_0.

These conditions correspond to assuming that, for any bicausal $U(z_1,z_2)$, there exist proper rational $K(z_1,z_2)$ and nonsingular Q_0 that solve

$$W(z_1,z_2)U(z_1,z_2) = C(I-\mathbf{A}-\mathbf{B}K(z_1,z_2))^{-1}\mathbf{B}Q_0 \tag{3.2}$$

For this, substitute in (3.2) $U_0 + U_S$ for U and the series expansion $C\mathbf{B} + C\mathbf{A}\mathbf{B} + C\mathbf{A}^2\mathbf{B} + \ldots$ for W. Then, looking for a solution of (3.2) in terms of $K(z_1,z_2)$ and Q_0, one starts by choosing $Q_0 = U_0$. Since

$$C(I-\mathbf{A})^{-1}\mathbf{B}(I+U_SU_0^{-1}) = C[I-\mathbf{A}-\mathbf{B}K(z_1,z_2)]^{-1}\mathbf{B} = C(I-\mathbf{A})^{-1}\mathbf{B}[I-K(z_1,z_2)(I-\mathbf{A})^{-1}\mathbf{B}]^{-1} \tag{3.3}$$

we are reduced to solving the following equation in $K(z_1,z_2)$

$$(I+U_SU_0^{-1})^{-1} = I - K(z_1,z_2)(I-\mathbf{A})^{-1}\mathbf{B} \tag{3.4}$$

Introduce a realization $\widetilde{\Sigma} = (\widetilde{A}_1,\widetilde{A}_2,\widetilde{B}_1,\widetilde{B}_2,\widetilde{C},I)$ of the left hand side of (3.4) and let

$$\widetilde{K}(z_1,z_2) := K(z_1,z_2)(I-\mathbf{A})^{-1} \qquad \widetilde{\mathbf{A}} = \widetilde{A}_1z_1 + \widetilde{A}_2z_2 \qquad \widetilde{\mathbf{B}} = \widetilde{B}_1z_1 + \widetilde{B}_2z_2$$

Therefore the search for a causal $K(z_1,z_2)$ that solves (3.4) reduces to find a causal $\widetilde{K}(z_1,z_2)$ that solves

$$\widetilde{C}(I-\widetilde{\mathbf{A}})^{-1}\widetilde{\mathbf{B}} = -\widetilde{K}(z_1,z_2)\mathbf{B} \tag{3.5}$$

We shall show that, if the matrix $[B_1 \mid B_2]$ is injective, there exists a causal solution of (4.5). In fact in this case there exists a left inverse L of $[B_1 \mid B_2]$ and the matrix $F := [\widetilde{B}_1 \mid \widetilde{B}_2]L$ satisfies

$$F\,[B_1 \mid B_2] = [\widetilde{B}_1 \mid \widehat{B}_2] \tag{3.6}$$

or, equivalently, $F(B_1z_1+B_2z_2) = B_1z_1+B_2z_2$.

This implies that a solution of (3.5) is given by the proper rational matrix $\widetilde{K}(z_1,z_2) = -\widehat{C}(I-\widetilde{A})^{-1}F$ so that

$$K(z_1,z_2) = -\widehat{C}(I-\widehat{A})^{-1}F(I-A) \tag{3.7}$$

constitutes the transfer matrix of a causal decoupling compensator.

The injectivity does not impose any constraint on the structure of the transfer matrix, since any 2D transfer matrix admits a realization with $[B_1 \mid B_2]$ injective [9]. So, when dealing with the decoupling problem starting from the state space equations, we will always assume $[B_1 \mid B_2]$ be injective.

Example Consider the 2D system Σ given by

$$A_1 = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \qquad B_1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C - \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The transfer matrix of Σ is

$$W(z_1,z_2) = C(I-A_1z_1-A_2z_2)^{-1}(B_1z_1+B_2z_2)$$

$$= \begin{bmatrix} \dfrac{z_1+z_1^2+z_1^2z_2-z_1z_2^2}{1-z_1^2-z_1z_2} & \dfrac{2z_1-z_1^2-z_1z_2}{1-z_1^2-z_1z_2} \\ -z_2^2 & z_2 \end{bmatrix}$$

The matrix

$$M_0 = C\mathbf{B} = \begin{bmatrix} z_1 & 2z_1 \\ 0 & z_2 \end{bmatrix}$$

satisfies properties i) and ii) of Proposition 1. In fact, assuming

$$Q_0 = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$$

the matrix M_0Q_0 is diagonal and $M_0^{-1}W$ is proper rational.

A decoupling bicausal precompensator is then given by

$$\hat{U} = W^{-1}M_0Q_0 = \frac{1}{1+z_1+2z_2-2z_2^2}\begin{bmatrix} 1-z_1^2-z_1z_2 & -2+z_1+z_2 \\ z_2(1-z_1^2-z_1z_2) & 1+z_1+z_1z_2-z_2^2 \end{bmatrix}$$

In order to simplify computations, it will be convenient to modify the precompensator by postmultiplying its transfer matrix by a diagonal bicausal factor

$$T = \begin{bmatrix} \dfrac{1}{1-z_1^2-z_1z_2} & 0 \\ 0 & 1 \end{bmatrix}$$

In this way $U = \hat{U}T$ is still a decoupling bicausal precompensator for Σ and

$$U^{-1} = \begin{bmatrix} 1+z_1+z_1z_2-z_2^2 & 2-z_1-z_2 \\ -z_2 & 1 \end{bmatrix}$$

is a polynomial matrix. The feedback decoupling scheme includes a static precompensator $U_0 = Q_0$. For applying (3.7) to obtain the transfer matrix $K(z_1,z_2)$ of a causal dynamic compensator, a realization $(\widetilde{A}_1, \widetilde{A}_2, \widetilde{B}_1, \widetilde{B}_2, \widetilde{C}, I)$ of

$$U_0U^{-1} = \begin{bmatrix} 1+z_1+2z_2+z_1z_2-z_2^2 & -z_1-z_2 \\ -z_2 & 1 \end{bmatrix}$$

is needed. It is immediate to verify that the realization given by

$$\tilde{A}_1 = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \quad \tilde{A}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \tilde{B}_1 = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \quad \tilde{B}_2 = \begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix} \quad \hat{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

satisfies $\tilde{C}(I-\tilde{A}_1z_1-\tilde{A}_2z_2)^{-1}(\tilde{B}_1z_1+\tilde{B}_2z_2) + I = U_0U^{-1}$

In this example the matrix F is given by

$$F = [\tilde{B}_1 \mid \tilde{B}_2]\,[B_1 \mid B_2]^{-1} = \begin{bmatrix} 1 & -2 & 2 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

so that

$$K(z_1,z_2) = -\tilde{C}(I-\tilde{A})^{-1}F(I-A) = \begin{bmatrix} z_1+2z_2-1 & z_1+2 & 2z_2-2 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

4. STABLE DECOUPLING

In the previous section we have shown how to realize a noninteracting control of a 2D system using dynamic compensators. In general, the decoupled system we obtain needs not to be internally stable for all choices of the decoupling compensator. So, it is important to decide if, given a 2D system, there exist stabilizing decoupling compensators and then to have procedures for their construction.

In this section we will show that, if the system is stabilizable and there exists a noninteracting control, it is possible to select a decoupling compensator which stabilizes the closed loop system.

More precisely, let the system (1.1) satisfies the following conditions:

(i) *stabilizability condition:* the matrix $[I-A_1z_1-A_2z_2 \quad B_1z_1+B_2z_2]$ has full rank for all (z_1,z_2) belonging to the unitary polydisk $\mathcal{P}_1 = \{(z_1,z_2): |z_1| \leq 1, |z_2| \leq 1\}$

(ii) *decouplability condition:* (i) and (ii) of Proposition 1

Then the class of stabilizing compensators contains compensators which are decoupling (for the system (1.1)). This property depends on the following facts:

1. the stabilization by state feedback preserves the decouplability of the system

2. if an internally stable system is decouplable, it can be decoupled without loosing internal stability

The proof of the first fact is immediate, since the matrix M_0 defined by (2.1) and relative to the original system coincide with the matrix M_0 that corresponds to the closed loop system. So that both systems satisfy condition i) of Proposition 1. As far as condition ii) of Proposition 1 is concerned, it is sufficient to note that the transfer matrix W_F of the feedback system differs from $C(I-\mathbf{A})^{-1}\mathbf{B}$ in a bicausal multiplicative factor, so $M^{-1}W_F$ is proper rational since $M^{-1}C(I-\mathbf{A})^{-1}\mathbf{B}$ is.

To prove the second part, observe that, if U is a decoupling bicausal precompensator, $\Delta := WU$ is a proper rational diagonal matrix. Now let h be the l.c.m. of the denominators of the elements of U^{-1}. Then also Uh^{-1} is a bicausal decoupling precompensator and $W(Uh^{-1}) = \Delta h^{-1}$.

Consequently, we can assume in (3.4) that the matrix $(I-U_SU_0^{-1})^{-1} = U_0U^{-1}$ is polynomial and that its realization $\widehat{\Sigma} = (\widetilde{A}_1, \widetilde{A}_2, \widetilde{B}_1, \widetilde{B}_2, \widehat{C}, D)$ satisfies the condition $\det(I-\widetilde{A}_1z_1-\widetilde{A}_2z_2) = 1$. In this case the transfer matrix $K(z_1, z_2)$ of the compensator, given by (3.8), is a polynomial matrix.

In order to obtain an internally stable closed loop system, we construct a coprime realization of $K(z_1, z_2)$, i.e. a 2D system $\overline{\Sigma} = (\overline{A}_1, \overline{A}_2, \overline{B}_1, \overline{B}_2, \overline{C}, \overline{D})$, where the matrices

$$[I-\overline{A}_1z_1-\overline{A}_2z_2 \quad \overline{B}_1z_1+\overline{B}_2z_2]$$

and

$$\begin{bmatrix} I-\overline{A}_1z_1-\overline{A}_2z_2 \\ \overline{C} \end{bmatrix}$$

are full rank for any (z_1, z_2) in C x C [9,11].

The state space model resulting from the feedback connection of Σ and Σ is internally stable, as a consequence of the following properties:

- the plant Σ is internally stable
- the compensator $\overline{\Sigma}$ is a coprime realization of a polynomial matrix. Therefore $\det(I-\overline{A}_1z_1-\overline{A}_2z_2) = 1$, which implies that $\overline{\Sigma}$ is internally stable
- the closed loop system is externally stable, since its transfer matrix is the product of the stable matrix $W(z_1, z_2)$ and the polynomial matrix $U(z_1, z_2)$

5. REFERENCES

[1] P.L.Falb, W.A.Wolovich *Decoupling in the design and synthesis of multivariable control systems* IEEE Trans. on Aut. Contr., vol AC-12, n.6, pp.651-59, 1967

[2] A.S.Morse, M.W.Wonham *Decoupling and pole assignment by dynamic compensation* SIAM J. Control, vol.8, n.3, pp.317-37, 1970

[3] M.J.L.Hautus, M.Heymann *Linear feedback decoupling – Transfer function analysis* IEEE Trans. on Aut. Contr., vol AC-28, n.8, pp.823-32, 1983

[4] K.B.Datta, M.L.J.Hautus *Decoupling of multivariable control systems over unique factorization domains* SIAM J. Contr. and Opt., vol.22, n.1, pp.28-39, 1984

[5] J.M.Dion, C.Commault *On linear dynamic state feedback decoupling* Proc. 24th Conf. on Decision and Control, Fort Lauderdale, FL, Dec.1985

[6] A.Willsky *Relationships between signal processing and control and estimation theory* Proc IEEE, vol.66, n.9, pp.996-1017, 1978

[7] W.Marszalek *Two dimensional state space models for hyperbolic differential equations* Appl. Math. Modelling, vol.8, pp.11-14, 1984

[8] M.Bisiacco *State and output feedback stabilizability of 2D systems* IEEE Trans. Circ. and Sys., vol.CAS-32, n.12, pp1246-54, 1985

[9] M.Bisiacco, E.Fornasini, G.Marchesini *Controller design for 2D systems* in "Frequency Domain and State Space Methods for Linear Systems" C.I.Byrnes and A.Lindquist eds., Elsevier Publ., pp.99-113, 1985

[10] J.P.Guiver, N.K.Bose *Causal and weakly causal 2D filters with applications in stabilization* in "Multidimensional Systems Theory" N.K.Bose ed.,pp.52-100, Reidel, 1985

[11] M.Bisiacco, E.Fornasini, G.Marchesini *Dynamic regulation of 2D systems: a state space approach* to appear in Linear Algebra and Appl., 1989

[12] E.Fornasini, G.Marchesini *On some connections between 2D systems theory and the theory of systems over rings*, Springer L.N. Contr.& Info. Sci., 58, pp.331-346, 1983

[13] R.Eising *2D systems: an algebraic approach* Thesis – Mathematisch Centrum, Amsterdam, 1979

[14] E.Fornasini, G.Marchesini *Noninteracting control of 2D systems* to appear in IEEE Trans. Aut. Contr., 1990