IV. Conclusions

It is shown that by shifting the imaginary axis by an infinitesimally small amount in either direction, the number of roots on the imaginary axis and RHP can be correctly determined. The main advantage is that it avoids finding the common factor between the even and odd parts of a polynomial which must be done if the correct root distribution is to be ascertained. The transformation suggested is simple and easy to apply. Thus it is demonstrated that correct root distribution can be obtained from the Routh's array itself even in respect of singular cases in a most direct and straightforward way. Therefore the transformation renders the Routh's procedure valid for determining the root distribution, inclusive of the roots on the imaginary axis, of any polynomial.

References

[1] F. R. Gantmacher, *Applications of the Theory of Matrices*. New York: Interscience, 1959.
[2] A. Salles Campos, "Further comments on 'on the Routh–Hurwitz criterion'", *IEEE Trans. Automat. Contr.* (Tech. Notes and Corresp.), vol. AC-20, p. 296, Apr. 1975.
[3] G. H. Hostetter, "Additional comments on 'on the Routh-Hurwitz criterion'", *IEEE Trans. Automat. Contr.* (Tech. Notes and Corresp.), vol. AC-20, pp. 296–297, Apr. 1975.

A Controllability Criterion for Continuous Linear Time-Invariant Systems

ETTORE FORNASINI AND ANTONIO LEPSCHY

Abstract—**A controllability criterion is suggested referring to the numerator coefficients of the input/state transfer functions. The criterion is comparable to the usual one from a computational point of view, and it seems to be useful from a didactic point of view because it can be proved in a very simple way.**

Let us consider a linear, time invariant, n-dimensional, strictly proper, one-input–one-output system

$$\dot{x} = Ax + Bu$$
$$y = Cx, \qquad x^T = (x_1, x_2, \cdots, x_n).$$

The input/state transfer function is therefore

$$W(s) = (sI - A)^{-1}B$$

and its components $W_i(s) \triangleq X_i(s)/U(s)$ $(x_i(0) = 0)$ assume the form

$$W_i(s) = \frac{\sum_{j=0}^{n-1} p_{ij}s^j}{\det(sI - A)}$$

where the numerator coefficients p_{ij} are obtained from A and B; the computation procedure can be easily implemented on an electronic computer.

The state space is not completely controllable (or reachable) if and only if there exists a nonzero constant vector K such that

$$K^TX = 0, \qquad K^T = (k_1, k_2, \cdots, k_n)$$

whatever the value of state vector x at any given time $t > 0$, if we assume $x(0) = 0$.

This condition holds also if we consider x as a function of time; therefore, it can be presented in terms of L-transforms:

$$0 = K^TX(s) = K^T \begin{bmatrix} p_{1j} & s^j \\ \cdots & \\ \cdots & \\ p_{nj} & s^j \end{bmatrix} U(s)/\det(sI - A).$$

This equation has to hold for each $U(s)$ and for each s; consequently, the linear equation set

$$\sum k_j p_{ij} = 0, \qquad i = 1, 2 \cdots, n$$

exhibits only the trivial solution if and only if the system is completely controllable.

In conclusion, the complete controllability (reachability) condition can be presented in the form

$$\operatorname{rank}(p_{ij}) = n.$$

This condition is obviously independent of the choice of the basis in the state space and could be extended to the multiinput case.

From a computational point of view, the suggested criterion seems to be comparable[1] to the usual one:

$$\operatorname{rank}(B, AB, \cdots, A^{n-1}B) = n.$$

Its proof, however, is very simple and it is based on an intuitive notion of controllability (that is, on the fact that the input can modify each state vector component, independently from the other ones).

The suggested criterion, therefore, seems to be useful from a didactic point of view.

[1]For some canonical forms (e.g., with a companion A matrix) the computation of the matrix (p_{ij}) is not so cumbersome as that of matrix $(B, AB, \cdots A^{n-1}B)$; in other words, each p_{ij} is a function of the elements of A and B "simpler" than those corresponding to the elements of the usually considered controllability matrix.

Manuscript received May 30, 1975.
The authors are with the Istituto di Elettrotecnica e di Elettronica, Università di Padova, Padua, Italy.

Observing the States of Systems with Unmeasurable Disturbances

SHIH-HO WANG, E. J. DAVISON, AND PETER DORATO

Abstract—**In order to estimate the state of a linear time-invariant multivariable system by using a Luenberger observer, it is generally assumed that all system inputs are measurable. This correspondence presents a simple and useful procedure for the construction of minimal-order state estimators for systems with unmeasurable disturbances.**

In order to estimate the state of a system by using a Luenberger observer [1], it is generally assumed that all system inputs are measurable. For systems with some unmeasurable inputs, Hostetter and Meditch [2] have considered this problem by assuming that the unmeasurable disturbances satisfy a given constant coefficient differential equation. Then they are able to build a Luenberger observer for the system augmented by the differential equation generating the unmeasurable disturbances. A deeper investigation of this problem has been made by Basile and Marro [3] and Guidorzi and Marro [4]. Without any knowledge of the disturbances, they have derived some necessary and sufficient conditions for the existence of observers for systems with

Manuscript received June 5, 1975. This work was supported in part by the National Research Council of the United States and the National Aeronautics and Space Administration, Langly Research Center, Hampton, Va. under a Postdoctoral Resident Research Associateship and under Contract NAS1-11707-52, and by the National Research Council of Canada under Grant A4396.
S.-H. Wang and P. Dorato are with the Department of Electrical Engineering, University of Colorado, Colorado Springs, Colo. 80907.
E. J. Davison is with the Department of Electrical Engineering, University of Toronto, Toronto, Ont., Canada.