

where P_A is vapor pressure of pure water A and P_B is vapor pressure of pure B . Then the vapor composition is

$$y_A = \frac{P_A}{P} \quad y_B = \frac{P_B}{P} \quad (11.3-14)$$

As long as the two liquid phases are present, the mixture will boil at the same temperature, giving a vapor of constant composition y_A . The temperature is found by using the vapor-pressure curves of pure A and pure B .

Note that by steam distillation, as long as liquid water is present, the high-boiling component B vaporizes at a temperature well below its normal boiling point without using a vacuum. The vapors of water (A) and high-boiling component (B) are usually condensed in a condenser and the resulting two immiscible liquid phases separated. This method has the disadvantage that large amounts of heat must be used to simultaneously evaporate the water with the high-boiling compound.

The ratio moles of B distilled to moles of A distilled is

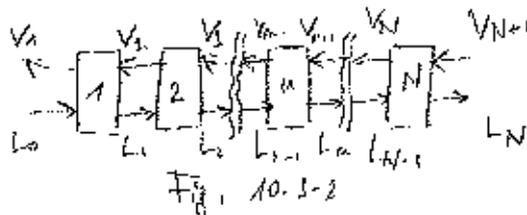
$$\frac{n_B}{n_A} = \frac{P_B}{P_A} \quad (11.3-15)$$

Steam distillation is sometimes used in the food industry for the removal of volatile taints and flavors from edible fats and oils. In many cases vacuum distillation is used instead of steam distillation to purify high-boiling materials. The total pressure is quite low so that the vapor pressure of the system reaches the total pressure at relatively low temperatures.

Van Winkle (VI) derives equations for steam distillation where an appreciable amount of a nonvolatile component is present with the high-boiling component. This involves a three-component system. He also considers other cases for binary batch, continuous, and multicomponent batch steam distillation.

Gas. B
$$f_{i,i+1} = \frac{L_i x_{i+1}}{V_{i,i+1}} + \frac{V_i y_{i+1} - L_{i+1} y_i}{V_{i,i+1}}$$

11.4 DISTILLATION WITH REFLUX AND McCABE-THIELE METHOD



11.4A Introduction to Distillation with Reflux

Rectification (fractionation) or stage distillation with reflux, from a simplified point of view, can be considered to be a process in which a series of flash-vaporization stages are arranged in a series in such a manner that the vapor and liquid products from each stage flow countercurrently to each other. The liquid in a stage is conducted or flows to the stage below and the vapor from a stage flows upward to the stage above. Hence, in each stage a vapor stream V and a liquid stream L enter, are mixed and equilibrated, and a vapor and a liquid stream leave in equilibrium. This process flow diagram was shown in Fig. 10.3-1 for a single stage and an example given in Example 11.2-1 for a benzene-toluene mixture.

For the countercurrent contact with multiple stages in Fig. 10.3-2, the material-balance or operating-line equation (10.3-13) was derived which relates the concentrations of the vapor and liquid streams passing each other in each stage. In a distillation column the stages (referred to as sieve plates or trays) in a distillation tower are arranged vertically, as shown schematically in Fig. 11.4-1.

The feed enters the column in Fig. 11.4-1 somewhere in the middle of the column. If the feed is liquid, it flows down to a sieve tray or stage. Vapor enters the tray and bubbles

through the liquid on this tray as the entering liquid flows across. The vapor and liquid leaving the tray are essentially in equilibrium. The vapor continues up to the next tray or stage, where it is again contacted with a downflowing liquid. In this case the concentration of the more volatile component (the lower-boiling component *A*) is being increased in the vapor from each stage going upward and decreased in the liquid from each stage going downward. The final vapor product coming overhead is condensed in a condenser and a portion of the liquid product (distillate) is removed, which contains a high concentration of *A*. The remaining liquid from the condenser is returned (refluxed) as a liquid to the top tray.

The liquid leaving the bottom tray enters a reboiler, where it is partially vaporized, and the remaining liquid, which is lean in *A* or rich in *B*, is withdrawn as liquid product. The vapor from the reboiler is sent back to the bottom stage or tray. Only three trays are shown in the tower of Fig. 11.4-1. In most cases the number of trays is much greater. In the sieve tray the vapor enters through an opening and bubbles up through the liquid to give intimate contact of the liquid and vapor on the tray. In a theoretical tray the vapor and liquid leaving are in equilibrium. The reboiler can be considered as a theoretical stage or tray.

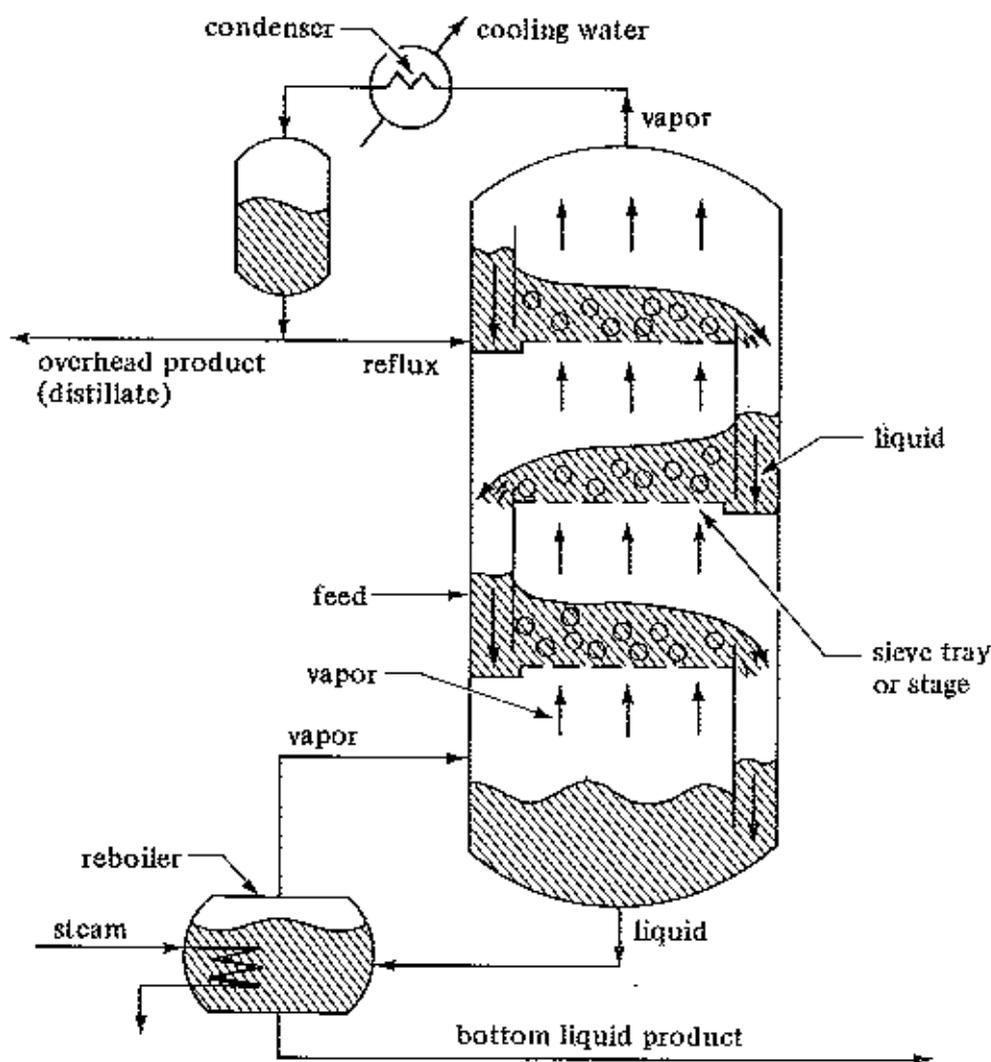


FIGURE 11.4-1. Process flow of a fractionating tower containing sieve trays.

11.4B McCabe-Thiele Method of Calculation for Number of Theoretical Stages

1. *Introduction and assumptions.* A mathematical-graphical method for determining the number of theoretical trays or stages needed for a given separation of a binary mixture of A and B has been developed by McCabe and Thiele. The method uses material balances around certain parts of the tower, which give operating lines somewhat similar to Eq. (10.3-13), and the xy equilibrium curve for the system.

The main assumption made in the McCabe-Thiele method is that there must be equimolar overflow through the tower between the feed inlet and the top tray and the feed inlet and bottom tray. This can be shown in Fig. 11.4-2, where liquid and vapor streams enter a tray, are equilibrated, and leave. A total material balance gives

$$V_{n+1} + L_{n-1} = V_n + L_n \quad (11.4-1)$$

A component balance on A gives

$$V_{n+1}y_{n+1} + L_{n-1}x_{n-1} = V_n y_n + L_n x_n \quad (11.4-2)$$

where V_{n+1} is mol/h of vapor from tray $n + 1$, L_n is mol/h liquid from tray n , y_{n+1} is mole fraction of A in V_{n+1} , and so on. The compositions y_n and x_n are in equilibrium and the temperature of the tray n is T_n . If T_n is taken as a datum, it can be shown by a heat balance that the sensible heat differences in the four streams are quite small if heats of solution are negligible. Hence, only the latent heats in stream V_{n+1} and V_n are important. Since molar latent heats for chemically similar compounds are almost the same, $V_{n+1} = V_n$ and $L_n = L_{n-1}$. Therefore, we have constant molar overflow in the tower.

2. *Equations for enriching section.* In Fig. 11.4-3 a continuous distillation column is shown with feed being introduced to the column at an intermediate point and an overhead distillate product and a bottoms product being withdrawn. The upper part of the tower above the feed entrance is called the *enriching section*, since the entering feed of binary components A and B is enriched in this section, so that the distillate is richer in A than the feed. The tower is at steady state.

An overall material balance around the entire column in Fig. 11.4-3 states that the entering feed of F mol/h must equal the distillate D in mol/h plus the bottoms W in mol/h.

$$F = D + W \quad (11.4-3)$$

A total material balance on component A gives

$$F x_F = D x_D + W x_W \quad (11.4-4)$$

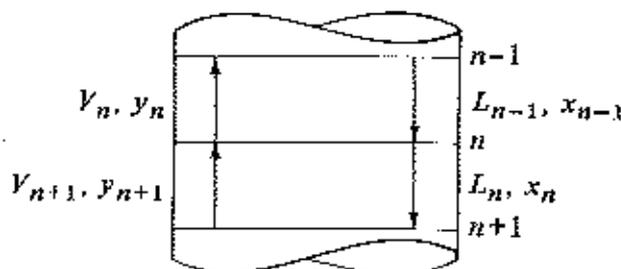


FIGURE 11.4-2. Vapor and liquid flows entering and leaving a tray.

continuous distillation column

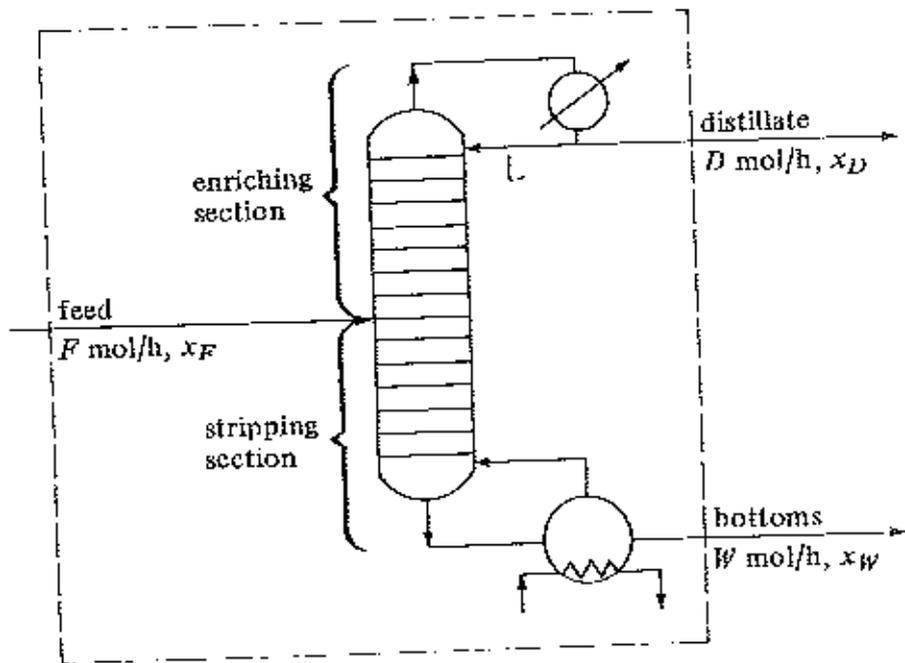


FIGURE 11.4-3. Distillation column showing material-balance sections for McCabe-Thiele method.

In Fig. 11.4-4a the distillation tower section above the feed, the enriching section, is shown schematically. The vapor from the top tray having a composition y_1 passes to the condenser, where it is condensed so that the resulting liquid is at the boiling point. The reflux stream L mol/h and distillate D mol/h have the same composition, so $y_1 = x_D$. Since equimolal overflow is assumed, $L_1 = L_2 = L_n$ and $V_1 = V_2 = V_n = V_{n+1}$. Making a total material balance over the dashed-line section in Fig. 11.4-4a,

$$V_{n+1} = L_n + D \quad (11.4-5)$$

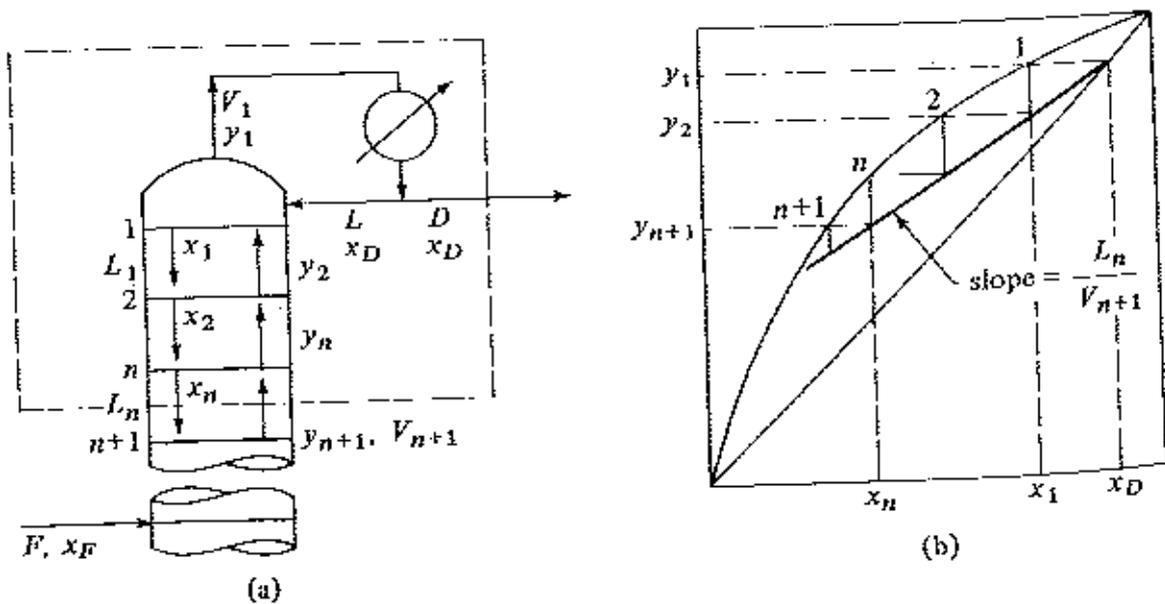


FIGURE 11.4-4. Material balance and operating line for enriching section: (a) schematic of tower, (b) operating and equilibrium lines.

Making a balance on component A,

$$V_{n+1}y_{n+1} - L_n x_n + Dx_D \quad (11.4-6)$$

Solving for y_{n+1} , the enriching-section operating line is

$$y_{n+1} = \frac{L_n}{V_{n+1}} x_n + \frac{Dx_D}{V_{n+1}} \quad (11.4-7)$$

Since $V_{n+1} = L_n + D$, $L_n/V_{n+1} = R/(R+1)$ and Eq. (11.4-7) becomes

$$y_{n+1} = \frac{R}{R+1} x_n + \frac{x_D}{R+1} \quad (11.4-8)$$

where $R = L_n/D = \text{reflux ratio} = \text{constant}$. Equation (11.4-7) is a straight line on a plot of vapor composition versus liquid composition. It relates the compositions of two streams passing each other and is plotted in Fig. 11.4-4b. The slope is L_n/V_{n+1} or $R/(R+1)$, as given in Eq. (11.4-8). It intersects the $y = x$ line (45° diagonal line) at $x = x_D$. The intercept of the operating line at $x = 0$ is $y = x_D/(R+1)$.

The theoretical stages are determined by starting at x_D and stepping off the first plate to x_1 . Then y_2 is the composition of the vapor passing the liquid x_1 . In a similar manner, the other theoretical trays are stepped off down the tower in the enriching section to the feed tray.

3. *Equations for stripping section.* Making a total material balance over the dashed-line section in Fig. 11.4-5a for the stripping section of the tower below the feed entrance,

$$V_{m+1} = L_m - W \quad (11.4-9)$$

Making a balance on component A,

$$V_{m+1}y_{m+1} = L_mx_m - Wx_W \quad (11.4-10)$$

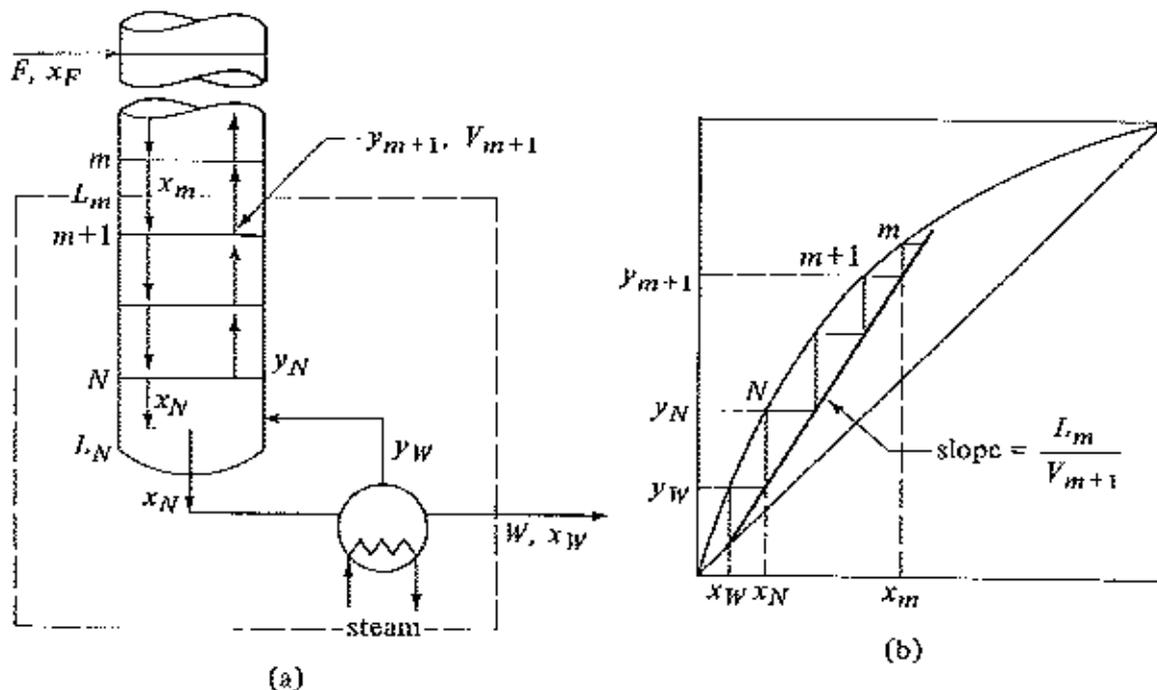


FIGURE 11.4-5. Material balance and operating line for stripping section: (a) schematic of tower, (b) operating and equilibrium lines.

Solving for y_{m+1} , the stripping-section operating line is

$$y_{m+1} = \frac{L_m}{V_{m+1}} x_m - \frac{Wx_W}{V_{m+1}} \quad (11.4-11)$$

Again, since equimolar flow is assumed, $L_m - L_N = \text{constant}$ and $V_{m+1} = V_N = \text{constant}$. Equation (11.4-11) is a straight line when plotted as y versus x in Fig. 11.4-5b, with a slope of L_m/V_{m+1} . It intersects the $y = x$ line at $x = x_W$. The intercept at $x = 0$ is $y = -Wx_W/V_{m+1}$.

Again the theoretical stages for the stripping section are determined by starting at x_W , going up to y_W , and then across to the operating line, etc.

4. *Effect of feed conditions.* The condition of the feed stream F entering the tower determines the relation between the vapor V_m in the stripping section and V_n in the enriching section and also between L_m and L_n . If the feed is part liquid and part vapor, the vapor will add to V_m to give V_n .

For convenience, we represent the condition of the feed by the quantity q , which is defined as

$$q = \frac{\text{heat needed to vaporize 1 mol of feed at entering conditions}}{\text{molar latent heat of vaporization of feed}} \quad (11.4-12)$$

If the feed enters at its boiling point, the numerator of Eq. (11.4-12), is the same as the denominator and $q = 1.0$. Equation (11.4-12) can also be written in terms of enthalpies.

$$q = \frac{H_V - H_F}{H_V - H_L} \quad (11.4-13)$$

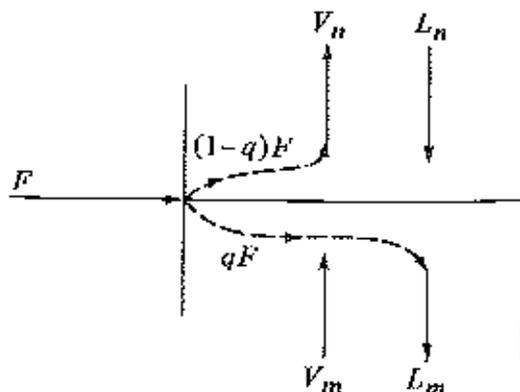
where H_V is the enthalpy of the feed at the dew point, H_L the enthalpy of the feed at the boiling point (bubble point), and H_F the enthalpy of the feed at its entrance conditions. If the feed enters as vapor at the dew point, $q = 0$. For cold liquid feed $q > 1.0$, for superheated vapor $q < 0$, and for the feed being part liquid and part vapor, q is the fraction of feed that is liquid.

We can look at q also as the number of moles of saturated liquid produced on the feed plate by each mole of feed added to the tower. In Fig. 11.4-6 a diagram shows the relationship between flows above and below the feed entrance. From the definition of q , the following equations hold:

$$L_m = L_n + qF \quad (11.4-14)$$

$$V_n = V_m + (1 - q)F \quad (11.4-15)$$

FIGURE 11.4-6. Relationship between flows above and below the feed entrance.



The point of intersection of the enriching and the stripping operating-line equations on an xy plot can be derived as follows. Rewriting Eqs. (11.4-6) and (11.4-10) as follows without the tray subscripts:

$$V_n y = L_n x + Dx_D \quad (11.4-16)$$

$$V_m y = L_m x - Wx_W \quad (11.4-17)$$

where the y and x values are the point of intersection of the two operating lines. Subtracting Eq. (11.4-16) from (11.4-17),

$$(V_m - V_n)y = (L_m - L_n)x - (Dx_D + Wx_W) \quad (11.4-18)$$

Substituting Eqs. (11.4-4), (11.4-14), and (11.4-15) into Eq. (11.4-18) and rearranging,

$$F x_F = D x_D + W x_W \quad \boxed{y = \frac{q}{q-1} x - \frac{x_F}{q-1}} \quad (11.4-19)$$

This equation is the q -line equation and is the locus of the intersection of the two operating lines. Setting $y = x$ in Eq. (11.4-19), the intersection of the q -line equation with the 45° line is $y = x = x_F$, where x_F is the overall composition of the feed.

In Fig. 11.4-7 the q line is plotted for various feed conditions given below the figure. The slope of the q line is $q/(q-1)$. For example, for the liquid below the boiling point, $q > 1$, and the slope is > 1.0 , as shown. The enriching and operating lines are plotted for the case of a feed of part liquid and part vapor and the two lines intersect on the q line. A convenient way to locate the stripping operating line is to first plot the enriching operating line and the q line. Then draw the stripping line between the intersection of the q line and enriching operating line and the point $y = x = x_W$.

5. Location of the feed tray in a tower and number of trays. To determine the number of theoretical trays needed in a tower, the stripping and operating lines are drawn to intersect on the q line as shown in Fig. 11.4-8. Starting at the top at x_D , the trays are stepped off. For trays 2 and 3, the steps can go to the enriching operating line, as shown

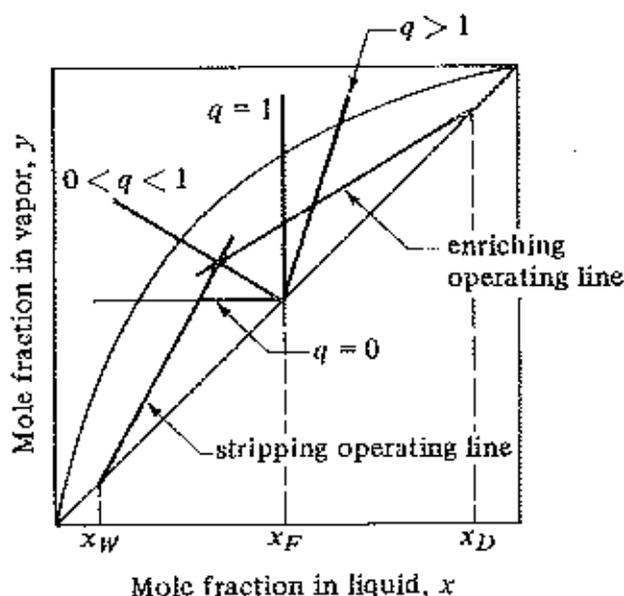


FIGURE 11.4-7. Location of the q line for various feed conditions: liquid below boiling point ($q > 1$), liquid at boiling point ($q = 1$), liquid + vapor ($0 < q < 1$), saturated vapor ($q = 0$).